# DEPARTMENT
# OF
# ELECTRONICS & COMMUNICATION ENGINEERING

# Communication Theory
# 18EC4DCCOT

## (Theory Notes)

## Autonomous Course

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| Module – 2 Contents |
|---|
| **Angle modulation:** Basic concepts, Relationship between FM and PM .Single tone FM, Spectral analysis of Sinusoidal FM, Types of FM: NBFM and WBFM, Transmission bandwidth of FM waves, Generation of FM: Indirect FM and Direct FM, Zero crossing detector |

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# UNIT-II : ANGLE MODULATION

## BASIC DEFINATIONS:

Amplitude modulation methods are also called linear modulation method. Another class of modulation is called angle modulation, in which either the phase or frequency of the carrier wave is varied according to the message signal, where the amplitude of the carrier wave is maintained constant.

The most important feature of angle modulation is that it can provide better discrimination against noise & interference than AM. This improvement is achieved at the expense of increased transmission bandwidth.

Expressing the modulated wave in general form.

$$s(t) = A_c \cos[\theta_i(t)] \quad \text{---} (1)$$

where $\theta_i(t)$ denote the angle of a modulated sinusoidal carrier, which is the function of the message. And $A_c$ is carrier amplitude.

In any event, a complete oscillation occurs whenever $\theta(t)$ changes by $2\pi$ radians. If $\theta(t)$ increases monotonically with time, the average frequency over an interval from $t$ to $t+\Delta t$ is given by

$$f_{\Delta t}(t) = \frac{\theta(t+\Delta t) - \theta(t)}{2\pi \cdot \Delta t}$$

The instantaneous frequency of the angle modulated wave $s(t)$ is given by

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$= \lim_{\Delta t \to 0} \left[ \frac{\theta(t+\Delta t) - \theta(t)}{2\pi \Delta t} \right]$$

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt} \quad \text{---} (2)$$

According to eqn (1) we may interpret the angle modulated wave $s(t)$ as a rotating phasor of length $A_c$ and angle $\theta_i(t)$. The angular velocity of such a phasor is $\frac{d\theta_i(t)}{dt}$ in accordance with eqn (2).

In case of an unmodulated carrier, the angle $\theta_i(t)$ is

$$\theta_i(t) = 2\pi f_c t + \phi_c \quad \text{---} (3)$$

and the corresponding phasor rotates with a constant angular velocity equal to $2\pi f_c$. The constant $\phi_c$ is the value of $\theta_i(t)$ at

There are an infinite number of ways in which the arg $\theta_i(t)$ may be varied in some manner with the baseband signal. We shall consider only two commonly used methods phase modulation and frequency modulation.

**Phase modulation [PM]**: Is that form of angle modulation in which the angular argument $\theta(t)$ is varied linearly with the message signal $m(t)$ as shown by

$$\theta(t) = 2\pi f_c t + k_p m(t) \quad \text{---} (4)$$

Where, $2\pi f_c t$ - represents the angular argument of the unmodulated carrier.

$k_p$ - represents the phase sensitivity of the modulator expressed in rad/V

The phase modulated wave $s(t)$ is thus described in the time domain by [Substituting eqn (4) in (1)]

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)] \quad \text{---} (5)$$

**Frequency modulation [FM]**: Is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal $m(t)$, as shown by

$$f_i(t) = f_c + k_f m(t) \quad \text{---} (6)$$

Where $f_c$ represents the frequency of the unmodulated carrier

$k_f$ represents the frequency sensitivity of the modulator expressed in Hz/V.

From eqn (2), $2\pi f_i(t) = \frac{d\theta}{dt}$

$$\theta(t) = \int_0^t 2\pi f_i(t)\,dt$$

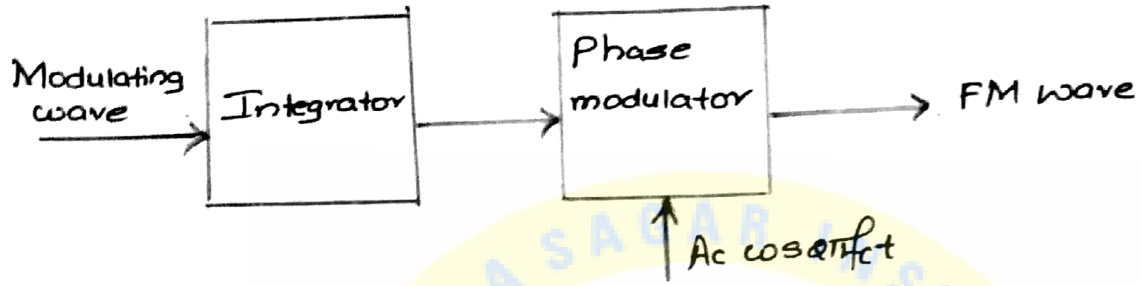
$$= 2\pi \int_0^t [f_c + k_f m(t)]\,dt$$

$$\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt \quad \text{---} (7)$$

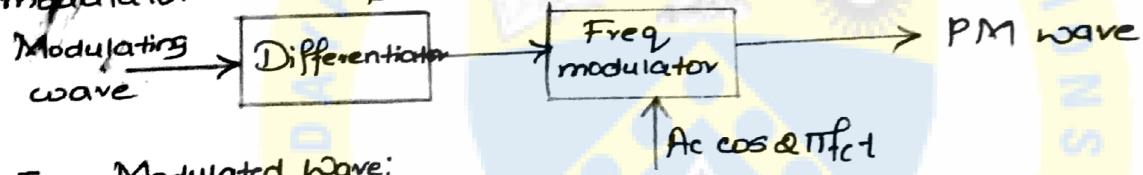
In the time domain, the frequency modulated wave can be written as

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

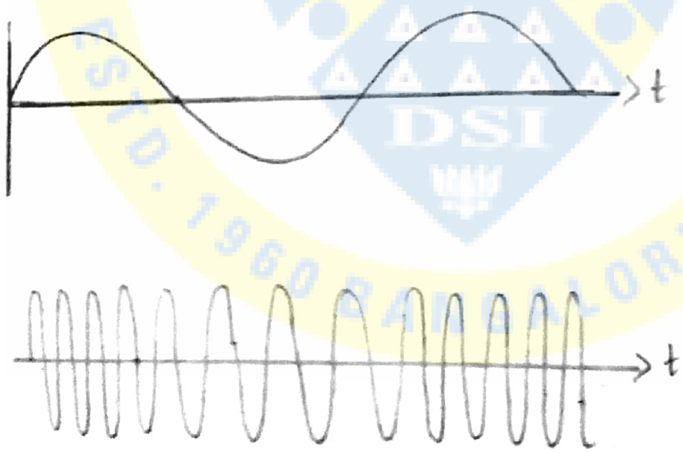
Comparing eqn 5 & 8 reveals that FM wave may be regarded as a PM wave in which the modulating wave is $\int^t m(t)\,dt$ in Place of $m(t)$. This means that FM wave can be generated by first intergrating $m(t)$ and then using the result as the input to a phase modulator as shown in fig below.




Conversly a PM wave can be generating by first differentiating $m(t)$ and then using the result as the input to a frequency modulator as in fig below

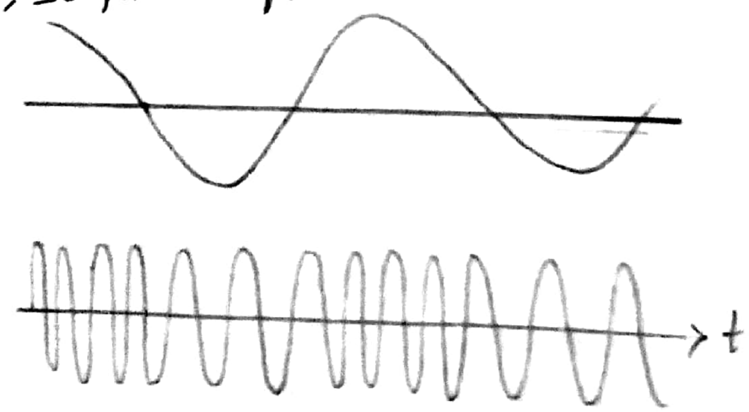



Freq Modulated Wave:




Phase modulated wave:

$\frac{dm(t)}{dt}$ lφ, 90° phase shift



# Frequency Modulation (FM)

The FM wave s(t) defined by eqn (8) is a non linear function of the modulating wave m(t). Hence FM is a non-linear modulation process.

## Single Tone Frequency Modulation:

Consider a sinusoidal modulating wave is defined by

$$m(t) = A_m \cos(2\pi f_m t) \quad \text{--- (9)}$$

The instantaneous frequency of the resulting FM wave is

$$f_i(t) = f_c + k_f\, m(t)$$

$$= f_c + k_f A_m \cos(2\pi f_m t) \quad \text{--- (10)}$$

$$f_i(t) = f_c + \Delta f \cos(2\pi f_m t)$$

Where $\Delta f = k_f A_m$, which is called the frequency deviation, representing the maximum departure of the instantaneous frequency of the FM wave from the carrier frequency $f_c$. $\Delta f$ is proportional to the amplitude of the modulating wave & is independent of the modulation frequency.

W.K.T $\theta_i(t) = 2\pi \int_0^t f_i(t)\, dt$

$$= 2\pi \int_0^t \left(f_c + \Delta f \cos 2\pi f_m t\right) dt$$

$$= 2\pi \int_0^t f_c\, dt + 2\pi \Delta f \cdot \int_0^t \cos 2\pi f_m t \cdot dt$$

$$= 2\pi f_c t + 2\pi \Delta f \cdot \frac{\sin 2\pi f_m t}{2\pi f_m}$$

$$\theta_i(t) = 2\pi f_c + \frac{\Delta f}{f_m} \cdot \sin 2\pi f_m t$$

The ratio of the frequency deviation $\Delta f$ to the modulation frequency $f_m$ is called the modulation index of the FM wave.

i.e. $\beta = \dfrac{\Delta f}{f_m} = \dfrac{k_f \cdot A_m}{f_m}$

$$\therefore \theta_i(t) = 2\pi f_c t + \beta \sin 2\pi f_m t$$

The FM wave is given by

$$s(t) = A_c \cos[\theta_i(t)]$$

$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right)$$

$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t\,dt\right)$$

$$= A_c \cos\left(2\pi f_c t + 2\pi k_f \cdot A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right)$$

$$s(t) = A_c \cos\left[2\pi f_c t + \beta \sin 2\pi f_m t\right]$$

Depending on the value of the modulation index $\beta$, there are 2 cases:

* Narrow band FM for which $\beta$ is small compared to one radian.
* Wideband FM for which $\beta$ is large compared to one radian.

