Controll systems

→ System

A system is a combination of different physical components which act together as an entire unit to achieve certain objective

ex: Classroom, kite, lamp, ~~etc~~ bike
bench, black board ... lecturer | child | ON & OFF switch

physical system.

lamp

→ Controll

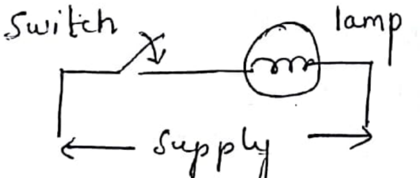
It means to regulate, direct or command a system so that the desired objective is attained

→ Controll System (CS)

It is arrangement of different physical components connected in such a manner so as to regulate, direct or command to attain a certain objective.

ex: In classroom, professor is delivering his lecture combination of CS, he tries to regulate, command the students to achieve objective which is to give good knowledge to student.

lamp → ON & OFF use like switch



controll system

Requirements of good controll systems

(1) **Accuracy**

Accuracy is very high as any error arising should be corrected. Accuracy is improved by using feedback element.

(2) Sensitivity,

A controll system has to sense the change in the output due to environmental or any other parameters and correct the same

(3) Speed, :

should be very high

(4) stability,

Stability means bounded → well defined Input and bounded output. A good controll system response is stable in all Variations..

(5) Noise,

A good controll system should be Insensitive to noise.

(6) Bandwidth,

for frequency response of good controll system bandwidth should be large.

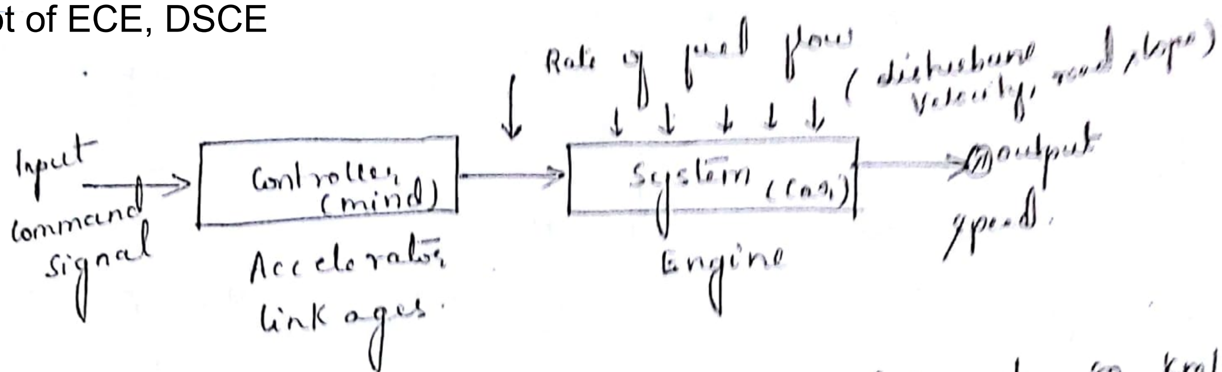
AS^3NB

→ Classification of Controll system

(1) open loop controll system.
(2) closed loop controll system

→ open loop controll system

A system in which the control action is totally Independent of the output of the system is called open loop system.



ex: driver wants to drive the car at 80 km/hr. To achieve the desired speed, he applies required pressure on the accelerator pedal & the car starts moving at desired speed.

But after sometimes due to the disturbances like wind velocity, road slope, the speed of car deviates from desired speed. The car does not run at the desired speed even though there is no change in pressure applied to accelerator pedal.

