DEPARTMENT
OF
ELECTRONICS & COMMUNICATION ENGINEERING

Communication Theory
18EC4DCCOT

(Theory Notes)

Autonomous Course

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Module – 3 Contents
Noise Characterization: Introduction, shot noise, thermal noise, white noise, Noise equivalent bandwidth, Narrow band noise, Noise Figure, Equivalent noise temperature, cascade connection of two-port networks. Figure of merit for AM[DSBSC], Threshold effect, Figure of merit for FM, Threshold effect, Pre and de-emphasis for FM

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MODULE 3 - NOISE CHARACTERIZATION

→ An unwanted signal at the channel or the receiver

→ Two types of noise based on the type of source

1) External ~~noise~~ source → external sources (channel)
  - → Atmospheric noise (static noise)
  - → Extra-terrestrial noise (solar noise and cosmic noise)
  - → Industrial noise – noise produced by machines, aircraft ignitions, switch gears,

2) Internal source → produced by the receiver components.
  - → Thermal agitation noise (Johnson noise or Electrical noise)
  - → Shot noise (random movement of electrons & holes)
  - → Transit-time noise
  - → other noises → flicker, mixer generated noise

Atmospheric noise → natural source of disturbance caused by lightening discharge in thunderstorm

→ Natural electrical disturbances
called as
→ Static noise

Industrial noise → man made noise

Extra terrestrial noise
↳ solar noise

→ Originates from sun
→ There are constant radiation from Sun due to its high temp.
→ Also, electrical disturbances such as coronal discharges & sunspots can produce additional noise

Cosmic noise → stars generate noise
→ Individual effect will be less but large number leads to collective effects
→ noise range - 8MHz to 1.43GHz

Thermal noise - Johnson-Nyquist noise

↳ Generated by the random thermal motion of charge carriers

↳ Thermal noise is white noise [∵ its power spectral density is nearly equal throughout the freq. spectrum]

↳ The commn system affected by thermal noise is often modelled as an AWGN channel.

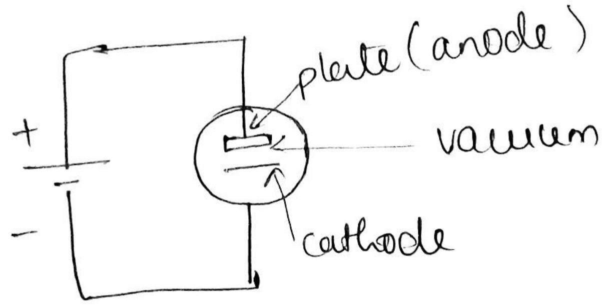
Shot noise

↳ due to random statistical fluctuations of the electric current when the charge carriers traverse a gap.

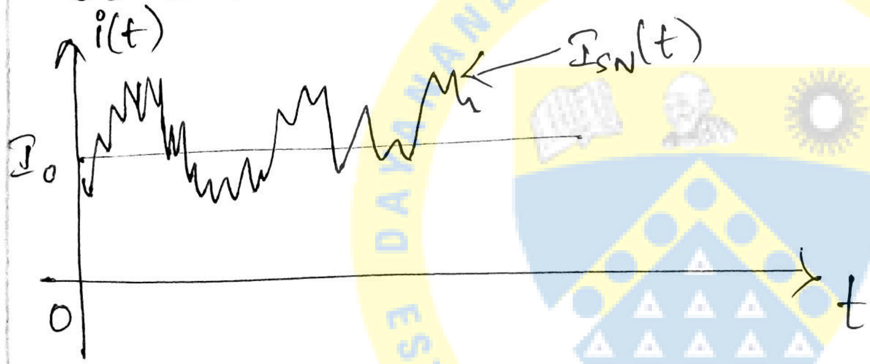
↳ If electrons flow across a barrier, then they have discrete arrival times. This discrete arrivals results in shot noise

→ eg. noise created by rain falling on a tin roof. The flow of rain may be relatively constant, but the individual rain drops arrive discretely

→ Vacuum tubes exhibit shot noise because the electrons randomly leave the cathode & arrive at the anode (plate)



→ The nature of current variation with time is as shown below.



→ I_0 = mean value of current.

→ The current which wiggles around the mean value I_0 is called as <u>Shot noise</u>.

→ This wiggling nature cannot be visualized using normal measuring instrument. that is why it looks constant current I.

→ It can be seen only in <u>fast sweep oscillators</u>.

$$i(t) = I + I_{SN}(t)$$

Power density spectrum of shot noise in diode

↳ It gives the strength of the variations as a function of frequency. That means it shows at which frequencies variations are strong & or weak

→ $I_{SN}(t)$ of the $i(t)$ is random in nature i.e it is indeterministic function. So it cannot be expressed as a function of f.

→ ∴ we use power density spectrum (of $I_{SN}(t)$) is

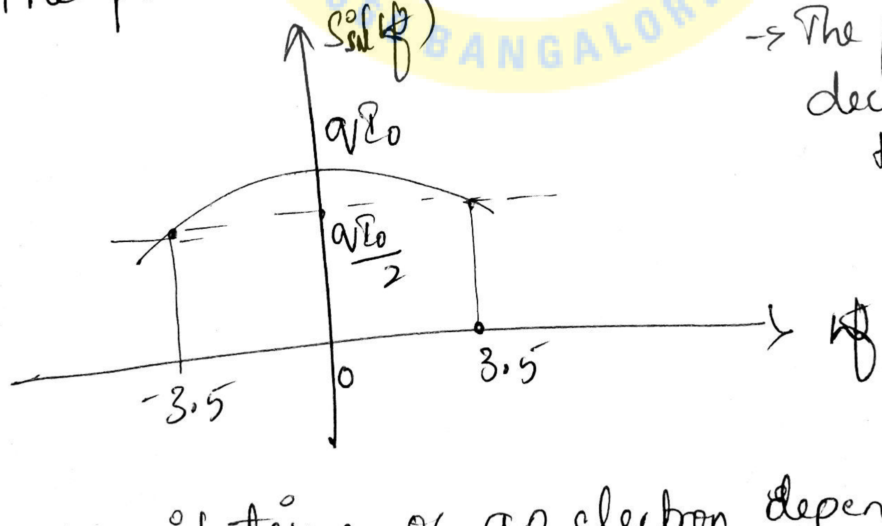
$$S_{i_{SN}}(f) = q I_0$$

$q \rightarrow$ Electron charge 1.6×10^{-19} coulombs

$I_0 \rightarrow$ mean value

→ The above equation is freq. independent,

→ The power density varies with freq. as shown below



→ The freq. range is decided by transit time

→ The transit time of an electron depends on anode voltage V

$d \rightarrow$ spacing b/w anode & cathode

$$\tau = \frac{3.36 \times d}{\sqrt{V}} \ \mu sec$$

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The mean square value of I_{SN} is given by

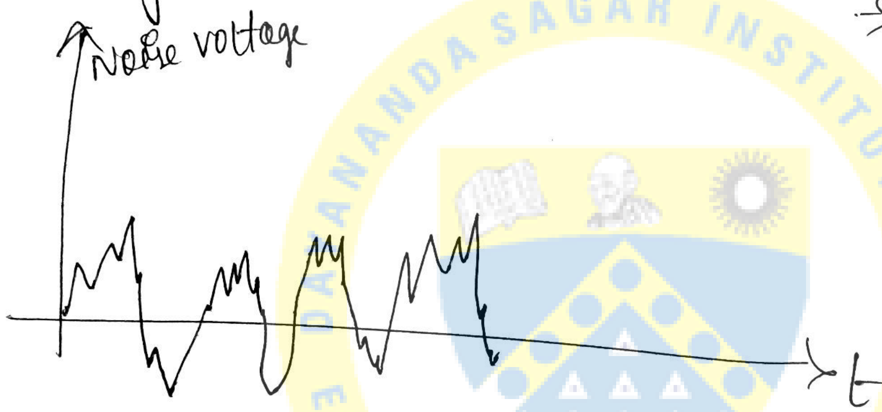
$$\overline{I_{SN}^2} = 2qI_0B_N = 2qI_0\Delta f \text{ amps}^2$$

$B_N \rightarrow$ Noise bandwidth in Hz.

$2\Delta f \rightarrow$

Thermal noise (Johnson noise)

→ random movement of electrons inside the conductor resulting in a randomly varying voltage across the conductor



→ Even in the absence of electric field, due to thermal energy, the electrons move randomly.

→ Thermal noise increases with temperature.

→ The power ~~dens~~ spectral density of thermal noise is given by, for resistor conductor.

$$S_{TN}(f) = \frac{2h|f|}{e^{\frac{h|f|}{KT}} - 1}$$

$T \rightarrow$ Temp in ^oK

$K \rightarrow$ Boltzman constant 1.38×10^{-23} J/°K

$h \rightarrow$ plank's constant 6.63×10^{-34} J/sec

when $f << \frac{KT}{h}$,

$$e^{\frac{h|f|}{KT}} = 1 + \frac{h|f|}{KT}$$

$$\therefore S_{TN}(f) = \frac{2h|f|}{\cancel{1} + \frac{h|f|}{KT} - \cancel{1}} = 2KT$$

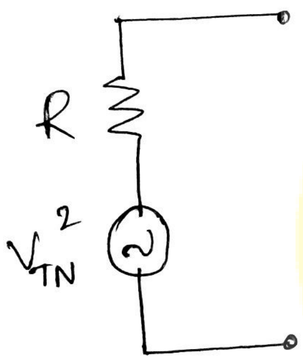
→ The mean square value of Thermal noise voltage is given by

$$V_{TN}^2 = 2RB_N \, S_{TN}(f)$$

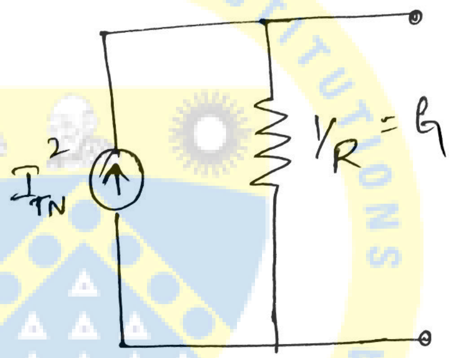
$$= 2RB_N(2KT)$$

$$\boxed{V_{TN}^2 = 4KTB_NR \ \text{volts}^2}$$

→ The model of a noisy resistor is shown below



Thevenin equivalent circuit



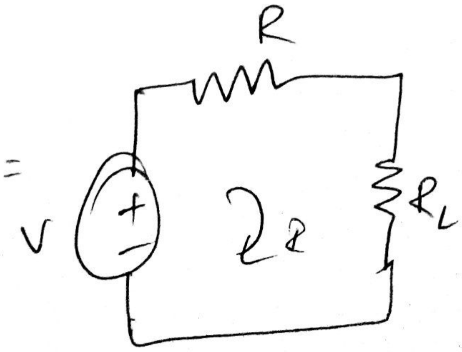
Norton equivalent circuit

$$I_{TN}^2 = \frac{V_{TN}^2}{R^2} = \frac{4KTB_N\cancel{R}}{R^{\cancel{2}}}$$

$$I_{TN}^2 = 4KTB_N \cdot G \leftarrow \text{conductance}$$

→ If we have load resistor, and the max. power transferred to the load is given by

$$P_n = \cancel{\frac{V_{RMS}^2}{R}} \cancel{=} \frac{\cancel{V_{TN}^2}}{R} =$$



$$I(R+R_L) = V$$

$I = \frac{V}{R+R_L}$, max. power transfer theorem

$R = R_L$

$I = \frac{V}{2R}$, W.k.T $P_o = I^2 R_L$ $(R_L = R)$

$= \frac{V^2}{4R^2} \cdot R = \frac{V^2}{4R}$

$P_n = \frac{V_{TN}^2}{4R} = \frac{\cancel{4}KTB_N \cancel{R}}{\cancel{4R}} = KTB_N$

Calculate the rms noise voltage and thermal noise power appearing across a 20kΩ resistor at 25°C temp. with an effective noise bandwidth of 10kHz.