3 NARROW BAND FM.

For small values of the modulation index $\beta$ compared to one radian, the FM wave assumes a narrow band form consisting essentially of a carrier an upper side-frequency component and a lower side frequency component.

FM signal is given as,

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

W.K.T $\cos(A+B) = \cos A \cos B - \sin A \sin B$.

Expanding this relation, we get

$$s(t) = A_c \cos 2\pi f_c t \cdot \cos[\beta \sin 2\pi f_m t] - A_c \sin 2\pi f_m t \cdot \sin[\beta \sin(2\pi f_m t)] \quad \text{---(11)}$$

In case of narrow band, $\beta$ is small. ∴ We can approximate

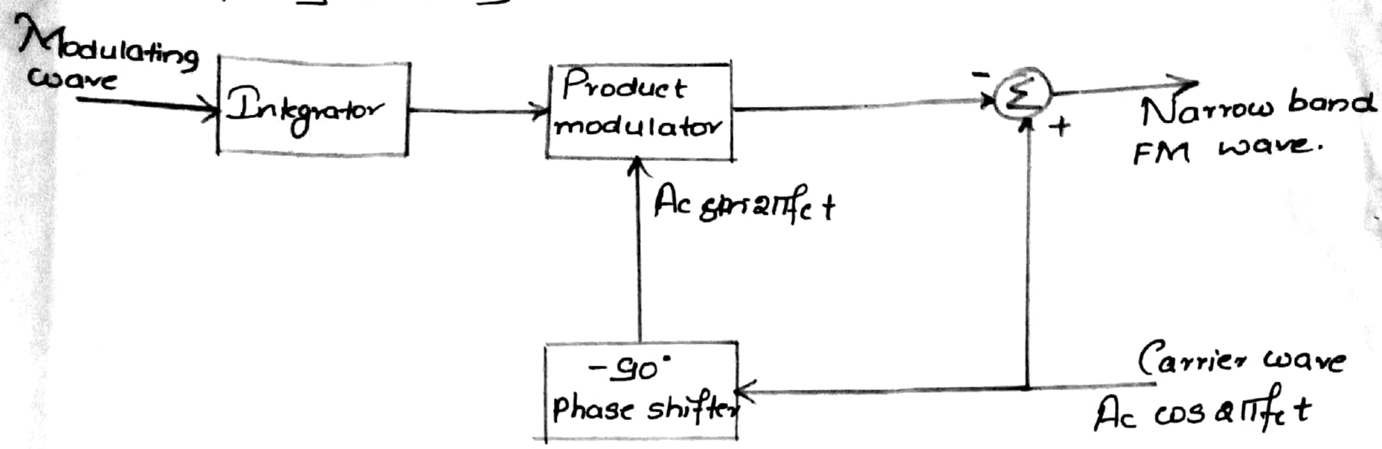
$$\cos[\beta \sin 2\pi f_m t] \cong 1 \text{ \& } \sin[\beta \sin 2\pi f_m t] \cong \beta \sin 2\pi f_m t.$$

∴ Eqn (11) becomes,

$$s(t) = A_c \cos 2\pi f_c t - A_c \sin 2\pi f_c t \, \beta \sin 2\pi f_m t$$

$$= A_c \cos 2\pi f_c t - A_c \cdot \beta \cdot \sin 2\pi f_c t \cdot \sin 2\pi f_m t. \quad \text{---(12)}$$

From this representation we show the block diagram of a method for generating a narrow band FM signal




The modulator involves spliting the carrier wave $A_c \cos 2\pi f_c t$ into two paths. One path is direct; the other path contains a −90° phase shifting network and a product modulator, the combination of which generates a DSB-SC modulated signal. The difference between these two signals produces a narrow band FM signal.

The eqn (12) can be expressed as

$$s(t) = A_c \cos 2\pi f_c t + \beta \frac{A_c}{2} \cos 2\pi(f_c + f_m)t - \beta \frac{A_c}{2} \cos 2\pi(f_c - f_m)t \quad (13)$$

This expression (13) is similar to the AM signal which is given by

$$s(t) = A_c \cos 2\pi f_c t + m \frac{A_c}{2} \cos 2\pi(f_c + f_m)t + m \frac{A_c}{2} \cos 2\pi(f_c - f_m)t$$

where, $m$ = modulation index of the AM signal.

The only difference we observe in the above 2 equations is that the algebraic sign of the lower side frequency in the NBFM is reversed. Thus a NBFM signal requires the same bandwidth ($2f_m$) as the AM signal.

We may represent the NBFM signal with a phasor diagram as shown in fig (a). Carrier phasor is used as reference. We see that the resultant of the two side-frequency phasors is always at right angles to the carrier phasor. i.e, the carrier is continuously phase shifted as a function of the modulating signal.

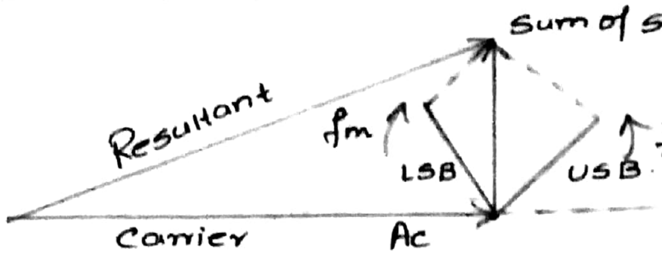



Fig (a)

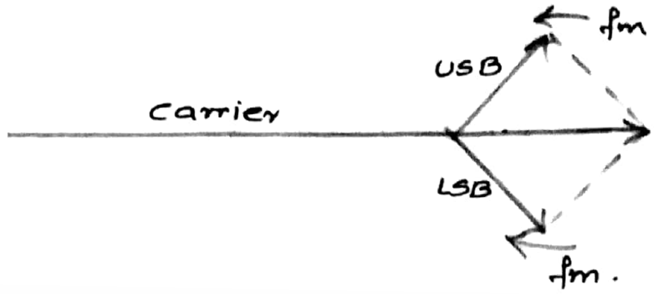



Fig (b)

Here we observe that the resultant of the two side bands is always along the direction of carrier changing the magnitude of the carrier continuously as a function of the modulating signal.

## 4 WIDE BAND FM

Let us determine the spectrum of the single tone FM signal of eqn (11) for an arbitrary value of the modulation index $\beta$.

From eqn (11), we can see that the in-phase and quadrature components of the FM wave $s(t)$ for the case of sinusoidal modulation are as follows:

$$s_I(t) = A_c \cos[\beta \sin(2\pi f_m t)]$$

$$s_Q(t) = A_c \sin[\beta \sin(2\pi f_m t)]$$

Hence the complex envelope of the FM wave,

$$\tilde{s}(t) = s_I(t) + j s_Q(t)$$

$$= A_c \cos[\beta \sin 2\pi f_m t] + j A_c \sin(\beta \sin 2\pi f_m t)$$

$$\tilde{s}(t) = A_c \cdot \exp[j\beta \sin 2\pi f_m t] \quad \text{--- (14)}$$

Expressing the FM wave $s(t)$ in terms of the complex envelope $\tilde{s}(t)$ by,

$$s(t) = \mathrm{Re}[\tilde{s}(t) \exp(j2\pi f_c t)] \quad \text{--- (15)}$$

From equation (14), we see that $\tilde{s}(t)$ is a periodic function of time with a fundamental frequency equal to the modulation frequency $f_m$. $\therefore$ $\tilde{s}(t)$ can be expanded in the form of a complex Fourier series as

$$\tilde{S}(t) = \sum_{n=-\infty}^{\infty} C_n \exp(j2\pi n f_m t) \quad \text{---(16)}$$

Where the complex Fourier coefficient $C_n$ equals

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} \tilde{s}(t)\exp(-j2\pi n f_m t)\,dt \quad \text{---(17)}$$

Substituting eqn (14) in (17), we get

$$C_n = f_m \cdot A_c \int_{-1/2f_m}^{1/2f_m} \exp(j\beta\sin 2\pi f_m t] \cdot \exp(-j2\pi n f_m t)\,dt$$

$$= f_m \cdot A_c \cdot \int_{-1/2f_m}^{1/2f_m} \exp[j\beta\sin 2\pi f_m t - j2\pi n f_m t]\,dt$$

Let $x = 2\pi f_m t$, $dx = 2\pi f_m \cdot dt \Rightarrow dt = \frac{dx}{2\pi f_m}$

When $t = \frac{-1}{2f_m} \Rightarrow x = \cancel{2\pi f_m} \times \frac{-1}{\cancel{2f_m}} = -\pi$, III$^{ly}$ $t = \frac{1}{2f_m} \Rightarrow x = \pi$

$$\therefore C_n = \cancel{f_m} \cdot A_c \cdot \int_{-\pi}^{\pi} \exp[j\beta\sin x - jnx] \cdot \frac{dx}{2\pi \cancel{f_m}}$$

$$C_n = \frac{A_c}{2\pi} \int_{-\pi}^{\pi} \exp j[\beta\sin x - nx]\,dx. \quad \text{---(18)}$$

The integral on the right hand side of above equation is $n^{th}$ order Bessel function of the first kind & argument $\beta$, denoted by $J_n(\beta)$.