The distinguished characteristic of an open loop system is that it cannot compensate or take corrective action for any disturbances that affect the system performance.

ex: (1) Automatic washing machine

(2) Traffic light controller

(3) Automatic door opening and closing system

(4) Fan Regulator

(5) Electric lift

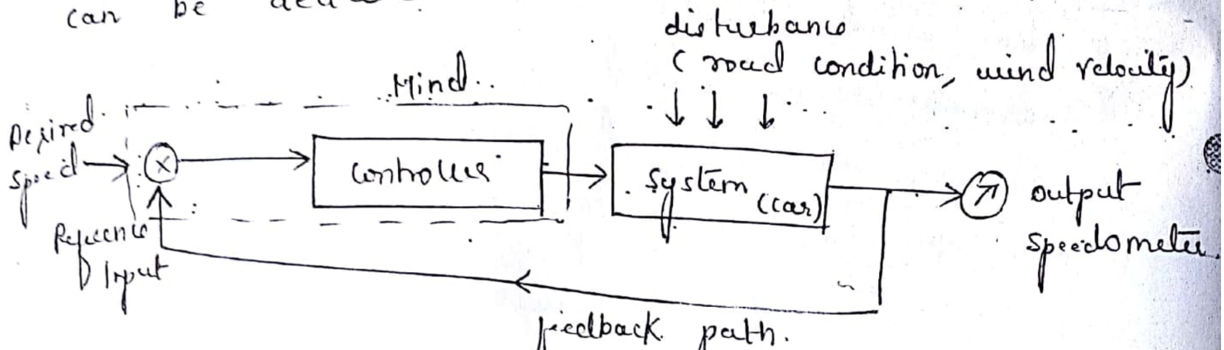
(6) Sprinkler used to water a lawn

→ **Closed loop controll System (feedback)**

A system in which the controlling action or Input is somehow dependent on the output or changes in output is called closed loop system.

Feedback

Is a property of the system by which it permits the output to be compared with the reference Input so that appropriate controlling action can be decided.



ex: Driving of a car at a desired speed is another example for closed loop controll. here the driver compares the speed of the car with desired speed (80 km/hr). If he finds any deviation in speed from the desired speed due to some disturbances then he may be Increase or decrease the speed by Increasing or decreasing the pressure on the accelerator pedal so that the deviation becomes Zero.

In this case pressure applied on the accelerator pedal is the controller output.

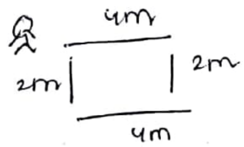
ex: Automatic electric Iron
Voltage stabilizers
D.C Motor speed controll

→ Comparision of open loop and closed loop C.S

	Open loop system		Closed loop system
(1)	No feedback	(1)	Feedback exists
(2)	In accurate	(2)	Accurate
(3)	No error detector	(3)	Error detector present
(4)	Highly sensitive to disturbances	(4)	less sensitive to disturbances
(5)	Economical	(5)	Costly. (Complicated design)
(6)	Small bandwidth	(6)	Large bandwith
(7)	stable	(7)	Stability is major consideration while designing.
(8)	Highly affected by noise	(8)	less noise

<u>Displacement</u> ($x(t)$)

Is a Vector quantity that refers to it is objects overall change in position.



distance = 12m

displacement = 0m

<u>Velocity</u> (v): Is a Vector physical quantity having both magnitude and direction.

" the rate at which an object changes its position.

<u>Acceleration</u>

Is the rate of change of Velocity with time

$$a = \frac{\Delta v}{\Delta t}$$

<u>force</u>,

A force is any Interaction which tends to change the motion of an object.

Kinetic energy $K.E = \frac{1}{2} m v^2$ — m = mass of object, v = speed of object.

Is the energy of motion, object that has motion, Vibrational, rotational, translat
Its scalar quantity.

Inductance (Magnetic field)

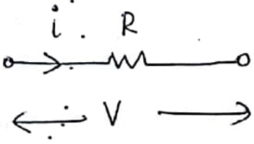
Is the property of a conductor by which
V) a change in current flowing through it induces
(Creates) a Voltage in both the conductor itself.

Capacitor (electrical field)

Is a passive 2 terminal electrical component
(I) used to store energy electrostatically in an
electric field.