Soln: $R = 20k\Omega$, $T = 273 + 25 = 298^\circ K$, $B_N = 10kHz$.

$V_{TN}^2 = 4KTB_N R$, $V_{TN} = \sqrt{4KTB_N R} = 1.81\,\mu V$

$P_n = \frac{V_{TN}^2}{4R} = KTB_N = 4.11 \times 10^{-17}$

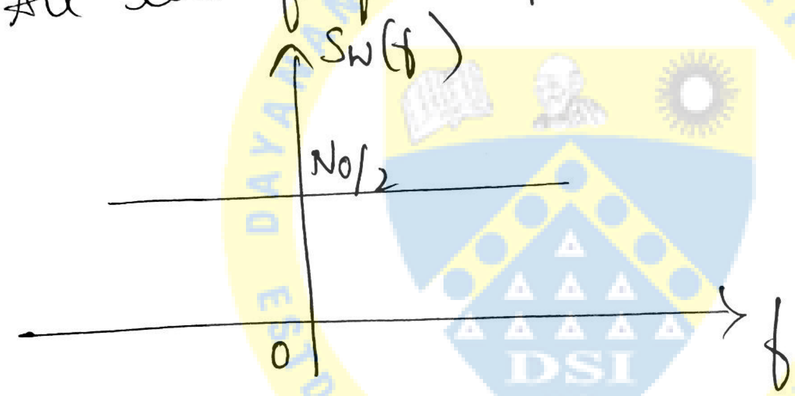
White noise

→ It is produced by combining sounds of all different frequencies together.

→ This is not the noise source

→ which has gaussian distribution & have flat spectral density over a wide range of frequencies.

→ All the freq. components in equal proportions.



$S_W(f) = \frac{N_0}{2}$

∴ Independent of freq.

Where $N_0 = KT_e$ T_e → equivalent noise temp. of receiver

$\frac{N_0}{2} = \frac{KT_e}{2}$ → K → Boltzman constant.

noise power due to white noise

$P_{WN} = N_0 . B$

$P_{WN} = KT_eB$

W.k.t from Fourier Transform theory, PSD & autocorrelation functions are FT pair

$$S_W(f) \xleftrightarrow{FT} R_W(\tau)$$

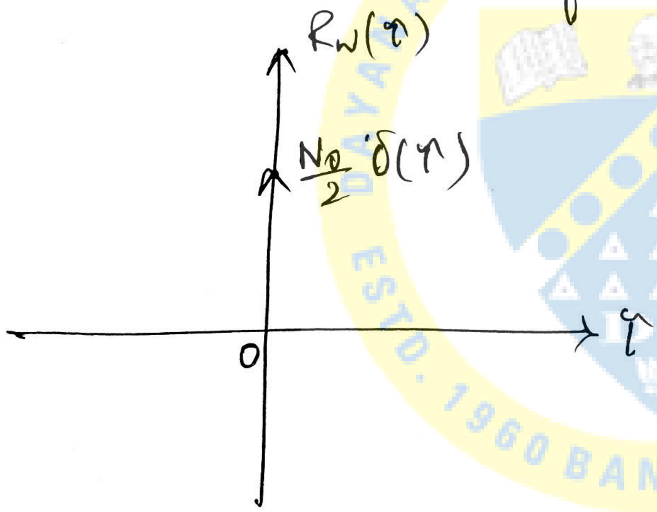
Here $R_W(\tau) = \int_{-\infty}^{\infty} e^{+j2\pi f\tau} \cdot S_W(f) \cdot df$

$$= \int_{-\infty}^{\infty} \frac{N_0}{2} e^{+j2\pi f\tau} \cdot df$$

$$= \frac{N_0}{2} \delta(\tau)$$

The autocorr. of white noise

→ It is a delta function occurring at $\tau = 0$



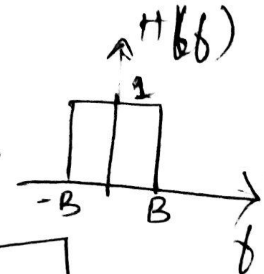
Filtered white noise

→ If I/P of filter is white noise, then o/p will not be the same white noise

→ Because the PSD is limited to filter bandwidth after passing it through a filter.

* Zero mean noise → does not present a net disturbance to the system.

→ Lets consider LPF with input as white gaussian noise, with zero mean & $PSD = \frac{N_0}{2}$ & Bandwidth of LPF → B Hz



→ The transfer function of ideal LPF is

$$H(f) = \begin{cases} 1 ; & -B < f < B \\ 0 ; & |f| > B \end{cases}$$

→ PSD at the o/p of LPF is

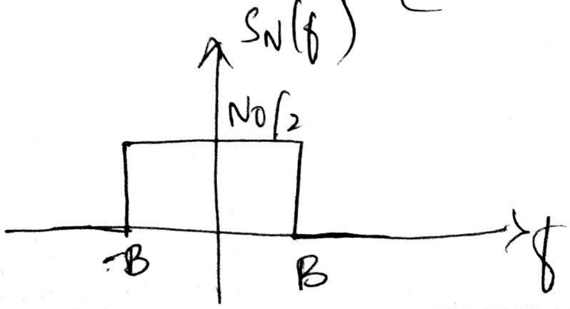
$$S_N(f) = |H(f)|^2 \cdot S_W(f)$$

$$= \begin{cases} N_0/2 ; & -B < f < B \\ 0 ; & |f| > B \end{cases}$$

$$P_N = S_N(f) \int_{-\infty}^{\infty} |H(f)|^2 \cdot df$$

$H(f) = 1$, $S_N(f) = \frac{N_0}{2}$

$$\boxed{P_N = N_0 B}$$



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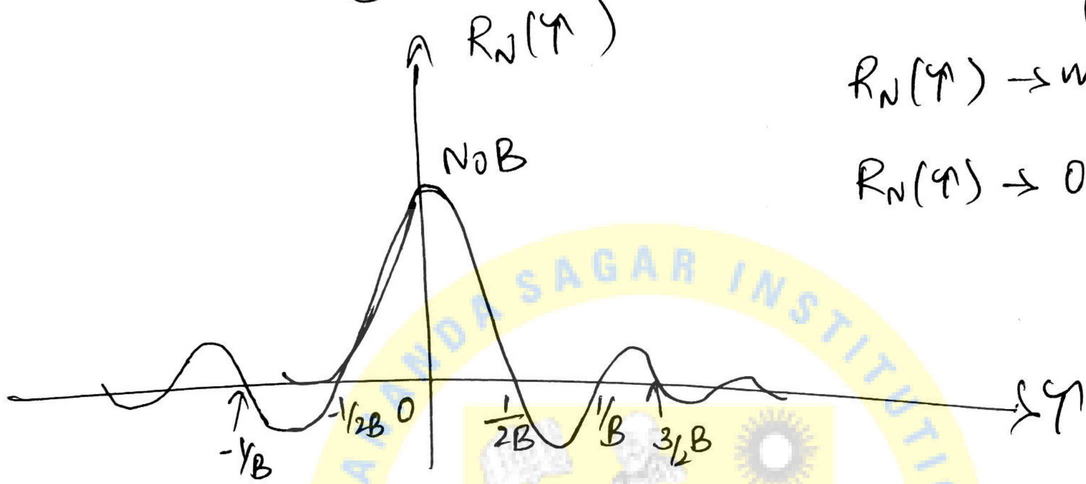
→ The autocorrelation function

$$R_N(\tau) = \int_{-\infty}^{\infty} S_N(f)\, e^{j2\pi f\tau}\cdot df$$

$$= \int_{-B}^{B} \frac{N_0}{2} e^{j2\pi f\tau}\cdot df = N_0 B \,\text{sinc}(2B\tau)$$



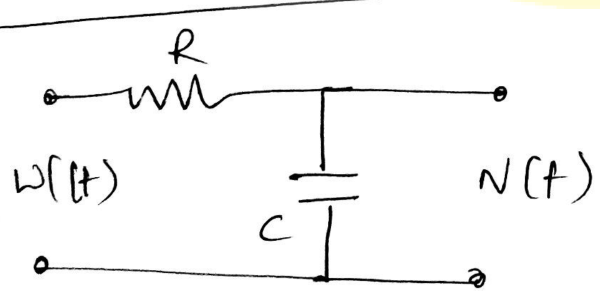
N_0B

$R_N(\tau) \rightarrow$ max. value at $\tau = 0$,

$R_N(\tau) \rightarrow 0,\ \tau = \pm \frac{n}{2B}$

→ It is observed that filter input was white guassian noise but filtered O/P is guassian but not a noise & is bandlimited to B Hz.

<u>RC low pass filtered noise</u>



white Guassian noise → Zero mean & PSD = $\frac{N_0}{2}$

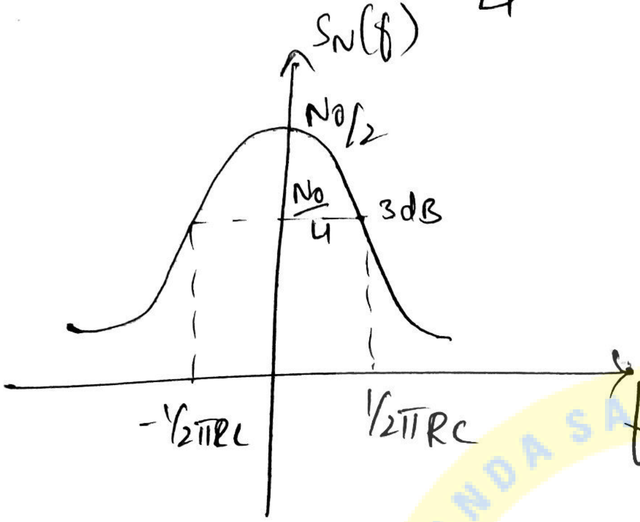
Freq. response of filter

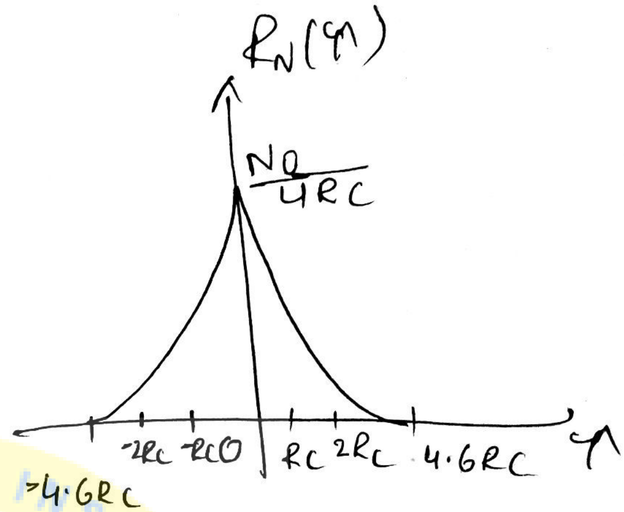
$$H(f) = \frac{1}{1+j2\pi fRC}$$

Cut off freq. of filter is $f_c = \frac{1}{2\pi RC}$

$$S_N(f) = \frac{N_0 a}{4} \cdot \frac{2a}{a^2 + (2\pi f)^2} \quad \text{where } a = \frac{1}{RC}$$

$$R_N(\tau) = \frac{N_0 a}{4} e^{-a|\tau|}$$





Autocorrelation function.