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[j(\beta\sin x - nx)]\,dx. \quad \text{---(19)}$$

Note: Bessel function: The mathematical analysis of the FM wave cannot be solved by equations with algebra & trignometric identities. But some bessel function identities are available that will yield solutions to equations & allow us to determine frequency components FM wave.

Substituting equation (19) in (18)

$$C_n = A_c \cdot J_n(\beta) \quad \text{---(20)}$$

Substituting equation (20) in (16)

$$\tilde{s}(t) = A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\exp(j2\pi n f_m t) \quad \text{---(21)}$$

Substituting eqn (21) in (15),

$$s(t) = A_c \cdot Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\exp(j2\pi n f_m t)\cdot \exp(j2\pi f_c t)\right]$$

$$= A_c \cdot Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\exp(j2\pi(f_c + nf_m)t)\right] \quad \text{--- (22)}$$

Interchanging the order of summation & evaluating the real part of the right side of eqn (22), we get

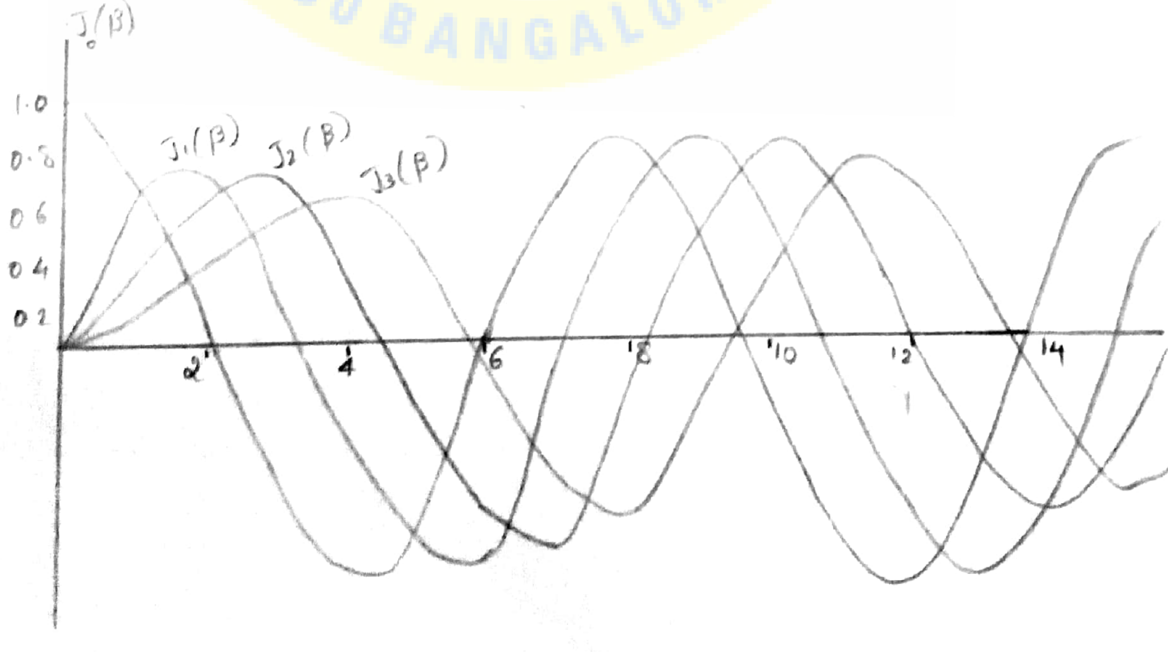
$$s(t) = A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t] \quad \text{--- (23)}$$

This is the desired form for the fourier series representation of the single tone FM. The discrete spectrum of $s(t)$ is obtained by taking the fourier transform of both sides of eqn (23)

$$S(f) = \frac{A_c}{2}\sum_{n=-\infty}^{\infty} J_n(\beta)\left[\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)\right] \quad \text{--- (24)}$$

As $n$ ranges from $-\infty$ to $\infty$, we can see that there are infinite number of side frequencies with the result, the theoretical B.W is $\infty$ in extent for FM wave.

Fig below, plotted the Bessel function $J_n(\beta)$ versus the modulation index $\beta$ for $n = 0, 1, 2, 3, 4$. These plots shows that for fixed $n$, $J_n(\beta)$ alternates between +ve & -ve values for increasing $\beta$ & $|J_n(\beta)|$ approaches zero as $\beta$ approaches infinity.



Some of useful properties of $J_n(\beta)$ are

* For fixed $\beta$,

$$J_n(\beta) = J_{-n}(\beta) \quad , n \text{ even}$$

$$J_n(\beta) = -J_{-n}(\beta), \; n \text{ odd}$$

Or in general, $J_n(\beta) = (-1)^n J_{-n}(\beta)$

* for $\beta \leq 0.3$

$$J_0(\beta) = 1 \;\; \& \;\; J_1(\beta) = \beta/2$$

$$J_n(\beta) = 0, \quad |n| > 1$$

* $$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1.$$

* Constant Average power:

The envelope of an FM wave is constant, so that the average power of such a wave dissipated in a $1\Omega$ resistor is also constant.

Average power dissipated by $s(t)$ in a $1\Omega$ resistor is given by

$$s^2(t) = \sum_{n=-\infty}^{\infty} \left[\frac{A_c}{2} J_n(\beta)\right]^2$$

$$= \frac{A_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

$$= \frac{A_c^2}{2}(1) = \frac{A_c^2}{2}.$$

From equation 23, the average power of a single tone FM wave $s(t)$ may be expressed in the form of a corresponding series as:

$$P = \left(E_c/\sqrt{2}\right)^2$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{A_c J_n(\beta)}{\sqrt{2}}\right]^2$$

$$= \frac{A_c^2}{2} \cdot \sum_{n=-\infty}^{\infty} J_n^2(\beta)$$

$$P = A_c^2/2.$$

Some of the important features of FM wave observed from equation (24) are

* The spectrum of an FM signal contains a carrier component & an infinite set of side frequencies [side bands] located symmetrically on either side of the carrier at frequency separations of $f_m$, $2f_m$, $3f_m$ .... unlike AM, where there are only 2 sidebands.
* For the special case of $\beta$, small compared with unity, only the Bessel coefficients $J_0(\beta)$ & $J_1(\beta)$ have significant values, so that the FM signal is composed of a carrier and a single pair of side frequencies at $f_c \pm f_m$. This situation corresponds to the special case of NBFM.
* In FM, unlike AM, the amplitude of the carrier component does not remain constant but varies with $\beta$ according to $J_0(\beta)$. i.e, the amplitude of the carrier component of an FM signal is dependent on the modulation index $\beta$.
* $P = \frac{1}{2} A_c^2$. When the carrier is modulated to generate the FM signal, the power in the side frequencies may appear only at the expense of the power originally in the carrier, there by making the amplitude of the carrier component dependent on $\beta$. [The values of $J_0(\beta)$, $J_1(\beta)$, $J_2(\beta)$ ... can be checked by looking into the Bessel functions table].

Power distribution in FM:

The amplitudes of the different frequency components are as follows:

| Frequency | Amplitude |
|---|---|
| $f_c$ | $A_c J_0(\beta)$ |
| 1st side freq, $f_c \pm f_m$ | $A_c J_1(\beta)$ |
| $f_c \pm 2f_m$ | $A_c J_2(\beta)$ |
| ⋮ | |
| nth freq, $f_c \pm nf_m$ | $A_c J_n(\beta)$ |

For a fixed load 'R', the total power $P_T$ is given by $P_T = \sum_{n=0} \frac{A_c^2}{2R} J_n^2(\beta)$

$$P_T = \frac{\left(\frac{A_c \cdot J_0(\beta)}{\sqrt{2}}\right)^2}{R} + \frac{2\left(\frac{A_c J_1(\beta)}{\sqrt{2}}\right)^2}{R} + 2\left[\frac{A_c J_2(\beta)}{\sqrt{2}}\right]^2 \Big/ R + \cdots$$

$$= \frac{A_c^2}{2R} J_0^2(\beta) + \frac{2A_c^2}{2R} J_1^2(\beta) + 2\frac{A_c^2}{2R} J_2^2(\beta) + \cdots$$

W.K.T carrier power $P_c$ is given by $P_c = \frac{A_c^2}{2R}$

$$\therefore P_T = P_c J_0^2(\beta) + 2P_c J_1^2(\beta) + 2P_c J_2^2(\beta) + \cdots$$

$$P_T = P_c\left[J_0^2(\beta) + 2\left(J_1^2(\beta) + J_2^2(\beta) + \cdots J_n^2(\beta)\right)\right]$$

But the property of Bessel function is that the sum $[J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + \cdots] = 1$. This shows that the total average power is equal to the unmodulated carrier power.

## 5. TRANSMISSION BANDWIDTH OF FM WAVES

In theory, an FM signal contains an infinite number of side frequencies so that the bandwidth required to transmit such a signal similarly infinite in extent. In practice however we find that the FM signal is effectively limited to a finite number of significant side frequencies.

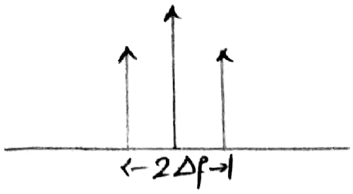
Consider the case of an FM signal generated by a single tone modulating wave of frequency fm. Here the side frequencies that are separated from the carrier frequency fc by an amount greater than the frequency deviation $\Delta f$, decrease rapidly toward zero. Therefore the bandwidth always exceeds the total frequency excursion: for large values of modulation index $\beta$, the bandwidth is slightly greater than the total frequency excursion $2\Delta f$. For small values of $\beta$, the spectrum of the FM signal is limited to the carrier frequency fc and one pair of side frequencies at $f_c \pm f_m$, so that the bandwidth approaches $2f_m$.

∴ To find the practical bandwidth a rule of thumb i.e., carson's rule is used. It states that the bandwidth of FM wave is twice the sum of the deviation and the highest modulating frequency.