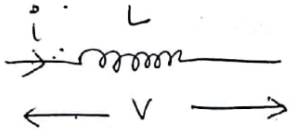
Electrical Systems

(a) Resistor



$V = iR$ or
$i = V/R$

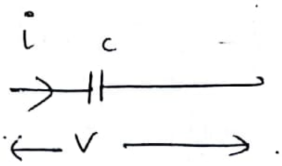
(b) Inductor



$V = L \frac{di}{dt}$ or

$i = \frac{1}{L} \int_0^t V.dt$.

(c) Capacitor



$V = \frac{1}{c} \int_0^t i.dt$. or

$i = c \cdot \frac{dv}{dt}$.

UNIT 1 : Modelling of Mechanical Systems

There are 2 types of mechanical systems they are

(1) Translational system

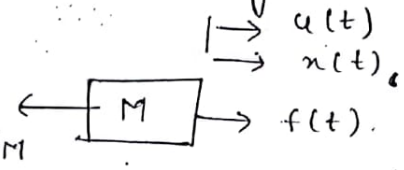
(2) Rotational system

(1) Translational system (Motion along straight line)

The basic elements of translational systems are

(1) Mass (2) Spring (3) Dashpot.

(1) Mass



F_M is the counter force or opposing force produced by the mass and its proportional to acceleration of the mass.

$$F_M \propto a$$

$$F_M = M \times a = M \cdot \frac{du(t)}{dt}$$

but $u(t) = \frac{dx(t)}{dt}$

$$\therefore F_M = M \times a = M \cdot \frac{du(t)}{dt} = M \cdot \frac{d^2 x(t)}{dt^2}$$

At equilibrium, according to Newton's III law

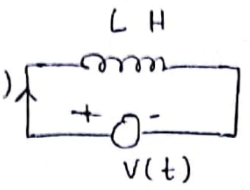
$$f(t) = F_M$$

$$\therefore \boxed{f(t) = M \cdot \frac{d^2 x(t)}{dt^2}} \longrightarrow (1)$$

Mass is an Interial element, its stores energy in form of Kinectic energy given by

$$\boxed{W = \frac{1}{2} M u^2} \quad J \longrightarrow (2)$$

Inductance



From the energy point of view, mass and Inductance behave in same manner.

$$V(t) = L \cdot \frac{d\,i(t)}{dt} \quad \text{—— (3)}$$

but $i(t) = \frac{d\,q(t)}{dt}$.

$$\therefore \; V(t) = L \cdot \frac{d\,i(t)}{dt} = L \cdot \frac{d^2 q(t)}{dt} \quad \text{—— (4)}$$

Inductance stores energy in the form of magnetic field given by.

$$\boxed{W = \frac{1}{2} L I^2} \;\; J \quad \text{—— (5)}$$

Two systems are said to analogues to each other if the mathematical equations of 2 system are Identical.

$$eq\ (1) = eq\ (4)$$

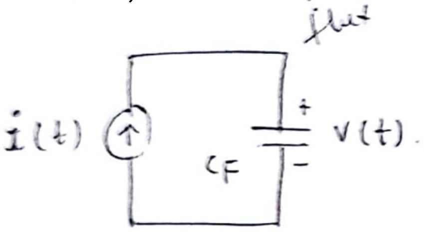
$$\therefore \; f(t) = V(t)$$

$$M = L$$

$$x(t) = q(t)$$

$$u(t) = i(t)$$

when force is compared with Voltage, the corresponding electrical Circuit is said to be force-Voltage (FV) electrical analogus ckt.



Current through capacitor

$$i(t) = C \frac{dv(t)}{dt}$$

according to faraday law,

$$v(t) \propto \frac{d\phi(t)}{dt}$$

$$\therefore \quad i(t) = C \frac{dv(t)}{dt} = C \cdot \frac{d^2\phi(t)}{dt^2} \quad \text{— (6)}$$

Comparing eq (6) with (1), the two equations are mathematically Identical if

$f(t) = i(t)$

$M = C$

$x(t) = \phi(t)$

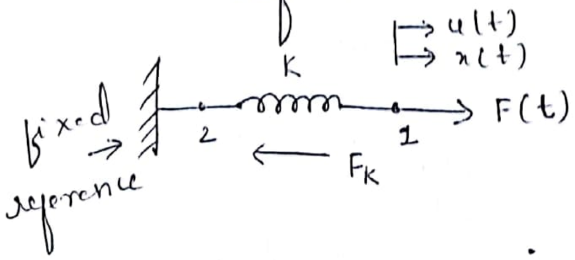
$u(t) = v(t)$

when force is compared with current, the resulting electrical ckt is known as force current (F-I) electrical analogous ckt.