PSD of filter o/p

$$S_N(f) = \frac{N_0}{2}, \quad H(f) = \frac{1}{1 + j2\pi f RC}, \quad P_N = \frac{N_0}{4RC}$$

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Noise Equivalent Bandwidth

→ We observed that white noise with $PSD = \frac{N_0}{2}$ and zero mean, the o/p of ideal LPF $[H(f) = 1 \text{ \& } B\,Hz]$
The average o/p noise power $P_N = N_0 B$

→ Also for RC LPF $P_N = N_0/4RC$. $\left[BW = \frac{1}{2\pi RC}\right]$

→ This implies that,
Average noise power is proportional to bandwidth
∴ Lets generalize this statement to all kinds of LPF.

∴ General eqn $S_N(f) = S_W(f)\,|H(f)|^2$

$$= \frac{N_0}{2}\,|H(f)|^2$$

Average noise power $P_N = \int_{-\infty}^{\infty} S_W(f)|H(f)|^2 df$

$$P_N = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 \cdot df$$

$|H(f)|^2$ is an even function of freq.

$$P_N = \frac{N_0}{\cancel{2}} \cdot \cancel{2}\int_{0}^{\infty} |H(f)|^2 \cdot df$$

$$P_N = N_0 \int_{0}^{\infty} |H(f)|^2 \cdot df$$

Lets consider that this ~~no~~ white noise is connected to an ideal LPF, ~~mean = 0~~, $H(0)$ and Bandwidth 'B' Hz.

$$P_N' = \cancel{N_0} \cdot \int_{-\infty}^{\infty} S_N(f) \cdot df$$

$$= \int_{-\infty}^{\infty} S_W(f) \cdot |H(f)|^2 \cdot df$$

$$|H(f)|^2 = \begin{cases} |H(0)|^2 & -B < f < B \\ 0 & f > B \end{cases}$$

$$P_N' = \int_{-B}^{B} \frac{N_0}{2} |H(0)|^2 \cdot df$$

$$= \frac{N_0}{\cancel{2}} |H(0)|^2 \cdot \cancel{2}B$$

$$= N_0 \cdot B |H(0)|^2$$

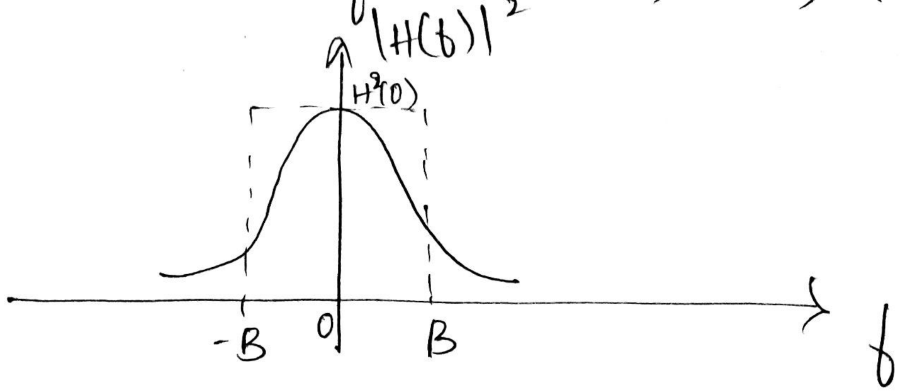
Equating P_N & P_N' we get

$$\cancel{N_0} \cdot B |H(0)|^2 = \cancel{N_0} \int_{0}^{\infty} |H(f)|^2 \cdot df$$

$$B = \frac{\int_{0}^{\infty} |H(f)|^2 \cdot df}{|H(0)|^2}$$

→ To calculate noise equivalent bandwidth,

* replace arbitrary LPF of $H(f)$ by an equivalent ideal LPF of ~~mean = 0~~, $H(0)$ & B Hz.



Determine noise equivalent BW of a RC LPF

$$H(f) = \frac{1}{1+j2\pi fRC}, \quad |H(f)|^2 = \frac{1}{1+(2\pi fRC)^2}$$

$$B_{noise} = \frac{\int_0^{\infty} |H(f)|^2 \cdot df}{1^2} = \int_0^{\infty} \frac{1}{1+(2\pi fRC)^2} \cdot df$$

Let $t = 2\pi RCf$ $\qquad df = \frac{dt}{2\pi RC}$

$dt = 2\pi RC \cdot df$,

$$B_0 = \int_0^{\infty} \frac{1}{1+t^2} \cdot \frac{1}{2\pi RC} \cdot dt$$

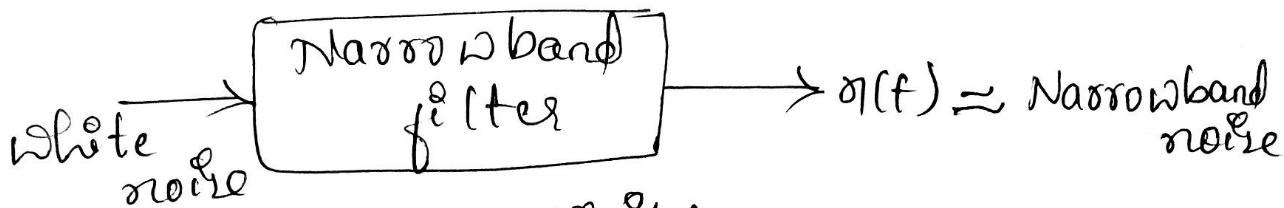
$$= \frac{1}{2\pi RC} \int_0^{\infty} \frac{1}{1+t^2} \cdot dt = \frac{1}{2\pi RC} \left[\tan^{-1} t\right]_0^{\infty}$$

$$= \frac{1}{2\cancel{\pi} RC} \cdot \frac{\cancel{\pi}}{2} = \frac{1}{4RC}$$

Narrow Band noise

→ An ideal lowpass filter at the receiver is called as narrow band filter

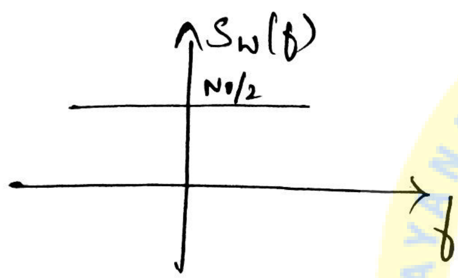
→

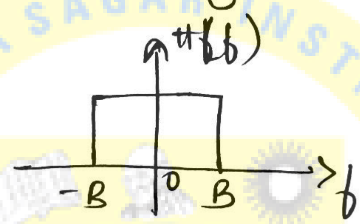


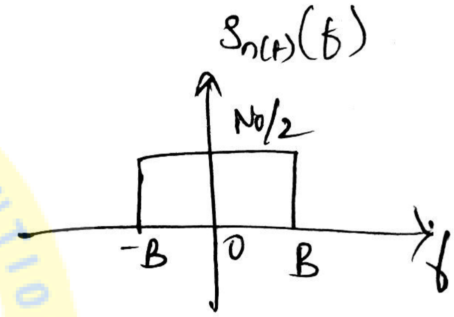
$S_w(f) = \frac{N_0}{2}$

Band limited

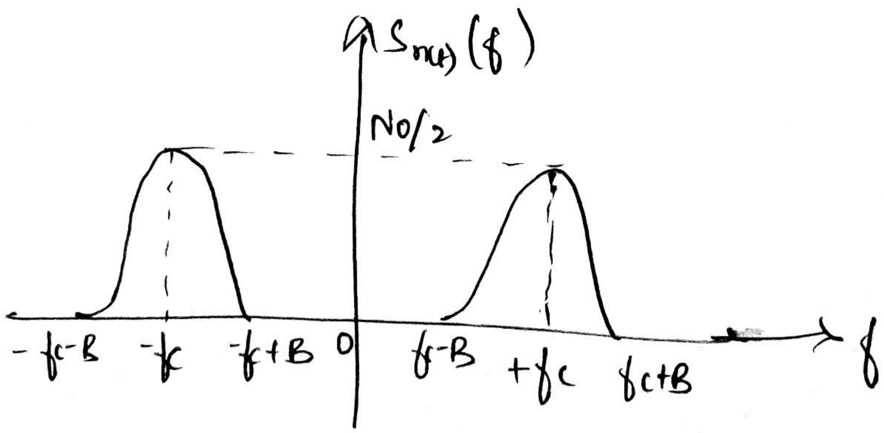
BW = B Hz



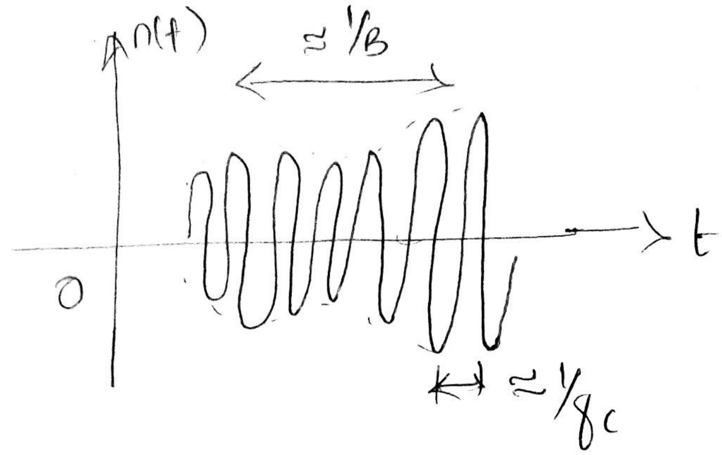




→ Practically, after modulation and other processes, the narrow band noise spectrum is centered at the mid-band freq. $\pm f_c$.



The o/p noise $n(t)$ appears as a sinusoidal waveform where its ampd & phase varies randomly.