B.W $B_T = 2\Delta f + 2 f_m$

$$B_T = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

$\beta = 1$,




W.K.T $\beta = \Delta f / f_m \Rightarrow \underline{\Delta f = \beta f_m}$.

**Deviation Ratio:** [Non sinusoidal or Arbitary modulation]

Let us consider the general case of an arbitary modulating signal $m(t)$ with its highest frequency component denoted by $W$. The bandwidth required to transmit an FM signal generated by this modulating signal is given by deviation ratio $D$. It is defined as the ratio of the frequency deviation $\Delta f$, which corresponds to the maximum possible amplitude of the modulation signal $m(t)$ to the highest modulation frequency $W$.

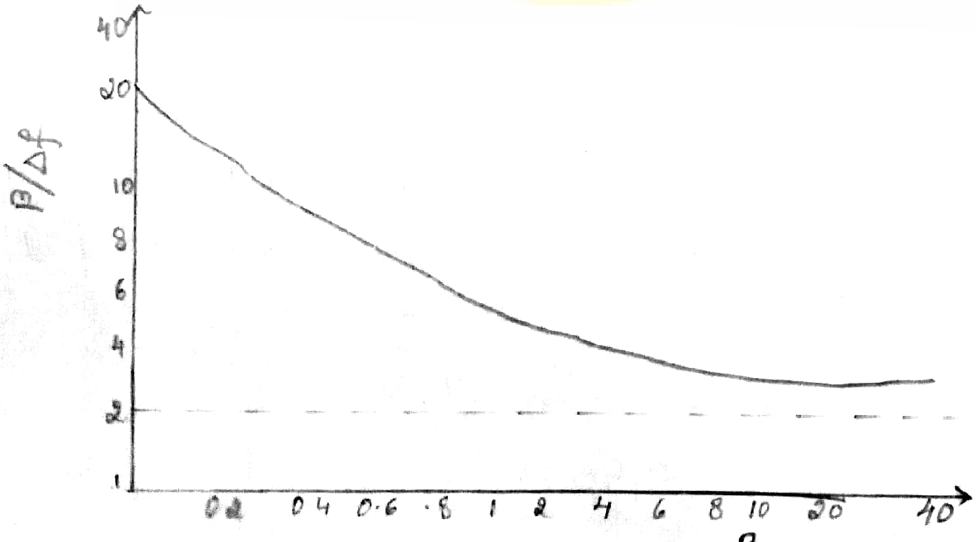
$$D = \Delta f / W \Rightarrow \Delta f = D \cdot W$$

By replacing $\beta$ by $D$ & $f_m$ with $W$. The bandwidth is given by

$$B_T = 2\Delta f\left(1 + 1/D\right)$$
$$= 2\beta f_m\left(1 + 1/D\right)$$
$$= 2DW\left(1 + 1/D\right)$$
$$= 2DW + 2W$$
$$\underline{B_T = 2W(1 + D)}$$

**Note:** The deviation ratio plays the same role for non sinusoidal modulation that the modulation index $\beta$ plays for the case of sinusoidal modulation.

Universal curve:



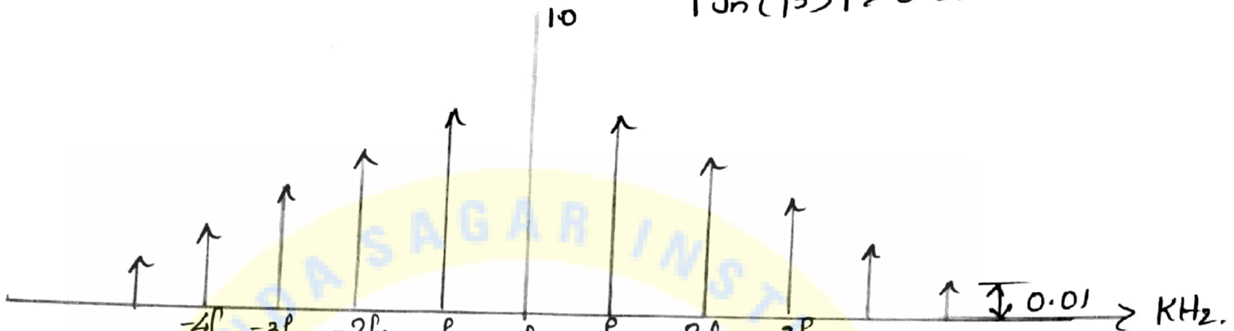

Universal curve for evaluating the 99% bandwidth of FM wave

Universal rule for evaluating Bandwidth:

Universal rule states that "the transmission bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side frequencies is greater than 1 percent of the carrier amplitude obtained when the modulation is removed. ie, transmission bandwidth is $= 2n_{max} f_m$.

where, $f_m$ = modulation frequency

$n_{max}$ = Largest value of the integer $n$ that satisfies $|J_n(\beta)| > 0.01$.




Prob

(1) An unmodulated carrier has amplitude 10V and frequency 100MHz. A sinusoidal waveform of frequency 1KHz, modulates this carrier such that the frequency deviation is 75KHz. The modulated waveform passes through zero & is increasing at time t = 0. Write the time-domain expression for the modulated carrier waveform.

Soln W.K.T $s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$

Given $A_c = 10V$ $\quad f_m = 1KHz$

$f_c = 100MHz$ $\quad \Delta f = 75KHz$

$$\beta = \frac{\Delta f}{f_m} = \frac{75K}{1KHz} = 75$$

$$\therefore s(t) = 10\cos[2\pi \times 100\times 10^6 t + 75 \sin 2\pi \times 10^3 t]$$

$$= 10\cos[2\pi \times 10^8 t + 75 \sin(2\pi \times 10^3) t]$$

(2) A FM wave is represented by the following equation, $s(t) = 10 \sin[5\times 10^8 t + 4\sin 1250t]$. Find i) Modulation index ii) Modulation frequency iii) Frequency deviation iv) Carrier frequency v) Power of the FM wave in a $5\Omega$ resistor.

Soln W.K.T $s(t) = A_c \sin[W_c t + \beta \sin W_m t]$.

Given $2\pi f_c = 5\times 10^8$ $\quad 2\pi f_m = 1250$ $\quad \beta = 4$

$f_c = 79.57MHz$ $\quad f_m = 199Hz$ $\quad A_c = 10$

$\Delta f = \beta \times f_m$

$= 196 Hz$

v) Power $= \frac{(A_c/\sqrt{2})^2}{R} = \frac{(10/\sqrt{2})^2}{5} = \underline{10W}$

3. A carrier is frequency modulated by a sinusoidal modulating signal of frequency 2KHz, resulting in a frequency deviation of 5KHz. i) What is the bandwidth occupied by the modulated waveform ii) If the amplitude of the modulating signal is increased by a factor of 2 & its frequency is lowered to 1KHz. What is the new bandwidth?

Soln:

i) Given $f_m = 2KHz$, $\Delta f = 5KHz$

$$B_T = 2(\Delta f + f_m)$$
$$= 2(5\times10^3 + 2\times10^3)$$
$$= \underline{14KHz}$$

ii) Since the amplitude of the modulating signal is increased by a factor of 2, the frequency deviation also increased by the same factor.

∴ New frequency deviation is

$$\Delta f = 2\times5k = \underline{10KHz}$$

Given $f_m = 1KHz$ ∴ $B_T = 2(\Delta f + f_m)$

$$= \underline{22KHz}$$

4. A sinusoidal modulating waveform of amplitude 5V and a frequency of 1kHz is applied to an FM generator that has a frequency sensitivity constant of 40 Hz/V. i) What is the frequency deviation ii) What is the modulation index.

Soln:

i) Frequency deviation is the maximum departure of the instantaneous frequency from the unmodulated carrier frequency.

W.K.T, $f_i(t) = f_c + k_f m(t)$

$$\Delta f = |k_f m(t)|_{max} = K_f \cdot A_m$$

$$\Delta f = 40\times5 = \underline{200Hz}$$

ii) $\beta = \Delta f / f_m = \frac{200}{1\times10^3} = \underline{0.2}$

NAWA HOLLA K

(5) An angle modulated signal is defined by $s(t) = 10\cos(2\pi \times 10^6 t + 0.2 \sin 2000\pi t)$ V. Find

i) The power in the modulated signal

ii) The frequency deviation $\Delta f$

iii) Phase deviation $\Delta\theta$

iv) The approximate transmission bandwidth.

Soln:

Given $A_c = 10$, $\beta = 0.2$, $f_c = 1\times10^6$, $f_m = 1000$

i) $\text{Power} = \left(A_c/\sqrt{2}\right)^2 = \left(10/\sqrt{2}\right)^2 = \underline{50W}$

ii) $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$

$f_i(t) = f_c + k_f m(t)$

$= f_c + \Delta f$

$\Delta f = |f_i(t) - f_c|_{max}$

$\theta_i(t) = 2\pi\times10^6 t + 0.2 \sin 2000\pi t$

$\frac{d\theta_i(t)}{dt} = 2\pi\times10^6 + 0.2\cos 2000\pi t \times 2000\pi$

$\therefore f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$

$= \frac{1}{2\pi}\left[2\pi\times10^6 + 0.2\cos 2000\pi t \times 2000\pi\right]$

$\Rightarrow \Delta f = |f_i(t) - f_c|_{max}$

$= \frac{1}{2\pi}\left(2\pi\times10^6 + 0.2\times2000\pi\right) - 1\times10^6$

$= \underline{200\text{Hz}}$

iii) $\Delta\theta = |\theta_i(t) - \theta_c|_{max}$ where $\theta_c = 2\pi\times10^6 t$

$= |\cancel{2\pi\times10^6 t} + 0.2\sin 2000\pi t - \cancel{2\pi\times10^6 t}|_{max}$

$= \cancel{2\pi\times10^6 t}.\ |0.2 \sin 2000\pi t|_{max}$

$= \underline{0.2 \text{ rad}}$

iv) $B_T = 2(\Delta f + f_m)$

$= 2[200 + 1000]$

$\underline{B_T = 2400\text{Hz}}$

NAVYA HOLLA K

(6) A sinusoidal modulating wave $m(t) = A_m \cos 2\pi f_m t$ is applied to a phase modulator with phase sensitivity $k_p$. The unmodulated carrier wave has frequency $f_c$ & amplitude $A_c$. Determine the spectrum of the resulting phase modulated wave, assuming the maximum phase deviation $\beta_p = k_p A_m$ does not exceed 0.3 rad.