* Mass has only one displacement.
* Counter force (F_M) produced by the mass is proportional to second derivative of displacement $x(t)$.
* when force is compared with Voltage analogy for ~~force~~ Mass is Inductance (F-V)

  Mass is Capacitance (F-I)

(ii) Spring case: when one end of spring is connected to reference.



for a linear spring counter force produced by the spring is proportional to net displacement of spring.

$$F_K \propto [x(t) - 0]$$

$$F_K \propto x(t)$$

$$F_K = K \cdot x(t).$$

where K is constant of proportionality, known as spring constant.

$$u(t) = \frac{d\,x(t)}{dt} \quad ; \quad \int u(t) \cdot dt = x(t).$$

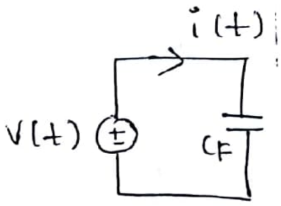
$$\therefore \boxed{F_K = K \cdot x(t) = K \int u(t)\, dt}$$

At equilibrium, according to Newtons III law

$$F(t) = F_K$$

$$F(t) = K \cdot x(t) = K \int u(t) \cdot dt \quad \text{---} (1)$$

from the energy point of view, Capacitance and spring behave in same manner.



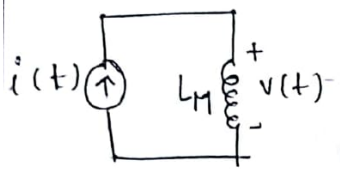
$$V(t) = \int \frac{1}{c}\, i(t) \cdot dt.$$

but $i(t) = \frac{d\,q(t)}{dt}$: $\int i(t)\, dt = q$

$$\therefore V(t) = \frac{1}{c} \int i(t)\, dt = \frac{1}{c} q(t)$$

Comparing equations (1) and (2) are mathematically Identical.

$$F(t) = V(t)$$
$$K = \frac{1}{C}$$
$$x(t) = q(t)$$
$$v(t) = i(t)$$



Current through Inductance is given by

$$i(t) = \frac{1}{L}\int v(t)\cdot dt.$$

$$v(t) = \frac{d\phi}{dt}.$$

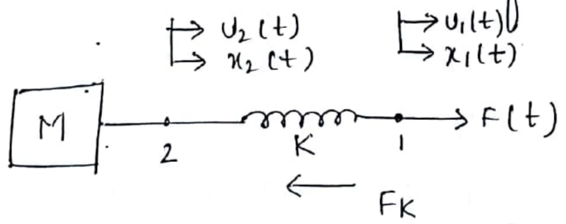
$$\int v(t)\cdot dt = \phi(t).$$

$$\therefore\ i(t) = \frac{1}{L}\int v(t)\cdot dt = \frac{1}{L}\phi(t) \quad \text{—(3)}$$