The PSD of above noise $n(t)$ is after passing through the filter

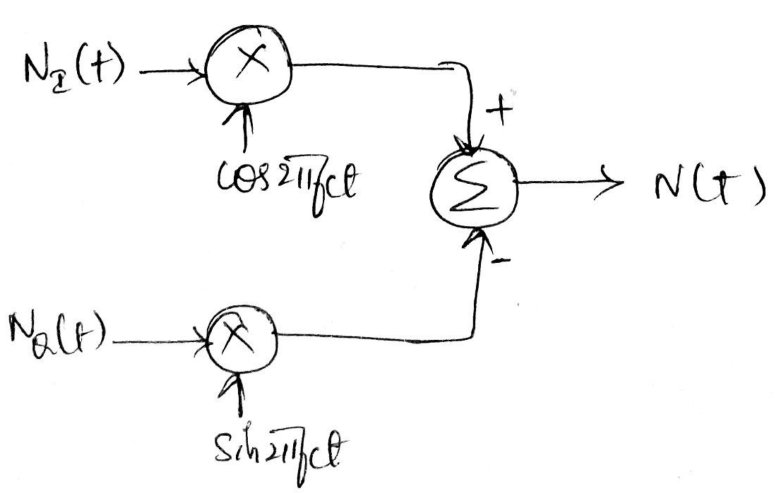
$$S_N(f) = |H(f)|^2$$

→ let us represent $n(t)$ in terms of inphase and quadrature components.

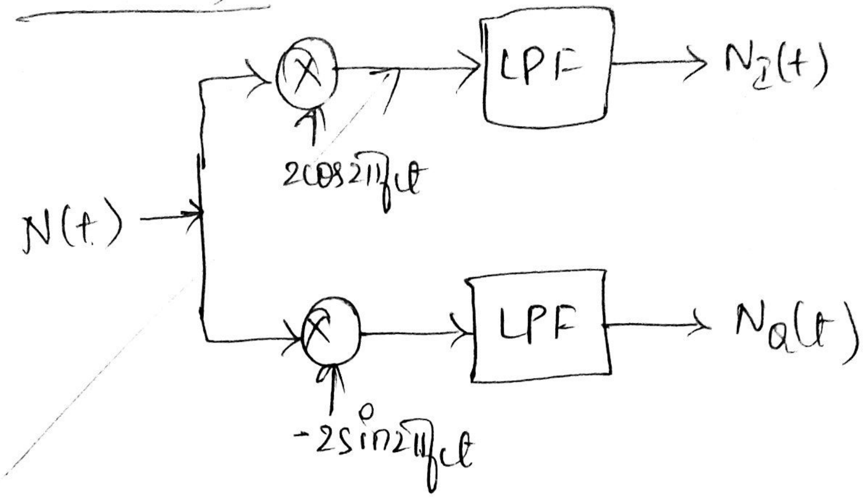
$$N(t) = N_I(t)\cos 2\pi f_c t - N_Q(t)\sin 2\pi f_c t$$

(BW = 2B centered at f_c)

→ Generation of NB noise



Extraction



$$N(t)\cdot 2\cos 2\pi f_c t = 2\left[N_I(t)\cos 2\pi f_c t - N_Q(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t$$

$$= 2N_I(t)\cos^2 2\pi f_c t - N_Q(t)\cos 2\pi f_c t\cdot\sin 2\pi f_c t$$

$$= \cancel{2}N_I(t)\,\frac{(1+\cos 4\pi f_c t)}{\cancel{2}} -$$

$$= N_I(t) + \underbrace{N_I(t)\cos 4\pi f_c t - \quad}_{\text{filter using LPF}}$$

$||^{ly}$ we get $N_Q(t)$.

Properties

-> $N_I(t)$ and $N_Q(t)$ have zero mean.

-> If $N(t)$ is guassian then $N_I(t)$ & $N_Q(t)$ are jointly guassian

-> If $N(t)$ is a wide-sense stationary then $N_I(t)$ & $N_Q(t)$ are also —||——

* **Wide sense stationary** -> a process mean & correlation functions do not change by shift in time.

→ Both $n_I(t)$ & $n_Q(t)$ have same PSDs, which is related to the PSD of original $n(t)$ as follows

$$S_{NI}(f) = S_{NQ}(f) = \begin{cases} S_N(f-f_c) + S_N(f+f_c) & -B \le f \le B \\ 0 & \text{elsewhere.} \end{cases}$$

→ $n_I(t)$ & $n_Q(t)$ have the same variance as $n(t)$.

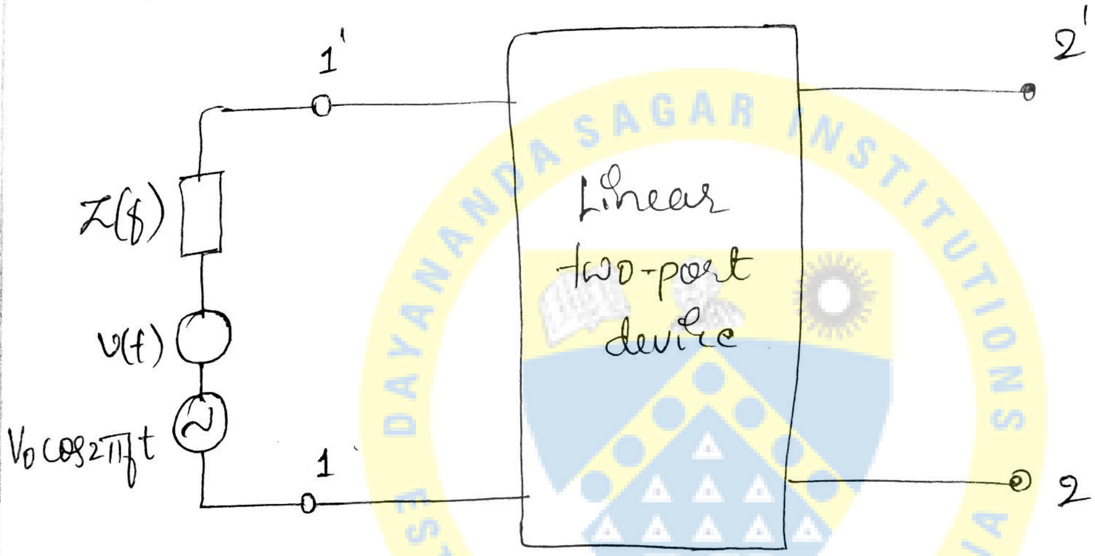
→ The cross PSD of $n_I(t)$ & $n_Q(t)$ components of narrowband noise is purely imaginary.

i.e. $$S_{nI}(f) = -S_{NQ}(f) = \begin{cases} j\,S_N(f+f_c) - S_N(f-f_c), & |f| \le B \\ 0 & |f| > B \end{cases}$$

Noise Figure

→ Noise Figure = $\dfrac{\text{Total o/p noise power due to the device \& source per unit bandwidth}}{\text{Available noise power due to source alone per unit bandwidth}}$

→ A convenient measure of the noise performance of a linear two-port device is by Noise figure.



→ Here internal impedance $Z(f) = R(f) + jX(f)$

→ $R(f)$ → internal resistance of the source gives a thermal noise voltage $v(t)$.

→ o/p noise has two components
- → One due to the source.
- → due to device itself.

→ Let $S_{N_o}(f)$ → power spectral density of the o/p noise power

$S_{N_s}(f)$ → PSD due to the source.

→ $G(f)$ → power gain of the two-port device

where $G(f) = \dfrac{\text{signal power at the output}}{\text{signal power at the source}}$

→ Noise figure of the device as

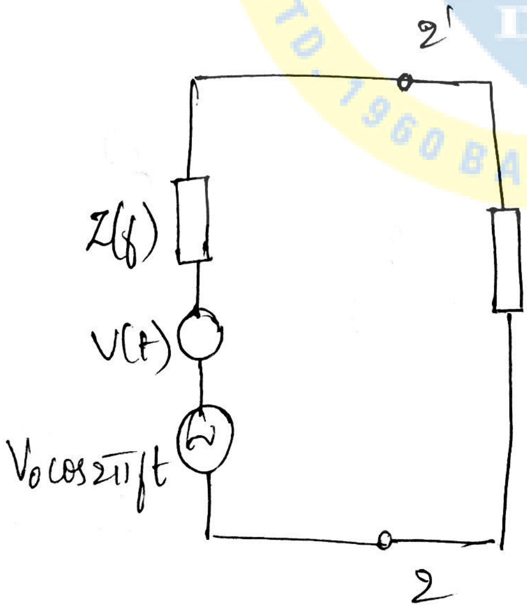
$$F(f) = \frac{S_{N_0}(f)}{S_{N_s}(f) \cdot G(f)}$$

→ If the two port device is noise free, → $F(f) = 1$

→ Noise figure is expressed in dB, $F(f)_{dB} = 10\log_{10} F(f)$

→ Max. power delivered by the source occurs when the load impedance connected to the source is the complex conjugate of the source impedance

$$Z_L(f) = Z^*(f) = R(f) - jX(f)$$



$$Z_L(f) = Z_{in}(f) = Z^*(f)$$

$$P_s(f) = \left(\frac{V_0}{2R(f)}\right)^2 \cdot R(f) = \frac{V_0^2}{4R_f}$$

* The available signal power at the output of the device is,

$$P_o(f) = G(f) \cdot P_s(f)$$

Therefore,

$$F(f) = \frac{S_{No}(f)}{G(f)\, S_{Ns}(f)} \cdot \frac{P_s(f) B_N}{P_s(f) \cdot B_N}$$

$$= \frac{P_s(f) \cdot S_{No}(f) \cdot B_N}{P_o(f) \cdot S_{Ns}(f) \cdot B_N} \begin{array}{l} \leftarrow 1/SNR_o(f) \\ \leftarrow SNR_s(f) \end{array}$$

$$= \frac{SNR_s(f)}{SNR_o(f)}$$

→ Signal to noise power ratio at the o/p is always less than the signal to noise power ratio at the input. This is because, the amplifier itself introduces the noise.

$$\therefore F(f) > 1.$$

→ If it is a noiseless amplifier, then

$$F(f) = 1.$$

$$\therefore \boxed{F(f) \geq 1}.$$

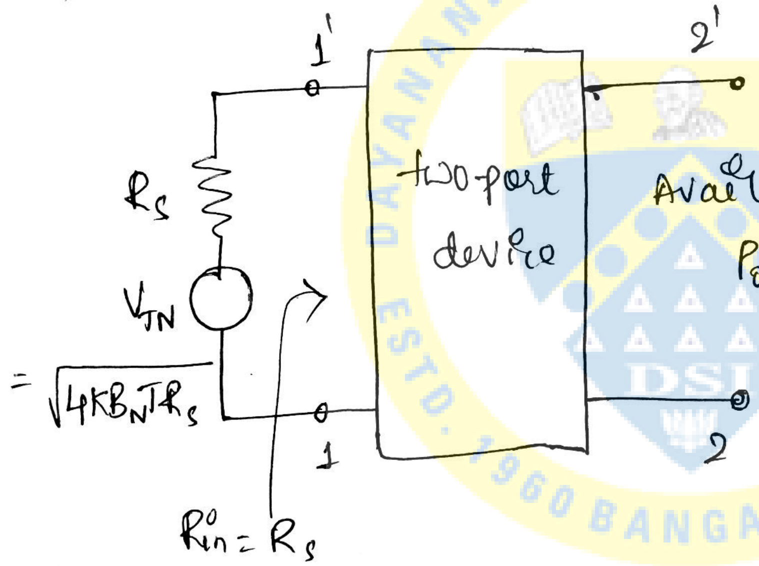
$$\therefore F = \frac{\int_{-\infty}^{\infty} S_{No}(f) \cdot df}{\int_{-\infty}^{\infty} G(f)\, S_{Ns}(f) \cdot df}$$

Equivalent noise temperature

→ If we use noise figure for comparing the noise performance of various devices it becomes difficult because, the values obtained are all close to unity. This is the major disadvantage.