Sol: The time-domain expression for the PM wave is

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

Substituting $m(t) = A_m \cos 2\pi f_m t$ in above equation,

$$s(t) = A_c \cos[2\pi f_c t + k_p A_m \cos 2\pi f_m t]$$

$$= A_c \cos[2\pi f_c t + \beta_p \cos 2\pi f_m t]$$

$$= A_c \cos 2\pi f_c t \cdot \cos(\beta_p \cos 2\pi f_m t) - A_c \sin 2\pi f_c t \cdot \sin(\beta_p \cos 2\pi f_m t)$$

Since $\beta_p \leq 0.3$ rad,

$$\cos(\beta_p \cos 2\pi f_m t) \simeq 1$$

$$\sin(\beta_p \cos 2\pi f_m t) \simeq \beta_p \cos 2\pi f_m t$$

$$\therefore s(t) = A_c \cos 2\pi f_c t - A_c \beta_p \sin 2\pi f_c t \cdot \cos 2\pi f_m t$$

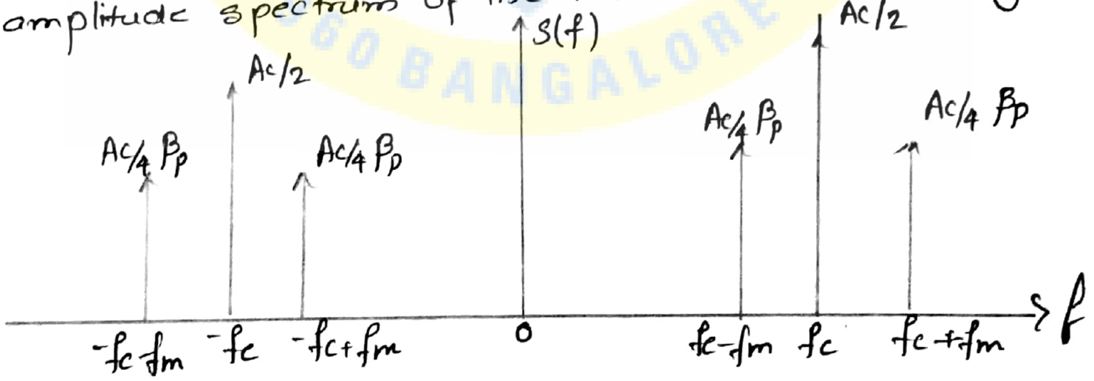
$$= A_c \cos 2\pi f_c t - \frac{A_c \cdot \beta_p}{2} \sin 2\pi (f_c + f_m) t - \frac{A_c \beta_p}{2} \sin 2\pi (f_c - f_m) t$$

Taking F.T,

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] - \frac{A_c \cdot \beta_p}{4j}\left[\delta(f - (f_c + f_m)) - \delta(f + (f_c + f_m))\right]$$

$$- \frac{A_c \cdot \beta_p}{4j}\left[\delta(f - (f_c - f_m)) - \delta(f + (f_c - f_m))\right].$$

The amplitude spectrum of the NBPM is drawn using above expression.




(7) Determine the instantaneous frequency in Hz for each of the following signals.

(i) $s(t) = 10\cos[200\pi t + \pi/3]$

ii) $s(t) = 10\cos[200\pi t + \pi t^2]$

iii) $s(t) = \cos(200\pi t)\cos(5\sin 2\pi t] + \sin(200\pi t)\sin(5\sin 2\pi t)$.

NAVYA HOLLA K

Soln: (i) $\theta_i(t) = 200\pi t + \pi/3$.

$$f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$= \frac{1}{2\pi} \times 200\pi = \underline{100 Hz}$$

ii) $\theta_i(t) = 200\pi t + \pi t^2$

$$f_i(t) = \frac{1}{2\pi} \times (200\pi + 2\pi t)$$

$$= \underline{(100 + t) Hz}$$

iii) W.K.T $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\therefore$ $s(t) = \cos[200\pi t - 5\sin 2\pi t]$

$$\theta_i(t) = 200\pi t - 5\sin 2\pi t$$

$$f_i(t) = \frac{1}{2\pi} \times [200\pi - 5\cos 2\pi t \times 2\pi]$$

$$= \underline{(100 - 5\cos 2\pi t)\ Hz}$$

8) A carrier wave of frequency 100MHz is frequency modulated by a sinusoidal wave of amplitude 20V & frequency 100KHz. The frequency sensitivity of the modulator is 25KHz/V.
(i) Find the bandwidth of FM signal, using Carson's rule.
(ii) Find the bandwidth by transmitting only those side frequencies whose amplitudes exceeds 1% of the unmodulated carrier amplitude. Use the universal curve.
iii) Repeat the calculations, assuming that the amplitude of the modulating signal is doubled.
iv) Repeat the calculations, assuming that the modulation frequency is doubled.

Soln: Given $f_c = 100MHz$, $A_m = 20V$, $f_m = 100KHz$, $K_f = 25KHz/V$.

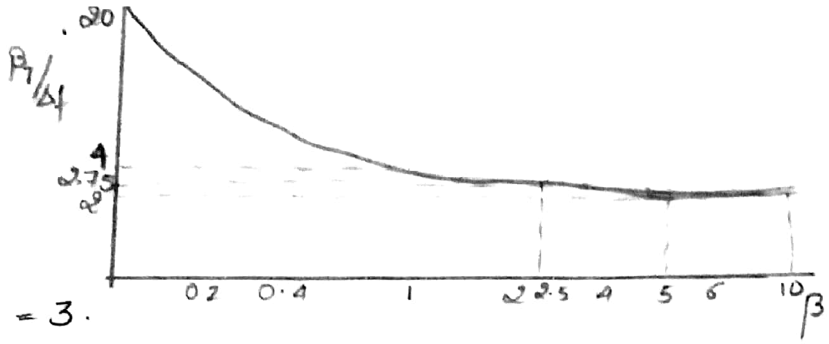
(i) $B_T = 2(\Delta f + f_m)$.

But $\Delta f = k_f \cdot A_m = 25\times10^3 \cdot 20 = \underline{5\times10^5\ Hz}$

$\therefore$ $B_T = 2(5\times10^5 + 100\times10^3) = \underline{1.2 MHz}$

ii) $\beta = \frac{\Delta f}{f_m} = \frac{5\times10^5}{10^5} = \underline{5}$

Using universal curve of below figure, we find for $\beta = 5$,




$\frac{B_T}{\Delta f} = 3.$

$\therefore B_T = 3 \times \Delta f$
$= 3 \times 5 \times 10^5 = \underline{1.5 MHz}$

iii) If the amplitude of the modulating wave is doubled, then

$A_m = 40V, \ \Delta f = 25 \times 10^3 \times 40 = \underline{1 MHz}$

$\Delta f = 1 MHz \quad \therefore \beta = \Delta f / f_m = \underline{10}$

Then $\quad B_T = 2(\Delta f + f_m)$
$= \underline{2.2 MHz}$

Using universal curve, $\quad B_T / \Delta f = 2.75$

$\therefore B_T = 2.75 \times 1 MHz = \underline{2.75 MHz}$

iv) If $f_m$ is doubled, $f_m = 200 KHz$.

$\beta = \Delta f / f_m = \frac{5 \times 10^5}{200 \times 10^3} = \underline{2.5}$

$B_T = 2(\Delta f + f_m). = \underline{1.4 MHz}$

Using universal curve, $B_T / \Delta f = 4$ for $\beta = 2.5$.

$\therefore B_T = 4 \times \Delta f$
$= \underline{2 MHz}$

(9) A 2KHz sinusoidal signal phase modulates a carrier at 100MHz with a phase deviation of 45°. Using carson's rule evaluate the approximate bandwidth of the PM signal.

Soln Given $f_m = 2 KHz, \quad \Delta\theta = 45° \Rightarrow \beta = 45° = 45 \times \pi / 180 = 0.78$

$f_c = 100 MHz, \quad \beta = \Delta f / f_m \Rightarrow \Delta f = \beta \times f_m = \underline{1.57 K}.$

$\therefore B_T = 2(\Delta f + f_m)$
$= \underline{7.14 KHz}$

6. GENERATION OF FM WAVES

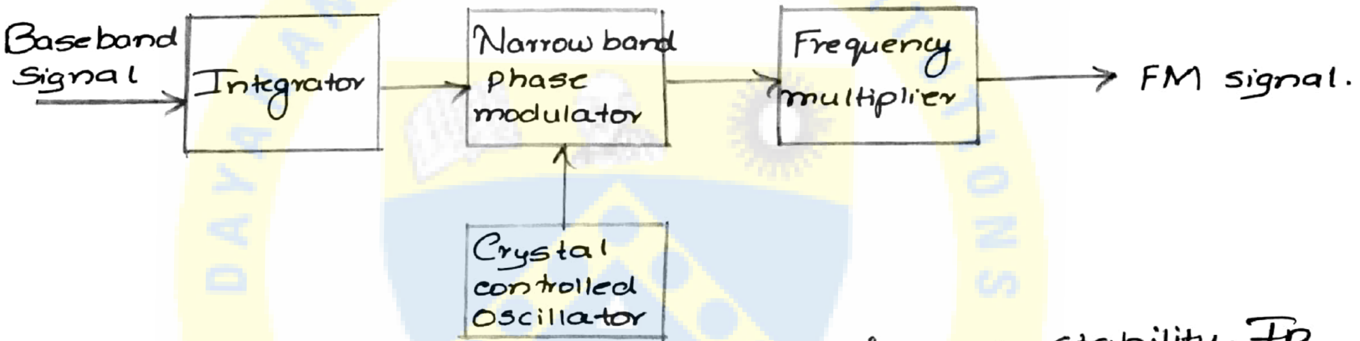
There are two basic methods of generating frequency modulated signals : Indirect FM & Direct FM.