eq (1) and (3) are mathematically Identical,

$$F(t) = i(t)$$
$$K = \frac{1}{L}$$
$$x(t) = \phi(t)$$
$$v(t) = V(t).$$

<u>Case ii</u> : when both ends of springs are free to move



force is equal to net displacement.
$F = K(x_1 - x_2)$

Counter force produced by spring is

$$F_K \propto (x_1(t) - x_2(t))$$

$$F_K = K[x_1(t) - x_2(t)]$$

$$F_K = K\left[\int v_1(t)\,dt - \int v_2(t)\,dt\right]$$

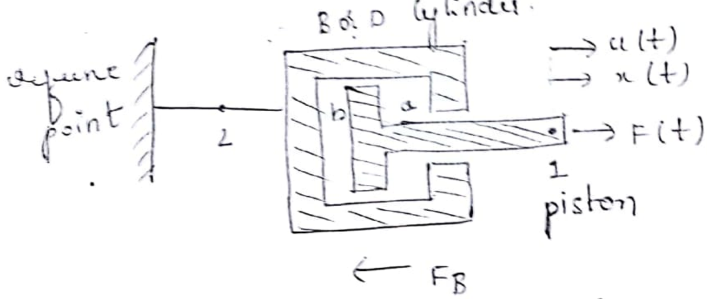
At equilibrium $F(t) = F_K.$

$$\therefore\ F(t) = K[x_1(t) - x_2(t)] = K\int v_1(t) - v_2(t)\,.\,dt.$$

* If one end of spring is connected to reference it has one displacement, if both ends are free to move it has 2 displacements.
* Counter force produced by spring is proportional to net displacement of spring.
* when force is compared with voltage
  - FV → capacitance is electrical analogy.
  - FI → Inductance is — " — for mechanical element spring.

(iii) <u>Dash pot (damper)</u>

<u>Case i</u> : when one end of dashpot is connected to ref.



Counter force (F_B) produced by dashpot is proportional to relative velocity b/w piston and cylinder.

$$F_B \propto (u(t) - 0)$$

$$F_B = B \cdot u(t) = B \cdot \frac{dx(t)}{dt}$$

B → constant of proportionality known as Viscuous friction constant.

Dashpot is energy dissipating element.

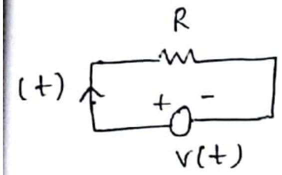
At equilibrium, According to newtons III law

$$F(t) = F_B.$$

$$F(t) = B \cdot u(t) = B \cdot \frac{dx(t)}{dt} \quad \text{—— (1)}$$

From energy point of View, Resistance is the electrical analogy for dashpot

R|K.

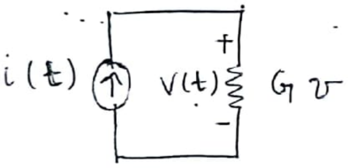


$$V(t) = R \cdot i(t)$$

$$V(t) = R \cdot \frac{d\,q(t)}{dt} \quad \text{— (2)}$$

eq (1) & (2) are mathematically Identical.

$$F(t) = V(t)$$
$$B = K$$
$$U(t) = i(t)$$
$$x(t) = q(t)$$



Current through Conductance $(G = \frac{1}{R})$

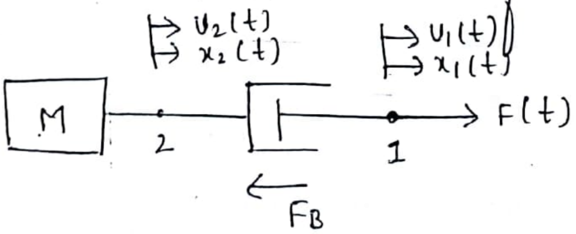
$$i(t) = G \cdot V(t)$$

$$i(t) = G \cdot \frac{d\,\phi(t)}{dt} \quad \text{— (3)}$$

eq (1) and (3) are mathematically Identical if the numerical values of

$$F(t) = i(t)$$
$$B = G$$
$$U(t) = V(t)$$
$$x(t) = \phi(t)$$

Case 2: when both ends of dashpot are free to move



Counter force produced by the dashpot is proportional to relative Velocity b/w piston and cylinder

$F_B \propto U_1(t) - U_2(t)$

$F_B = B[U_1(t) - U_2(t)]$

$F_B = B\left[\frac{d\,x_1(t)}{dt} - \frac{d\,x_2(t)}{dt}\right]$

$$\boxed{F_B = B\left[\frac{d}{dt}(x_1(t) - x_2(t))\right]}$$

At equilibrium, acc to N-III law

$F(t) = F_B$

$F(t) = B[U_1(t) - U_2(t)] = B\frac{d}{dt}(x_1(t) - x_2(t))$

* If one end of dashpot is connected to reference it has one displacement and if both ends are free to move it has 2 displacements.
* Counter force produced by dashpot is proportional to first derivative of net displacement.
* when force is compared with voltage.
  - FV → resistance is electrical analogy
  - FI → conductance —"— for dashpot.