→ It is advantageous to use equivalent noise temperature.

→ consider a linear two-port device as shown below



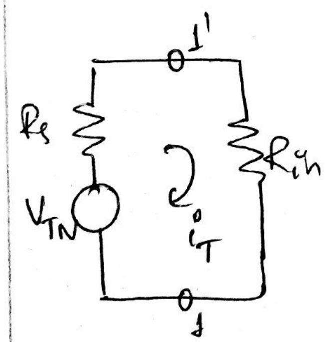
Available noise power

$P_{out} = GP_s + P_d$

P_s → Source noise power

P_d = device noise power

G = gain



$2R_s i_T = V_{TN}$,

$$i_T = \frac{V_{TN}}{2R_s}$$

∴ noise power at Source, $P_s = i_T^2 \cdot R_s = \dfrac{V_{TN}^2}{4R_s^2} \cdot R_s$

$= KB_N T$

→ noise power due to the two port device

$$P_d = kB_N T_e G.$$

→ ∴ The total o/p noise power

$$P_{out} = GP_s + P_d$$

$$= GkB_N(T+T_e)$$

$T_e \rightarrow$ noise equivalent temp. [illegible]

∴ noise figure $F = \dfrac{P_{out}}{GP_s}$

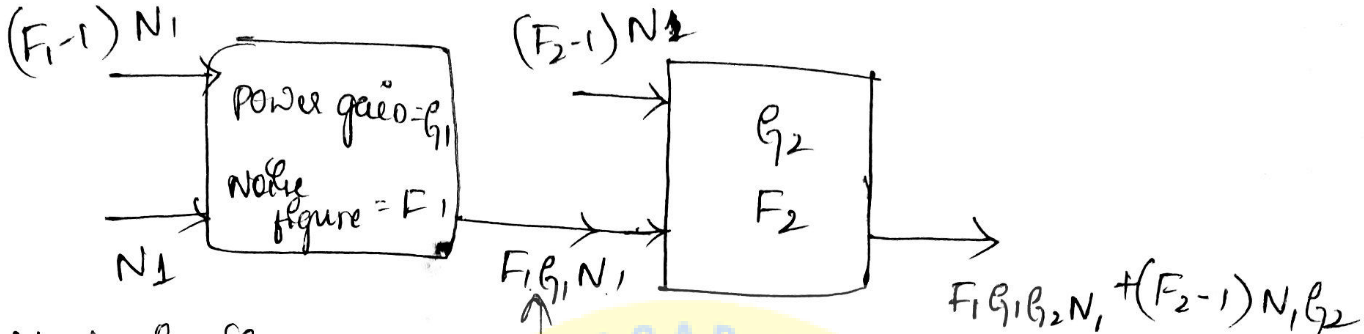
$$= \frac{GkB_N(T+T_e)}{GkB_N T}$$

$$F = 1 + \frac{T_e}{T}$$ or

$$\boxed{T_e = (F-1)T}$$

Cascade connection of two-port networks

→ cascading of two port networks whose individual noise figures are known.



N_1 → noise power by source

$(F_1-1)N_1$ → noise power by ~~device~~ network

→

$$= N_1G_1 + (F_1-1)N_1G_1$$

$$= \cancel{N_1G_1} + F_1N_1G_1 - \cancel{N_1G_1} = F_1N_1G_1$$

→ Noise figure $F = \dfrac{F_1G_1G_2N_1 + (F_2-1)N_1G_2}{F_1N_1G_1}$

$$\boxed{F = F_1 + \frac{F_2-1}{G_1}}$$

→ For any no. of two port n/w's

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \frac{F_4-1}{G_1G_2G_3} + \cdots$$

→ We can also express over all equivalent noise temperature of the cascade connection.

w.k.t $F = 1 + \frac{T_e}{T}$, $F_1 = 1 + \frac{T_1}{T}$, $F_2 = 1 + \frac{T_2}{T}$ - - -

→ substitute above eqn's in 'F' eqn.

$$\cancel{1} + \frac{T_e}{T} = \cancel{1} + \frac{T_1}{T} + \frac{\left(\cancel{1} + \frac{T_2}{T}\right) - \cancel{1}}{G_1} + \frac{\left(\cancel{1} + \frac{T_3}{T}\right) - \cancel{1}}{G_1 G_2} + \cdots$$

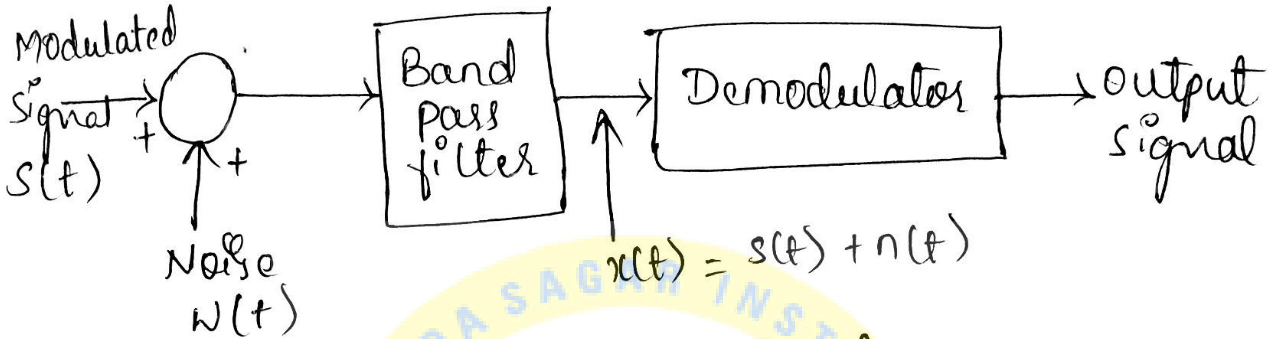
$$\frac{T_e}{\cancel{T}} = \frac{T_1}{\cancel{T}} + \frac{T_2}{\cancel{T} G_1} + \frac{T_3}{\cancel{T} G_1 G_2} + \cdots$$

$$\boxed{T_e = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \cdots}$$

The above equation is known as <u>Frii's formula</u>

→ To understand the analysis of noise in modulation systems, we need a receiver model.

→ Let us consider a simple receiver model as shown below.



$s(t)$ → incoming modulated signal

$w(t)$ → Front-end receiver noise

$n(t)$ → Filtered noise

→ Here there are three important parameters need to be concentrated

1) Input signal-to-noise ratio $(SNR)_I$

$$(SNR)_I = \frac{\text{Average power of the modulated signal } s(t)}{\text{Average power of the filtered noise } n(t)}$$

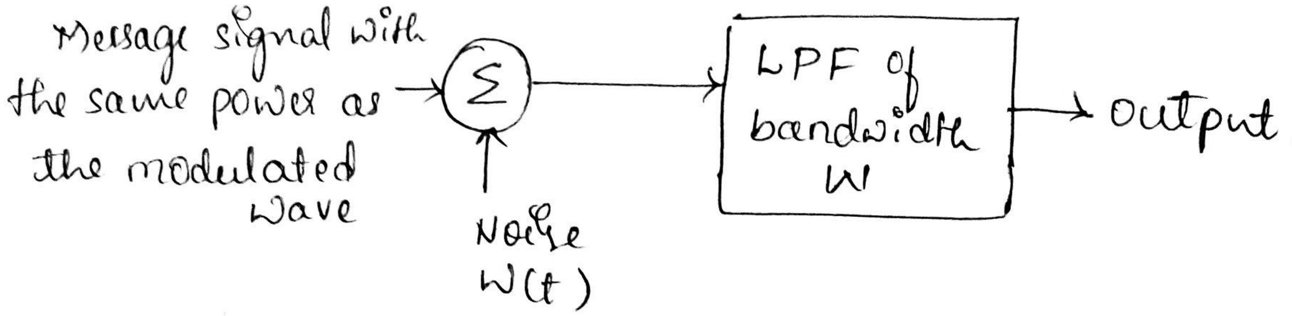
2) Output signal-to-noise ratio $(SNR)_o$

$$(SNR)_o = \frac{\text{Average power of the demodulated message signal}}{\text{Average power of the noise}}$$

3) Channel Signal-to-noise ratio $(SNR)_c$

$$(SNR)_c = \frac{\text{Average power of the modulated signal}}{\text{Average power of noise in the message Bandwidth}}$$

* $(SNR)_C$ may be viewed as the SNR that results from direct transmission of the message signal $m(t)$ without modulation, as shown below.



→ For the purpose of comparing different modulation schemes, we normalize the receiver performance by dividing the output signal to noise ratio by the channel signal to noise ratio. This ratio is called as <u>Figure of Merit</u> (FOM) (γ)

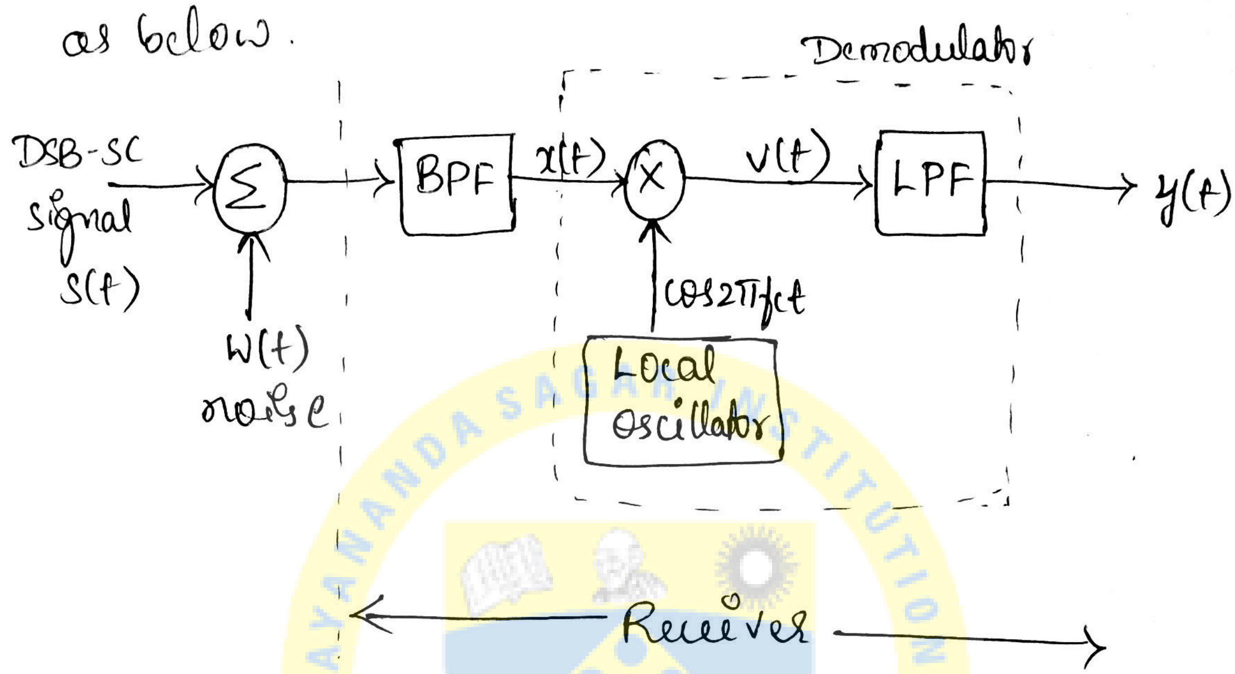
i.e. $$\text{Figure of Merit } (\gamma) = \frac{(SNR)_0}{(SNR)_C}$$

→ It is desired to have higher values of FOM. Its value depends on the modulation and on the type of detector used.