Indirect Method: The modulating signal is first used to produce a narrow-band FM signal & frequency multiplication is next used to increase the frequency deviation to the desired level.

Direct Method: The carrier frequency is directly varied in accordance with the input baseband signal.

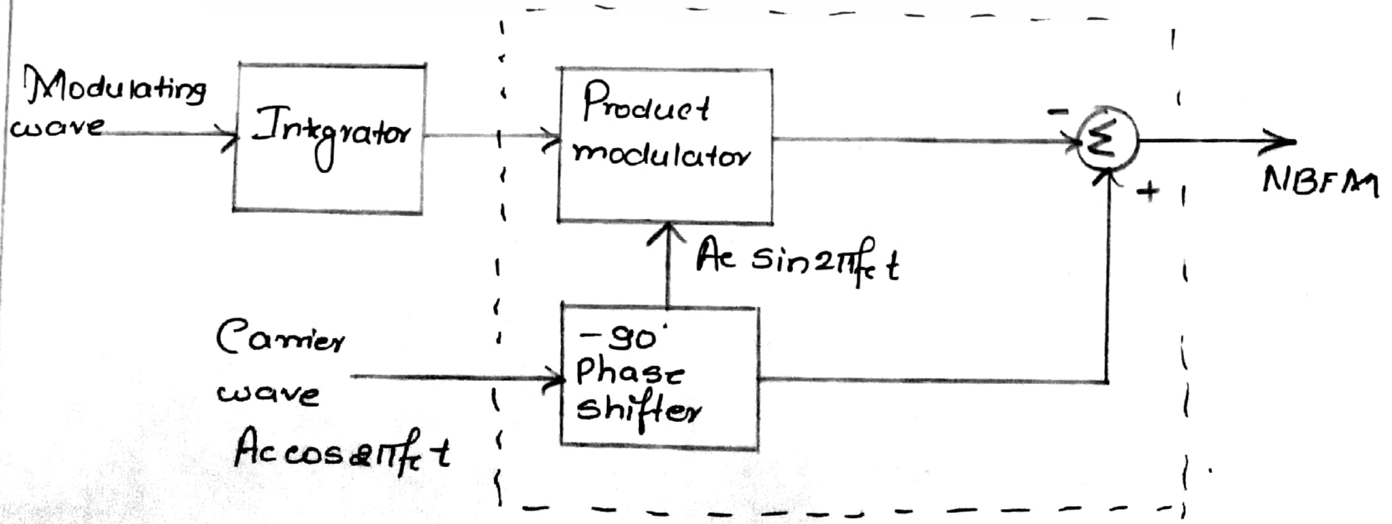
6.1 INDIRECT FM

A simplified block diagram of an indirect FM is shown in figure below. The message signal $m(t)$ is first integrated and then used to phase modulate a crystal-controlled oscillator.



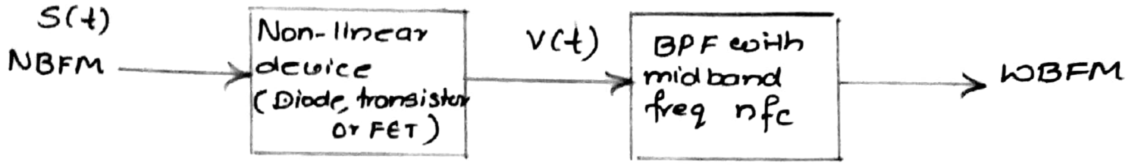

The use of crystal control provides frequency stability. In order to minimize the distortion inherent in the phase modulator, the maximum phase deviation or modulation index $\beta$ is kept small thereby resulting in a narrow-band FM signal.

Implementation of the narrow-band phase modulator is as shown in figure below [Dotted line].



The narrowband FM signal is next multiplied in frequency by means of a frequency multiplier so as to produce desired wide-band FM signal.

A frequency multiplier consists of a memoryless non-linear device followed by a band pass filter. Memory less non linear device implies that it has no energy-storage elements.




The input-output relation of such a device may be expressed in the general form.

$$V(t) = a_1 s(t) + a_2 s^2(t) + \cdots \cdots a_n s^n(t)$$

Where $a_1, a_2, \cdots a_n$ are constant coefficients determined by the operating point by the device.

The input $s(t)$ is a FM signal defined by

$$S(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

Whose instantaneous frequency is

$$f_i(t) = f_c + k_f\, m(t)$$

The mid-band frequency of BPF is set equal to $nf_c$. The BPF is designed to have a bandwidth equal to 'n' times the transmission bandwidth of $s(t)$. After band-pass filtering of the non-linear devices' output $V(t)$, we have new FM signal defined by

$$s'(t) = A_c' \cos\left(2\pi n f_c t + 2\pi n k_f \int_0^t m(t)\,dt\right]$$

Whose instantaneous frequency is

$$f_i'(t) = nf_c + nk_f\, m(t)$$

By comparing $f_i(t)$ & $f_i'(t)$ we see that non linear processing circuit acts as a frequency multiplier.

[Explanation for Fig. 2]: (Generation of NBFM)

Expression for an FM wave $s_1(t)$ is given as

$$s_1(t) = A_1 \cos\left(2\pi f_1 t + \varphi_1(t)\right]$$

Where $f_1$ is carrier frequency, $A_1$ is carrier amplitude.

The angular argument $\phi_1(t)$ of $s_1(t)$ is related to $m(t)$ by

$$\phi_1(t) = 2\pi k_1 \int_0^t m(t)dt$$

Where $k_1$ is frequency sensitivity of the modulator. Provided that the angle $\phi_1(t)$ is small compared to one radian for all $t$, then

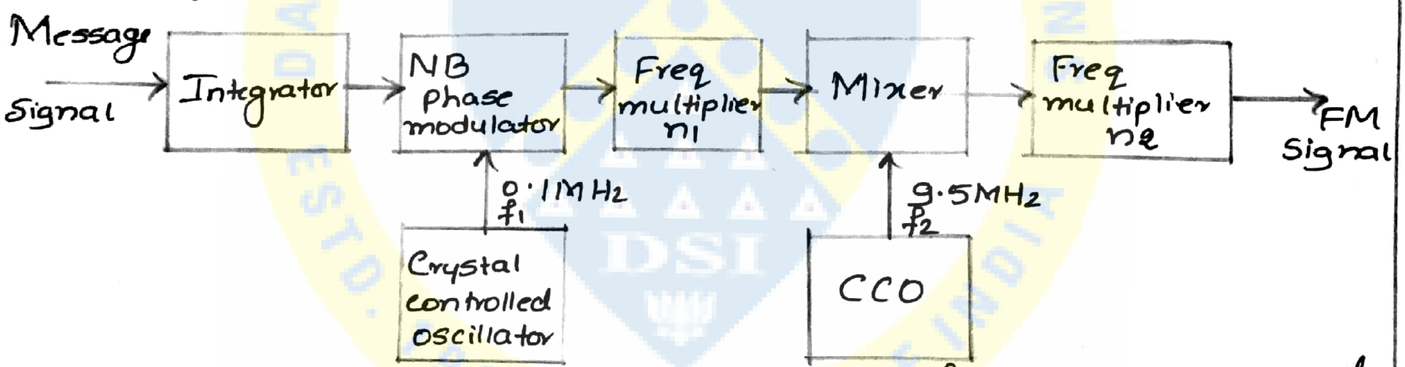
$$\cos[\phi(t)] \approx 1 \quad \& \quad \sin[\phi(t)] \simeq \phi(t).$$

$$\therefore s_1(t) = A_1 \cos 2\pi f_1 t - A_1 \sin 2\pi f_1 t \cdot \phi_1(t)$$

$$= A_1 \cos 2\pi f_1 t - A_1 \sin 2\pi f_1 t \cdot \int_0^t m(t)\cdot dt \cdot 2\pi k_1$$

Above equation defines a narrow band FM wave. Scalling factor $2\pi k_1$ is taken care of by the product modulator.

Ex: Figure shows the block diagram of an FM transmitter used to transmit audio signals containing frequencies in the range 100Hz to 15KHz. The narrow band phase modulator is supplied with a carrier wave of frequency $f_1 = 0.1MHz$ by a crystal controlled oscillator. The desired FM wave at the transmitter output has a carrier frequency $f_c = 100MHz$ & frequency deviation $\Delta f = 75KHz$. $\beta$ is less than 0.3 rad (Assume $\beta = 0.2$)




The lowest modulating frequency produces a frequency deviation of

$$\Delta f_1 = \beta \cdot f_m = 0.2 \times 100 = 20Hz$$

To produce frequency deviation of $\Delta f = 75KHz$, Frequency multiplication is required. To generate an FM wave having both the desired frequency deviation & carrier frequency, we need to use 2-stage frequency multiplier with an intermediate stage of frequency translation.