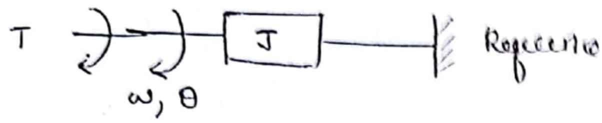
→

Mechanical system	Electrical System FV analogy	Electrical System FI analogy
(1) Force $[F(t)]$	(1) Voltage $[V(t)]$	(1) Current $[i$
(2) Velocity $[U(t)]$	(2) Current $[i(t)]$	(2) Voltage $[v$
(3) Displacement $[x(t)]$	(3) Electric charge $[q(t)]$	(3) flux $[\phi($
(4) Mass $[M]$	(4) Inductance $[L]$	(4) Capacitanc
(5) Dashpot constant $[B]$	(5) Resistance $[R]$	(5) Conductance $[G]$ =
(6) Spring constant $[K]$	(6) Reciprocal of Capacitance $[1/C]$	(6) Reciprocal of Inductance
(7) Reciprocal of spring constant $(1/K)$	(7) Capacitance (C)	(7) Inductan

Rotational Motion

Is the motion of the body about its own axis.

(1) Inertia (J)



$T \propto \alpha$

$T = J \cdot \frac{d\omega}{dt}$

$T = J \cdot \frac{d^2\theta}{dt^2}$

where T = Torque (force × distance)

$\omega = \frac{d\theta}{dt}$, angular velocity

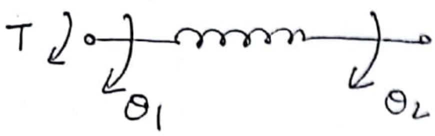
J = Inertia

θ = Angular displacement.

The property of system which stores kinetic energy in rotational system is called Inertia (J).

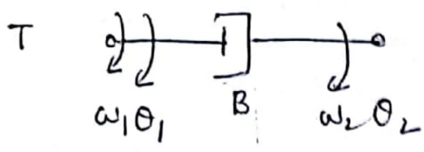
Opposing torque due to Inertia (J) is proportional to the angular acceleration (α) of that Inertia.

(2) Spring



$T = K(\theta_1 - \theta_2)$

(c) Damper



$T = B \frac{d}{dt}(\theta_1 - \theta_2)$

	Translation Motion	Rotational Motion
(1)	Mass (M)	Inertia (J)
(2)	~~friction~~ spring (K)	spring (K)
(3)	Damper (B)	Damper (B)
(4)	Force (F)	Torque (T)
(5)	displacement (x)	Angular displacement (
(6)	Velocity $v = \frac{dx}{dt}$	Angular Velocity, $\omega = \frac{d}{d}$
(7)	Acceleration $\frac{d^2x}{dt^2}$	Angular acceleration, $\alpha = \frac{d}{}$

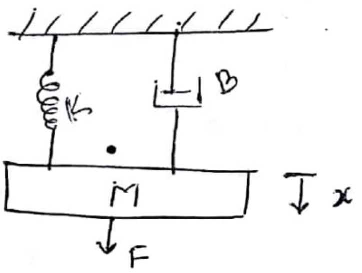
(2)

(3)

eleme

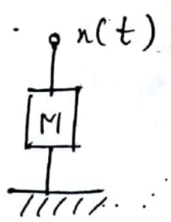
(4)

(1) For the system shown below write the equivalent system of equations.