* $\gamma > 1$, Thus this range of γ is desirable
* $\gamma = 1$, Thus this range of γ is permissible
* $\gamma < 1$, Thus this range of γ is not permissible.

Figure of Merit for AM (DSB-SC)

→ Let us consider DSB-SC receiver with coherent detection scheme. The blockdiagram is depicted as below.



→ We know that s(t) of DSB-SC is,

$$s(t) = m(t) \cdot c(t) = m(t) A_c \cos 2\pi f_c t.$$

→ The noise at the input of the BPF is white and Gaussian in nature. The noise after passing throug the BPF is converted into band pass noise.

To find $(SNR)_c$

$$(SNR_o)_c = \frac{\text{Average power of } s(t)}{\text{Avg. power of noise in } m(t)}$$

Here Average power of $s(t) = \overline{s^2(t)}$ ← mean square value

$$= \overline{(m(t) A_c \cos 2\pi f_c t)^2}$$

$$= \overline{m^2(t)}\ \overline{(A_c \cos 2\pi f_c t)^2}$$

Here $\overline{m^2(t)}$ = Average power of $m(t) = P_m$

$$\overline{(A_c \cos 2\pi f_c t)^2} = \left(\frac{A_c}{\sqrt{2}}\right)^2 = \frac{A_c^2}{2}$$

$\therefore$ Average power of $s(t) = P_m \cdot \frac{A_c^2}{2}$

→ Average power of noise in $m(t)$ of message bandwidth is $= N_0 W$. N_0 → Average power per unit bandwidth in watts/Hz.

$$\therefore (SNR)_c = \frac{P_m \cdot A_c^2}{2} \Big/ N_0 W = \frac{P_m A_c^2}{2 N_0 W}$$

To find $(SNR)_0$

$$(SNR_0) = \frac{\text{Avg. power of demodulated signal}}{\text{Avg. power of the noise}}$$

→ The time domain expression for filtered narrowband noise w.r.t

$\longrightarrow n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t.$

→ WKT, in coherent detection, the signal at the detector input is multiplied by a synchronous carrier $\cos 2\pi f_c t$,

$$\therefore v(t) = x(t)\cos 2\pi f_c t$$

$$v(t) = [s(t) + n(t)] \cos 2\pi f_c t$$ (note: observe the block diagram)

$$= s(t) \cos 2\pi f_c t + n(t) \cos 2\pi f_c t$$

$$= \underbrace{m(t) A_c \cos 2\pi f_c t}\cdot \cos 2\pi f_c t +$$

$$(n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t) \cos 2\pi f_c t$$

$$= m(t) A_c \cos^2 2\pi f_c t + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \cdot \cos 2\pi f_c t$$

$$= m(t) A_c \frac{(1+\cos 4\pi f_c t)}{2} + n_I(t) \frac{(1+\cos 4\pi f_c t)}{2} -$$

$$n_Q(t) \left[\frac{1}{2}(\sin 0 + \sin 4\pi f_c t)\right]$$

$$= \frac{A_c m(t)}{2} + \frac{A_c}{2} m(t) \cos 4\pi f_c t + \frac{n_I(t)}{2} + \frac{n_I(t)}{2} \cos 4\pi f_c t$$

$$- \frac{n_Q(t)}{2} \sin 4\pi f_c t$$

→ The above signal is passed through LPF, the spectrum above W Hz is filtered.

$$\therefore y(t) = \underbrace{\frac{A_c}{2} m(t)}_{\substack{\text{desired signal} \\ m_d(t)}} + \underbrace{\frac{1}{2} n_I(t)}_{\substack{\text{noise component} \\ n_d(t)}}$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \frac{A_c^2}{4} \overline{m^2(t)} = \frac{A_c^2}{4} \cdot P_m$$

Average power $n_d(t) = \overline{n_d^2(t)}$

$$= \frac{1}{4}\overline{n_I^2(t)}$$

$$= \frac{1}{4} \times \text{Area under power spectral density}$$

$$= \frac{1}{4} \times N_0 2W = \frac{N_0 W}{2}$$

Hence, $(SNR)_0 = \dfrac{\text{Avg. power of } m_d(t)}{\text{Avg. power of } n_d(t)}$

$$= \frac{A_c^2 P_m}{2 N_0 W}$$

Therefore, Figure of Merit $= \dfrac{(SNR)_0}{(SNR)_c} = 1$

$$\boxed{\therefore \gamma = 1}$$

AM Threshold Effect

-> When the noise is large compared to the signal at the input of the detector, the detected output has a message signal completed ~~added~~ Mingled with noise.

-> It means that if the $(SNR)_I$ is below a certain level called threshold level.

-> This effect at the output of the detector is called as threshold effect.

-> As we know, the input to the Envelope detector is an additive of $s(t)$ and narrow band noise $n(t)$, but the detected message signal is multiplied by noise which is random.

-> The message signal is hence hopelessly mutilated and its information has been lost. The mutilation or loss of information at low $(SNR)_I$ is called as Threshold effect.

-> If the detector is synchronous detector, the output signal and noise are always additive. Even with $(SNR)_I << 1$, $(SNR)_o$ can be still defined and is proportional to $(SNR)_I$.

→ Since $(SNR)_o$ is always proportional to $(SNR)_I$ in synchronous detection (coherent), there is no threshold effect exists.

→ In envelope detector also $(SNR)_o$ is proportional to $(SNR)_I$, when $(SNR)_I$ is large.

→ However when $(SNR)_I$ is decreased below a value the $(SNR)_o$ decreases rapidly as shown below.

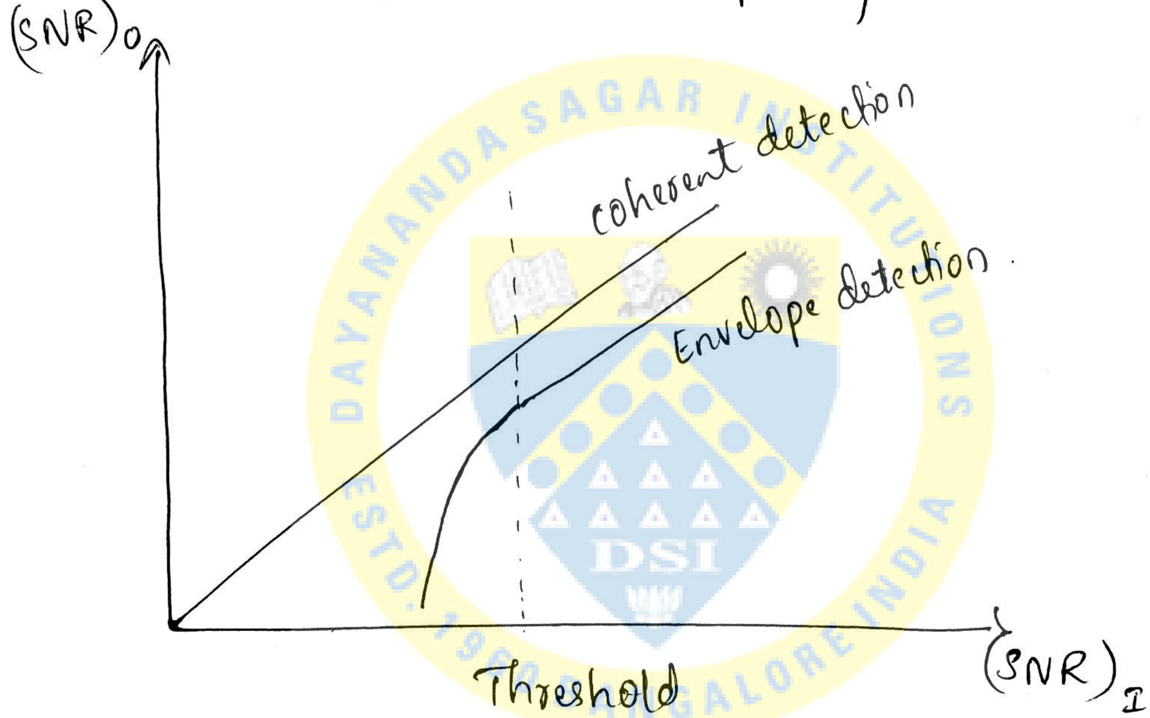
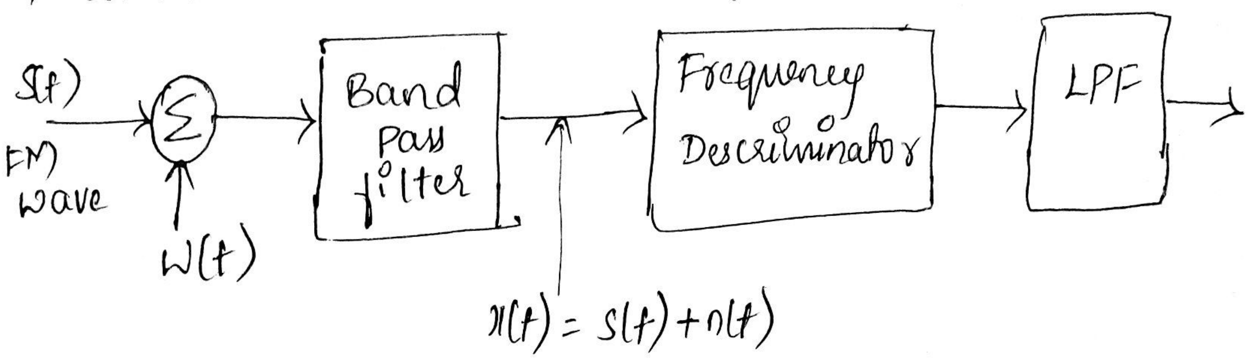


Figure of Merit for FM

→ Let us consider the FM system as shown below



→ Here WKt, $s(t) = A_c \cos(2\pi f_c t + \phi_c)$

$$= A_c \cos(\theta(t))$$

W.k.t $\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\cdot dt$

$$\therefore \phi_c = 2\pi k_f \int_0^t m(t)\cdot dt$$

$m(t)$ → message signal bandlimited to W Hz and

k_f → Freq. sensitivity.