Let $n_1$ & $n_2$ denote the respective frequency multiplication ratios, so that

$$\left[(\Delta f_1)n_1 \rightarrow (\Delta f \cdot n_1)n_2 = \Delta f\right]$$

$$n_1 n_2 = \frac{\Delta f}{\Delta f_1} = \frac{75{,}000}{20}$$

$$= 3750.$$

The carrier frequency at the first frequency multiplier output is translated downward in frequency to $(f_2 - n_1 f_1)$ by mixing it with a sinusoidal wave of frequency $f_2 = 95MHz$ which is supplied by a second crystal controlled oscillator. However the carrier frequency at the input of the second frequency multiplier is equal to $f_c/n_2$. Equating these two frequencies, we get

$[f_c = n_2 \cdot f_2]$
$\therefore f_2 = f_c/n_2$

$$f_2 - n_1 f_1 = f_c/n_2$$

With $f_1 = 0.1MHz$, $f_2 = 9.5MHz$ & $f_c = 100MHz$, we get

$$9.5M - 0.1M \times n_1 = \frac{100M}{n_2} \quad \text{--- (26)}$$

Solving (25) & (26)

$$n_1 = \frac{3750}{n_2}$$

$$9.5 - 0.1\left(\frac{3750}{n_2}\right) = \frac{100}{n_2}$$

$$9.5 - \frac{375}{n_2} = \frac{100}{n_2}$$

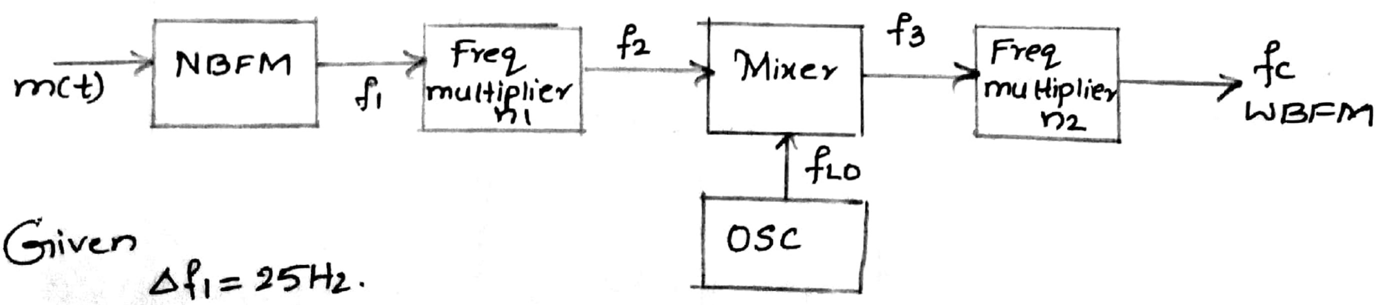
$$9.5 n_2 = 100 + 375$$

$$n_2 = \underline{\underline{50}}$$

$$\therefore n_1 = \frac{3750}{50} = \underline{\underline{75}}$$

(10) [For a wideband frequency modulator if narrowband carrier $f_1 = 0.1MHz$, 2nd carrier frequency $f_2 = 9.5MHz$, o/p carrier frequency 100MHz, frequency deviation = 75KHz. Calculate the multiplying factors $n_1$ & $n_2$ if the narrowband frequency deviation is 20Hz. Draw the block diagram of the modulator].

(11) For the block diagram shown in figure below, compute the maximum frequency deviation & output frequency of the transmitter. Take $f_1 = 200KHz$, $f_{LO} = 10.8MHz$, $\Delta f_1 = 25Hz$, $n_1 = 64$ & $n_2 = 48$.




Given
$\Delta f_1 = 25Hz$.

$f_2 = n_1 f_1$
$= \underline{\underline{12.8MHz}}$

* $f_3 = f_2 \pm f_{LO}$.

(i) $f_3 = f_2 + f_{LO} = 12.8M + 10.8M = \underline{23.6MHz}$

(ii) $f_3 = f_2 - f_{LO} = 12.8M - 10.8M = \underline{2MHz}$

* $f_c = n_2 f_3$
$= 48 \times 23.6M$
$= \underline{1.132GHz}$

OR

$f_c = n_2 f_3$
$= 48 \times 2M$
$= \underline{96MHz}$

* $\Delta f = \Delta f_1 \times n_1 \times n_2$
$= 25 \times 64 \times 48 = \underline{76.8kHz}$

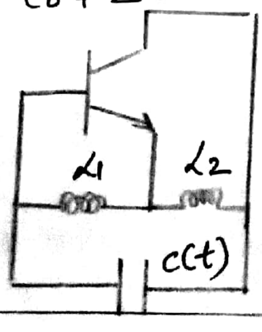
## • DIRECT FM

In a direct FM system, the instantaneous frequency of the carrier wave is varied directly in accordance with a message signal by means of a device known as voltage controlled oscillator. One way of implementing such a device is to use a sinusoidal oscillator having a highly resonant network in which the capacitance will vary in accordance with modulating signal. The capacitive component in the frequency selective network consists of a fixed capacitor in parallel with a voltage variable capacitor. The resultant capacitance is represented by $C(t)$. The voltage variable capacitor called varicap or varactor is one whose capacitance depends on the voltage applied across its electrodes. The capacitance of a reverse biased varactor diode depends on the voltage applied across its pn junction. The larger the reverse voltage applied to such a diode, the smaller will be its transition capacitance. An example of such a scheme is shown in figure using a Hartley Oscillator. The frequency of oscillation of Hartley oscillator is given by,

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C(t)}} \quad \text{---}(27)$$

Where $C(t)$ is the total capacitance of fixed capacitor & the variable voltage capacitor & $L_1$ & $L_2$ are the two inductances in the frequency determing network of the oscillator. Assume that for a sinusoidal modulating wave of frequency $f_m$. The capacitance $C(t)$ is expressed as

$$C(t) = C_0 + \Delta C \cos(2\pi f_m t) \quad \text{---}(28)$$



Substituting eqn (28) in (27)

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0+\Delta C\cos(2\pi f_m t)}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0 + (L_1+L_2)\Delta C\cos 2\pi f_m t}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1+\frac{(L_1+L_2)\Delta C\cos 2\pi f_m t}{(L_1+L_2)C_0}\right)}}$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}\sqrt{\left[1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right]}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}\cdot\left[1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right)^{1/2}}$$

$$f_i(t) = f_0\cdot\left[1+\frac{\Delta C}{C_0}\cos 2\pi f_m t\right]^{-1/2} \quad \text{--- (29)}$$

Where $f_0$ is the unmodulated frequency of oscillator. ie;

$$f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)C_0}}$$

Considering the maximum change in capacitance $\Delta C$ is small compared with the unmodulated capacitance $C_0$.

Eqn (29) becomes,

$$f_i(t) \simeq f_0\left(1-\frac{\Delta C}{2C_0}\cos 2\pi f_m t\right]$$

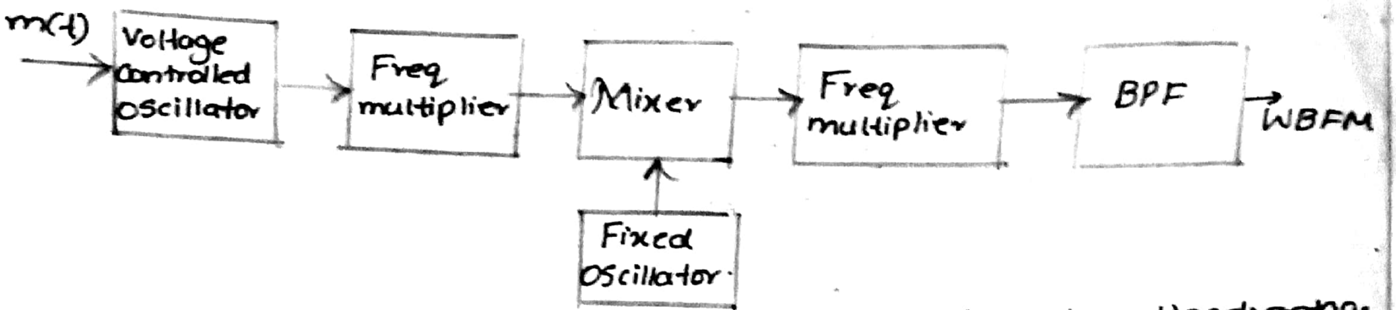
[Binomial fn: $(1+x)^{-1/2} \simeq 1-\frac{1}{2}x$ if $x \leq 1$]

Let $\frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_0}$

Hence the instantaneous frequency of the oscillator which is being frequency modulated by varying the capacitance of the frequency determining network is approximately given by

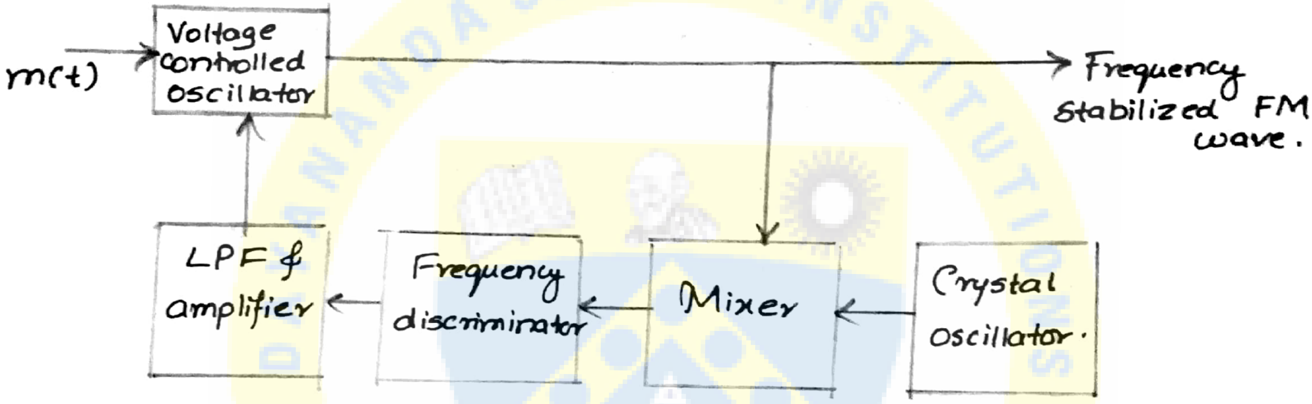
$$f_i(t) \simeq f_0 + \Delta f\cos 2\pi f_m t$$

To generate a wide-band FM wave with the required frequency deviation. The configuration as shown in fig below consisting of a voltage controlled oscillator followed by a series of frequency multipliers and mixers, is used. This configuration gives good oscillator stability, constant proportionality between output frequency change & i/p voltage change, And the necessary frequency deviation to achieve WBFM.