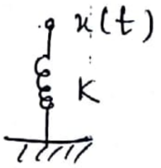


Sol^u: procedure

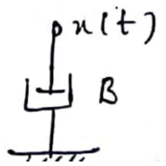
(1) Total number of nodes ~~Toto~~ is equal to Total number of displacements = Total no of masses [Take one reference node in addition]



for Mass $\rightarrow M \frac{d^2x}{dt^2} \rightarrow M \cdot s^2 X(s)$



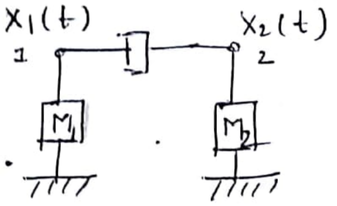
for spring $\rightarrow K \cdot x(t) \rightarrow K \; X(s)$



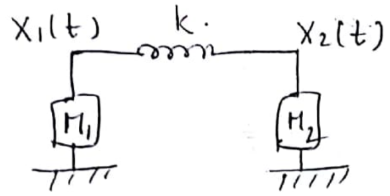
for damper $\rightarrow B \cdot \frac{dx}{dt} \rightarrow B \cdot s \cdot X(s)$

(2) Mass (M) or Inertia (J) has one displacement x or θ. Connect it between the node x & reference. [No mass b/w 2 Nodes]

(3) Spring and damper have 2 displacements. Connect element b/w X_1 and X_2 node.

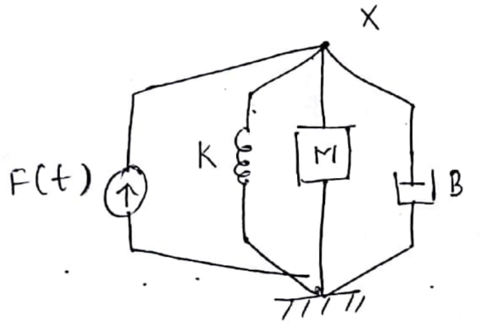


for damper $\rightarrow$ $B\left[\frac{dx_1}{dt} - \frac{dx_2}{dt}\right]$



for spring $\rightarrow$ $K[X_1 - X_2]$

(4) Once the mechanical n/w is drawn, the force/Torque equations is written for each node by equating the sum of force/Torque at each node is zero, a technique similar to nodal analysis used in electrical Circuit



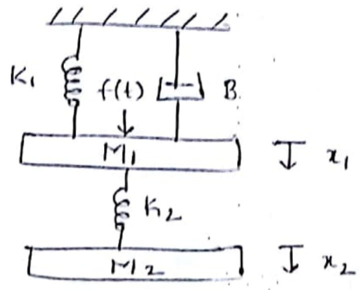
$$F(t) = M \cdot \frac{d^2 x}{dt^2} + B\frac{dx}{dt} + K \cdot x$$

Taking laplace transform, on both sides and setting all Initial condition.

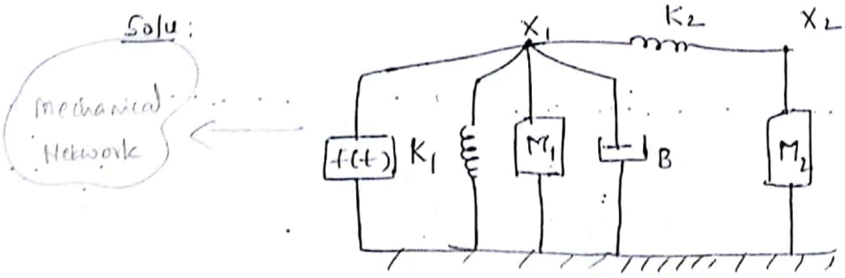
$$F(s) = M \cdot s^2 x(s) + B \cdot s \cdot x(s) + K \cdot x(s) \quad \text{—(1)}$$

$$\therefore F(s) = (s^2 M + sB + K)\, x(s)$$

(2) Draw the mechanical n/w,
Write the differential equation of n/w.
Find the transfer function $\frac{X_2(s)}{F(s)}$