$n(t)$ → channel noise at the input of frequency descriminator is a narrowband noise with PSD $S_N(f)$ & bandwidth $2(D+1)W$ Hz [from Carson's rule]

Wkt, Average power of $s(t) = \frac{A_c^2}{2}$

Average power of $n(t) = N_0 W$

$$\therefore (SNR)_c = \frac{A_c^2}{2N_0 W}$$

→ Let us consider the next stage

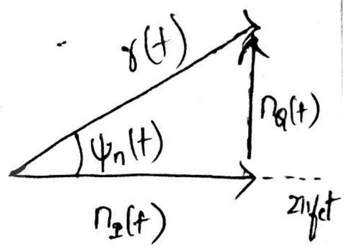
$x(t) = s(t) + n(t)$

$n(t) \rightarrow$ narrow band noise

wkt $n(t)$ can be represented as

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

→ Polar representation of $n(t)$ is given by

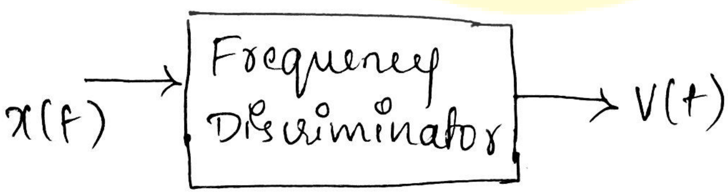


$$n(t) = \underset{\substack{\uparrow \\ \text{envelope}}}{r(t)} \cos[2\pi f_c t + \underset{\substack{\uparrow \\ \text{phase}}}{\psi_n(t)}]$$

where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$, $\psi_n(t) = \tan^{-1}\left[\frac{n_Q(t)}{n_I(t)}\right]$

$$\therefore\ x(t) = A_c\cos(2\pi f_c t + \phi_c) + r(t)\cos[2\pi f_c t + \psi_n(t)]$$

→ The next stage is frequency discriminator



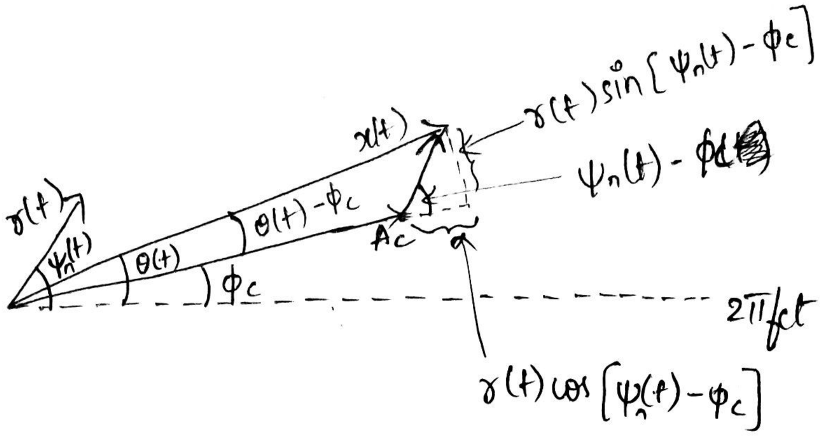
→ The frequency of $x(t)$ of the frequency discriminator,

$$f(t) = \frac{1}{2\pi}\frac{d[\theta(t)]}{dt}$$

→ From polar representation and phasor diagram, we get

$$\theta(t) = \phi_c + \frac{n_Q(t)}{A_c}$$

Phasor diagram for $x(t) = n(t) + s(t)$ is



From above diagram.

$$\tan[\theta(t) - \phi_c] = \frac{r(t)\sin[\psi_n(t) - \phi_c]}{A_c + r(t)\cos[\psi_n(t) - \phi_c]}$$

Here, $A_c > r(t)$ & $\psi_n(t) > \phi_c$

$$\therefore \tan[\theta(t) - \phi_c] \approx \frac{r(t)\sin\psi_n(t)}{A_c}$$

→ And from the phasor diagram of $n(t)$ we can write

$$n_Q(t) = r(t)\sin\psi_n(t)$$

$$\therefore \tan[\theta(t) - \phi_c] = \frac{n_Q(t)}{A_c}$$

Since $A_c >> n_Q(t)$, we can write

$$\theta(t) - \phi_c \approx \frac{n_Q(t)}{A_c}$$

$$\therefore \theta(t) = \frac{n_Q(t)}{A_c} + \phi_c$$

$$\therefore f(t) = \frac{1}{2\pi}\frac{d}{dt}\left[\phi_c + \frac{1}{A_c} n_Q(t)\right]$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f \int_0^t m(t)\,dt\right] + \frac{1}{2\pi A_c}\frac{d}{dt} n_Q(t)$$

→ The output of frequency discriminator $V(t)$ is given by

$$V(t) \simeq f(t)$$

$$= \underbrace{k_f m(t)}_{\text{desired signal } m_d(t)} + \underbrace{\frac{1}{2\pi A_c}\frac{d}{dt} n_Q(t)}_{\text{noise component } n_d(t)}$$

→ Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \overline{\left[k_f m(t)\right]^2} = k_f^2 P_m$$

→ Taking fourier transform of $n_d(t)$, we get

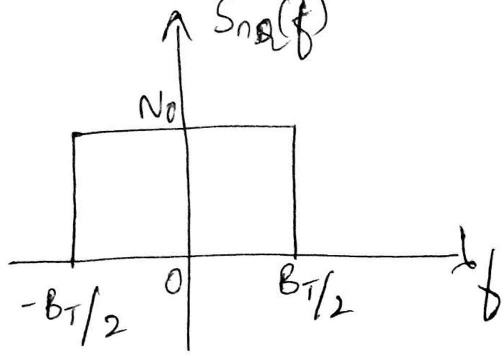
$$N_d(f) = \frac{1}{2\pi A_c} j2\pi f\, N_Q(f)$$

$$\Rightarrow |N_d(f)|^2 = \frac{f^2}{A_c^2}|N_Q(f)|^2$$

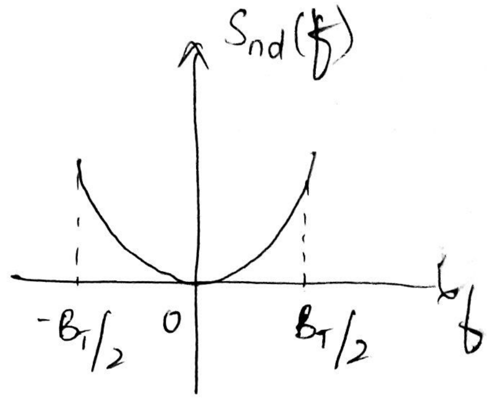
$$\Rightarrow S_{nd}(f) = \frac{f^2}{A_c^2} S_{nQ}^2(f)$$

$$= \begin{cases} \frac{f^2}{A_c^2} N_0 & |f| \le B_T \\ 0, & \text{otherwise} \end{cases}$$

where $N_0 \rightarrow$ PSD of $n_Q(t)$ over $|f| \leq B_T/2$.



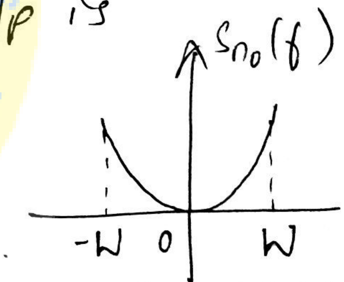
PSD of $n_Q(t)$



PSD of $n_d(t)$

-> The next stage is LPF with a bandwidth equal to W. For a wideband FM, since $W < B_T/2$, the out-of-band components of $n_d(t)$ will be rejected by LPF. Hence the PSD $S_{n_0}(f)$ of the noise $n_0(t)$ appearing at the receiver o/p is

$$S_{n_0}(f) = \begin{cases} \dfrac{f^2 N_0}{A_c^2} & |f| \leq W \\ 0 & \text{otherwise.} \end{cases}$$



PSD of $n_0(t)$

-> The average output noise power of $n_0(t)$ is

$$\overline{n_0^2(t)} = \frac{N_0}{A_c^2} \int_{-W}^{W} f^2 \cdot df = \frac{2 N_0 W^3}{3 A_c^2}$$

$$\therefore (SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_0^2(t)}} = \frac{k_f^2 P_m}{\frac{2 N_0 W^3}{3 A_c^2}} = \frac{3 A_c^2 k_f^2 P_m}{2 N_0 W^3}$$

Therefore, FOM of FM receiver is

$$\gamma = \frac{(SNR)_o}{(SNR)_c}$$

$$= \frac{3k_f^2 P_m}{W^2}$$

→ let m(t) be a single-tone modulating signal

$$m(t) = A_m \cos 2\pi f_m t$$

→ The average power of m(t) is

$$P_m = \overline{m^2(t)} = \left[\frac{A_m}{\sqrt{2}}\right]^2 = \frac{A_m^2}{2}$$

Hence $$\gamma = \frac{3k_f^2 \frac{A_m^2}{2}}{f_m^2} = \frac{3k_f^2 A_m^2}{2f_m^2} = \frac{3}{2}\left[\frac{k_f \cdot A_m}{f_m}\right]^2$$

$$= 1.5\,\beta^2$$

where $\beta = \frac{\Delta f}{f_m} = \frac{k_f \cdot A_m}{f_m}$ → modulation index.

Threshold effect in FM

→ CNR → carrier to noise ratio,

SNR ⇒ Signal to noise ratio of modulated signal

$$CNR = \frac{\text{Average power of FM signal } s(t)}{\text{noise power in the transmission BW}}$$

→ WKT, $(SNR)_0 = \dfrac{3A_c^2 k_f^2 P_m}{2N_0 W^3}$

This above equation is valid only if CNR >> 1

→ As the input noise increases, CNR decreases, the discriminator output becomes more and more corrupted by noise.

→ Initially, occasional click / cracking / spluttering sounds will be heard in FM receiver, if CNR further decreases, continous clicks are heard.

→ Near the breaking point, $(SNR)_0$ will be less than $3A_c^2 k_f^2 P_m / 2N_0 W^3$. This phenomenon is called as "threshold effect."