- The FM transmitter described above has the disadvantage that the carrier frequency is not obtained from a highly stable oscillator. It is necessary to provide some auxilary means by which a very stable frequency generated by a crystal will be able to control the carrier frequency.

One method to do this is as shown below.



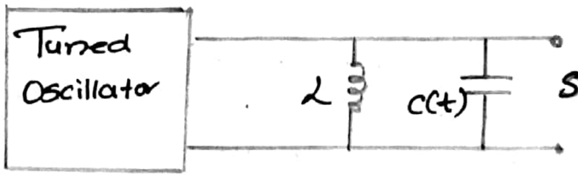

The output of the FM generator is applied to a mixer together with the output of a crystal controlled oscillator, & the difference frequency term is extracted. The mixer output is next applied to a frequency discriminator & then low pass filtered.

A frequency discriminator is a device whose output voltage has an instantaneous amplitude that is proportional to the instantaneous frequency of the FM signal applied to its input.

When the FM transmitter has exactly the correct carrier frequency, the LPF output is zero. However deviations of the transmitter carrier frequency from its assigned value will cause the frequency discriminator - filter combination to develop a dc output voltage with a polarity determined by the sense of the transmitter frequency drift. The dc voltage after suitable amplification, is applied to the voltage controlled oscillator of the FM transmitter in such a way as to modify the frequency of the oscillator in a direction that tends to restore the carrier frequency to its correct value.

(12) At low frequencies it may be possible to generate an FM wave by varying the capacitance of a parallel resonant circuit shown in figure below. Shows that the output $s(t)$ of the tuned circuit is an FM wave if the capacitance has the form:

$$C(t) = C_0 - km(t) \quad \& \quad \left|\frac{km(t)}{C_0}\right| << 1$$




Sol:

$$w_i(t) = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{L(C_0 - km(t))}}$$

$$= \frac{1}{\sqrt{LC_0 - Lkm(t)}} = \frac{1}{\sqrt{LC_0\left(1 - \frac{Lkm(t)}{LC_0}\right)}}$$

$$w_i(t) = \frac{1}{\sqrt{LC_0}} \cdot \left[1 - \frac{km(t)}{C_0}\right]^{-1/2} \quad \text{---} (30)$$

By binonimal theorem, $(1-x)^{-1/2} \cong 1 + \frac{1}{2}x$ , $|x| << 1$

Since it is given $\frac{km(t)}{C_0} << 1$, Eqn (30) can be written as

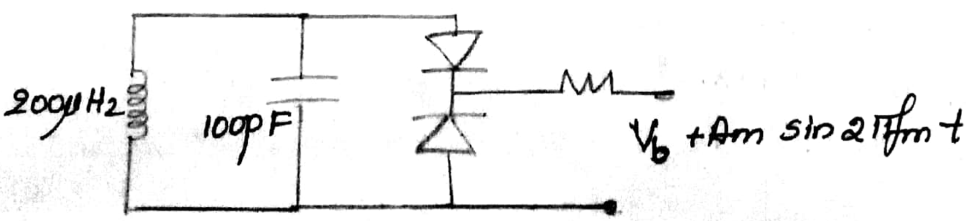
$$w_i(t) = \frac{1}{\sqrt{LC_0}}\left[1 + \frac{1}{2}\frac{k}{C_0}m(t)\right]$$

$$2\pi f_i(t) = \frac{1}{\sqrt{LC_0}} + \frac{1}{2}\frac{k}{C_0} \cdot \frac{1}{\sqrt{LC_0}} m(t)$$

$$f_i(t) = \frac{1}{2\pi\sqrt{LC_0}} + \frac{1}{4\pi} \cdot \frac{k}{C_0} \cdot \frac{1}{\sqrt{LC_0}} m(t).$$

This is the form of $f_i(t) = f_c + k_f m(t)$. $\therefore$ $s(t)$ is an FM wave

(13) Figure depicts a typical frequency discrimination circuit. Frequency modulation is produced by applying $m(t) = A_m \cos 2\pi f_m t$, plus a bias $V_b$ to a pair of varactor diodes. The capacitor of each varactor diode is related to the voltage $V_0$ applied across its electrodes by $C_0 = 100 V_b^{-1/2} \times 10^{-12}$ F. The unmodulated frequency of oscillation is 1MHz. (i) Find the magnitude of the bias voltage $V_b$.



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Soln: Frequency of oscillation is

$$f_o = \frac{1}{2\pi\sqrt{L(C+C_0/2)}}$$

$$1\times10^6 = \frac{1}{2\pi\sqrt{200\times10^{-6}(100\times10^{-12}+50\times V_b^{-1/2}\times10^{-12})}}$$

$$V_b = \underline{\underline{3.52V}}$$

ii) The VCO output is applied to a frequency multiplier to produce an FM signal with a carrier frequency of 64MHz & a modulation index of 5. Find the amplitude Am of the modulating wave given that fm = 10KHz.

Soln:

$$f_o \rightarrow \boxed{\begin{array}{c}\text{Freq}\\ \text{multipli}\\ n_1\end{array}} \rightarrow f_c$$

$$f_c = n_1 f_o \Rightarrow n_1 = f_c/f_o = \frac{64M}{1M}$$

$$n_1 = \underline{64}$$

Frequency multiplication ratio is 64. ∴ The modulation index of FM at multiplier input is $\beta = 5/64 = \underline{0.078}$

This indicates that the FM wave is NBFM, which means Am is small compared to $V_b$.

∴ The instantaneous frequency of this AM wave is

$$f_i(t) = \frac{1}{2\pi}\left[200\times10^{-6}\left(100\times10^{-12}+50\times10^{-12}(3.52+A_m\sin2\pi f_m t)^{-1/2}\right)\right]^{-1/2}$$

$$= \frac{1}{2\pi}\left(200\times10^{-6}\left(100\times10^{-12}+50\times10^{-12}\left(3.52\left[1+\frac{A_m}{3.52}\sin2\pi f_m t\right]^{-1/2}\right)\right)\right)^{-1/2}$$

$$= \frac{10^7}{2\sqrt{2\pi}}\left(1+0.266\left[1+\frac{A_m}{3.52}\sin2\pi f_m t\right]^{-1/2}\right)^{-1/2}$$

$$= \frac{10^7}{2\sqrt{2\pi}}\left(1+0.266\left[1-\frac{A_m}{7.04}\sin2\pi f_m t\right]\right)^{-1/2}$$

$$= 10^6(1-0.03A_m\sin2\pi f_m t)^{-1/2}$$

$$f_i(t) = 10^6\left(1+\tfrac{1}{2}\times0.03A_m\sin2\pi f_m t\right)$$

$$= 10^6 + 0.015\times10^6 A_m\sin2\pi f_m t$$

This is of the form,

$$f_i(t) = f_o + \Delta f\cos2\pi f_m t.$$

$$\therefore \Delta f = 0.015\times10^6 A_m.$$

W.K.T

$$\Delta f = \beta\cdot f_m.$$

$$= 0.078\times10\times10^3 = \underline{\underline{0.078\times10^4}}$$

Equating both equations, we get

$$0.015\times10^6\times A_m = 0.078\times10^4$$

$$\Rightarrow A_m = \underline{\underline{52\times10^{-3}\,V}}$$

## 2. Zero Crossing Detector:

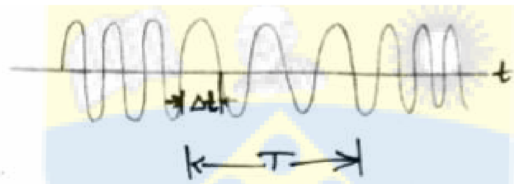
This detector exploits the property that the instantaneous frequency of the FM wave is approximately given by

$$f_i \simeq \frac{1}{2\Delta t} \quad \text{---(14)}$$

Where $\Delta t$ is the time difference between adjacent zero crossing of the FM wave as in fig (9). The interval $T$ is chosen according to following conditions

(i) The interval $T$ is small compared to the reciprocal of the message bandwidth $W$.

(ii) The interval $T$ is large compared to the reciprocal of the carrier frequency $f_c$ of the FM wave.




(9)

Condition (i) means that the message signal $m(t)$ is essentially constant inside the interval $T$.

Condition (ii) ensures that a reasonable number of zero crossing of the FM wave occurs inside the interval $T$. These conditions are illustrated by the waveform. [Fig(9)].

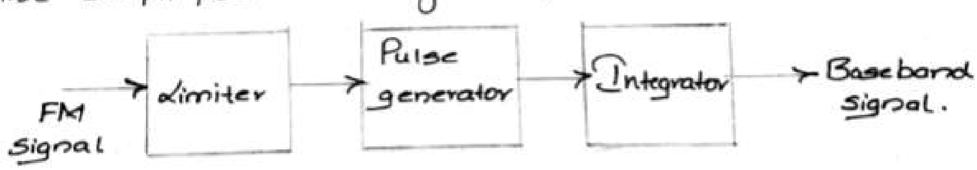
Let $n_0$ denote the number of zero crossing inside the interval $T$. ∴ The time $\Delta t$ between adjacent zero crossing is,

$$\Delta t = \frac{T}{n_0} \quad \text{---(15)}$$

Substituting eqn (15) in (14),

$$f_i \simeq \frac{1}{2(T/n_0)} = \frac{n_0}{2T} \quad \text{---(16)}$$

Since w.k.t the instantaneous frequency is linearly related to the message signal $m(t)$. From eqn (16) we see that $m(t)$ can be recovered from the knowledge of $n_0$. Figure below shows the simplified block diagram of zero crossing detector.




The limiter produces a square wave version of the Input FM wave. The pulse generator produces short pulses at the positive going as well as negative going edges of the limiter output. The integrator performs the averaging over the Interval $T$ as indicated in eqn (16). There by reproducing the original message signal $m(t)$ at its output.