Threshold = Min. CNR that yields an actual $(SNR)_0$

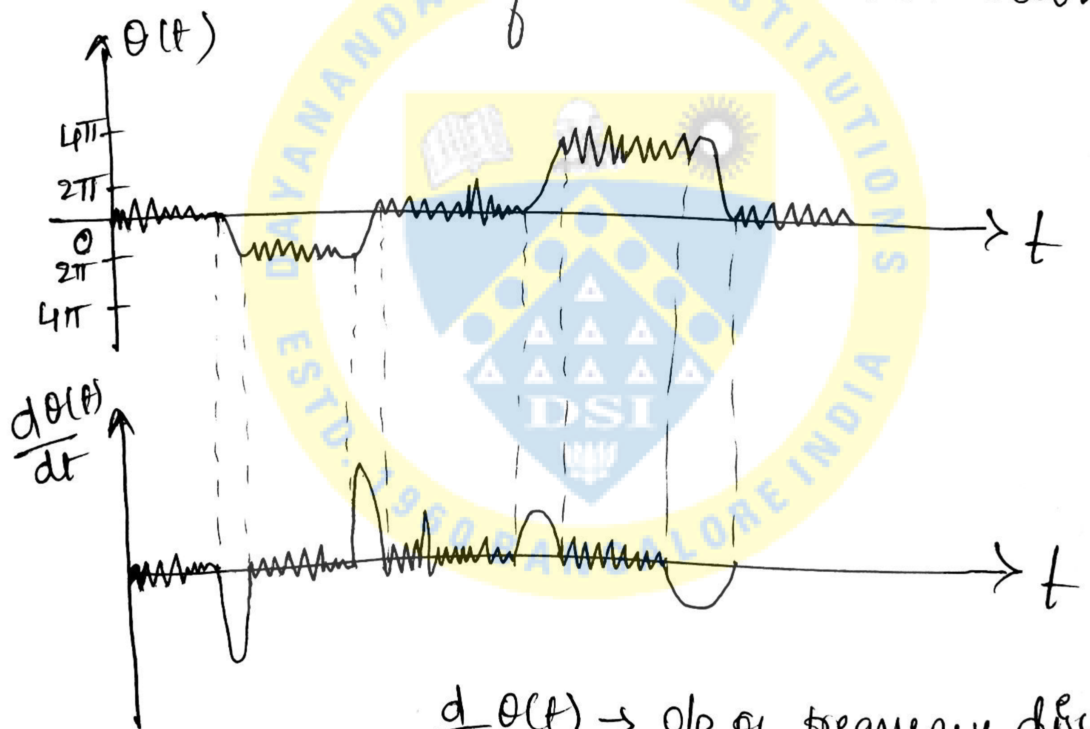
→ While deriving $(SNR)_0$ eqn, the noise power is assumed to be very small compared to signal (carrier) power.

→ consider an unmodulated FM wave corrupted by noise, then the input to FM discriminator is

$$x(t) = s(t) + n(t)$$
$$= A_c \cos 2\pi f_c t + r(t) \cos\left[2\pi f_c t + \psi_n(t)\right]$$

$$\therefore \theta(t) \simeq \frac{n_Q(t)}{A_c}$$

→ Under large noise condition, CNR << 1, the random variations of θ(t) are shown below.



$\frac{d}{dt}\theta(t)$ → o/p of frequency discriminator.

→ The impulse like components produce click sounds at the o/p receiver. Clicks are produced only when θ(t) changes by ±2π radians. This happens when CNR<<1.

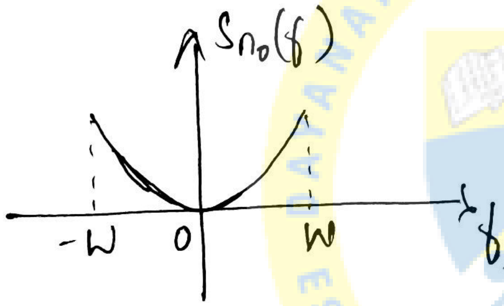
→ A positive click occurs when θ(t) changes by +2π rad. & a negative click occurs when θ(t) changes of -2π rad.

→ As CNR decreases, the average no. of clicks per unit time increases and the threshold is said to occur.

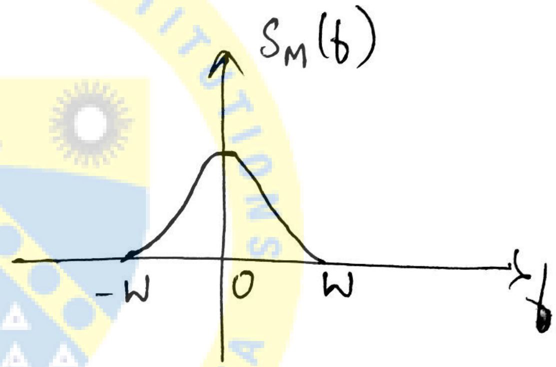
→ The threshold effect in an FM receiver may be avoided by having CNR = 13dB.

Pre-emphasis and De-emphasis in FM

→ The PSD of the noise at the output of the frequency discriminator $S_{n_0}(f)$ is ~~proportional~~ given below.



PSD of $S_{n_0}(f)$



PSD of message signal.

→ Observe the PSD of noise [$S_{n_0}(f)$], noise power is more at high frequencies, where as the message signal power is much lower at low frequencies [observe PSD of message ($S_M(f)$)]

→ Thus the high frequency components of the message signal are affected most by noise.

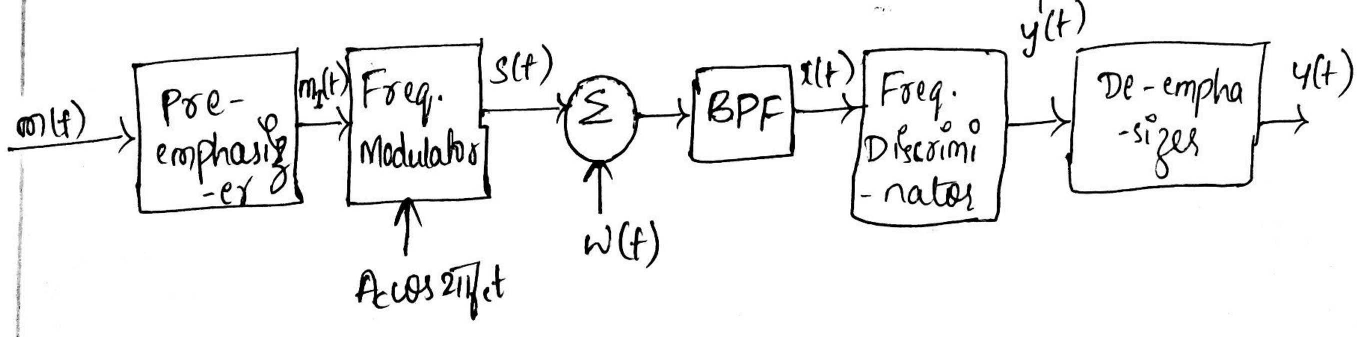
[eg: To enjoy high quality music (high freq. - 20KHz) there high freq. note should be present in received audio signal of FM radio]

→ To overcome this problem, SNR at receiver end should be high or should be maintained at a particular value. This can be achieved by pre-emphasis and De-emphasis.

→ At the transmitter side, high frequency components of the message signal m(t) are boosted using a filter called pre-emphasis. Hence the amplitudes of low & high freq. components of m(t) will be almost equal.

→ At the receiver side, while reproducing the m(t) at detector, the amplitude distribution of m(t) is disturbed by pre-emphasis process. Therefore an inverse operation is required to bring back the amplitude level of high freq. components to original level. This is done by the method called de-emphasis.

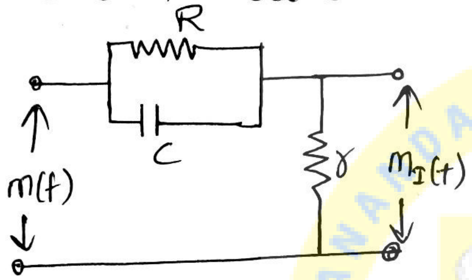
→ A typical FM system with pre-emphasis & de-emphasis filter is shown in figure below.



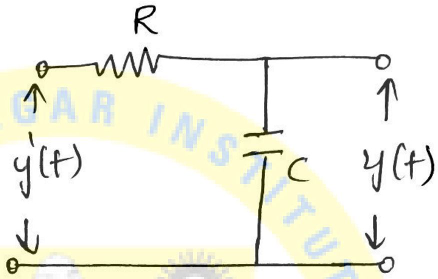
→ The transfer functions of Pre-emphasis and de-emphasis filters must have an inverse relationship. Let $H_p(f)$ is transfer function of pre-emphasis & $H_d(f)$ is for de-emphasis. Then,

$$H_d(f) = \frac{1}{H_p(f)}, \quad -W \le f \le W$$

→ A simple pre-emphasis and de-emphasis filters are shown below.



Pre-emphasis filter



de-emphasis filter.

$$H_p(f) = \frac{r}{R}\left[1 + \frac{jf}{f_1}\right] \quad H_d(f) = \frac{1}{1 + j(f/f_1)}, \quad f_1 = \frac{1}{2\pi RC}$$

→ $H_p(f)$ and $H_d(f)$ must satisfy the condition $H_p(f) \cdot H_d(f) = \frac{r}{R}$

→ In commercial FM broadcasting, $f_1 = 2.1$ kHz for a message signal BW, $W = 15$ KHz, improved SNR is about 7dB

→ Here $\int_{-\infty}^{\infty} S_M(f) \cdot df = \int_{-\infty}^{\infty} S_M(f) \cdot |H_p(f)|^2 \cdot df$

- i.e the average power of the emphasized signal will be same as that of original message signal m(t).

→ Next in the FM system, the signal will be frequency modulated with $A_c \cos 2\pi f_c t$ to get $s(t)$. There will be an addition of noise $w(t)$ to $s(t)$. This signal will be passed through the BPF, we get $x(t) = s(t) + n(t)$.

→ At the output of freq. discriminator,

$$y'(t) = m_d'(t) + n_d'(t)$$

$$= k_f m_d(t) + \frac{1}{2\pi A_c}\frac{d}{dt} n_Q(t)$$

And Average noise power of $n_d'(t)$ we get

$$\overline{n_d'(t)^2} = \frac{2N_0 W^3}{3A_c^2}$$

$$\cancel{\overline{n_d(t)^2} = \int_{-W}^{W} \frac{f^2 N_0}{A_c^2} \cdot |H_d(f)|^2 \cdot df}$$

refer derivation of FOM for FM for detailed derivation.

~~Thus the improvement factor~~

$$\cancel{I = \frac{\text{Average power of } n_d'(t)}{\text{Average}}}$$

→ At the output of de-emphasis filter,

$$y(t) = m_d(t) + n_d(t)$$

$$\text{PSD} = \begin{cases} \frac{f^2 N_0}{A_c^2} \\ 0 \end{cases} \longrightarrow \boxed{\text{De-emphasis filter}} \;(H_d(f)) \longrightarrow \text{modified PSD} = |H_d(f)|^2 \cdot \text{PSD}$$

$$\overline{n_d(t)^2} = \int_{-W}^{W} \frac{N_0 f^2}{A_c^2} \cdot (H_d(f))^2 \cdot df.$$

Thus the improvement factor is

$$I = \frac{\text{Avg. power of } n_d'(t) \leftarrow \text{without pre-emphasis \& de-emp.}}{\text{Avg. power of } n_d(t) \leftarrow \text{with pre \& de-emphasis}}$$

$$= \frac{\dfrac{2N_0W^3}{3A_c^2}}{\dfrac{N_0}{A_c^2}\displaystyle\int_{-W}^{W} f^2 |H_d(f)|^2 \cdot df} = \frac{2W^3}{3\displaystyle\int_{-W}^{W} f^2 |H_d(f)|^2 \cdot df}$$

→ Since $|H_d(f)| < 1$, we ~~gent a~~ get I greater than 1. Therefore higher the value of I, better the noise performance of the FM receiver.