Solu:



At node X_1,

$$M_1 \frac{d^2x}{dt^2} + B\frac{dx_1}{dt} + Kx_1 + K_2(x_1 - x_2) = f(t)$$

Taking laplace Transform (LT) on both sides.

$$M_1 s^2 X_1(s) + B\,s\,X_1(s) + K_1 X_1(s) + K_2 X_1(s) - K_2 X_2(s) = F(s)$$

$$[M_1 s^2 + Bs + K_1 + K_2]\,X_1(s) - K_2 X_2(s) = F(s) \qquad \text{— (1)}$$

At node X_2,

$$M_2 \frac{d^2x_2}{dt^2} + K_2(x_2 - x_1) = 0$$

Taking L.T both sides

$$s^2 M_2 X_2(s) + K_2 X_2(s) - K_2 X_1(s) = 0 \qquad \text{— (2)}$$

$$-K_2 X_1(s) + [M_2 s^2 + K_2]\,X_2(s)$$

putting ① and ② in the matrix form.

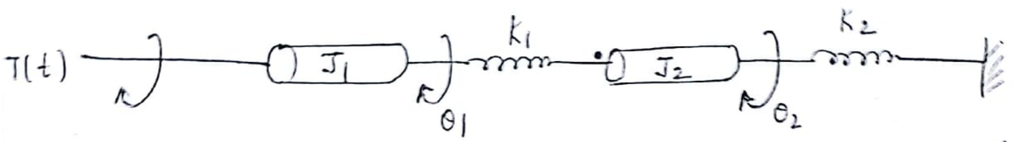
$$\begin{bmatrix} m_1 s^2 + B_1 s + K_1 + K_2 & -K_2 \\ -K_2 & M_2 s^2 + K_2 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$$

Solving for $X_2(s)$ using Cramer's rule, we get

$$X_2(s) = \frac{\begin{vmatrix} m_1 s^2 + B_1 s + K_1 + K_2 & F(s) \\ -K_2 & 0 \end{vmatrix}}{\Delta}$$

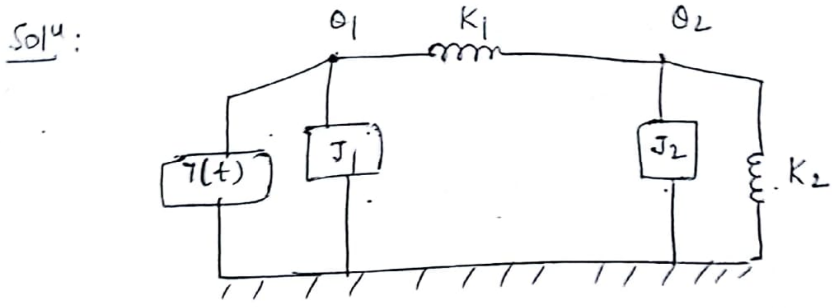
$$X_2(s) = -\frac{(-K_2 \; F(s))}{\Delta} = \frac{K_2 \; F(s)}{\Delta}$$

$$T.F = \frac{X_2(s)}{F(s)} = \frac{K_2}{\Delta}$$



obtain T.F $\frac{\theta_1(s)}{T(s)}$, draw the mechanical n/w.

Sol^n:



At node θ_1,

$$J_1 \frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta_2) = T(t)$$

Taking L.T

$$J_1 s^2 \theta_1(s) + K_1 \theta_1(s) - K_2 \theta_2(s) = T(s)$$

$$\left[J_1 s^2 + K_1 \right] \theta_1(s) - K_1 \theta_2(s) = T(s) \quad \text{——(1)}$$

At node θ_2

$$J_2 \frac{d^2\theta}{dt^2} + K_2 \theta_2 + K_1(\theta_2 - \theta_1) = 0$$

Taking L.T

$$J_2 s^2 \theta_2(s) + K_2 \theta_2(s) + K_1 \theta_2(s) - K_1 \theta_1(s) = 0$$

$$-K_1 \theta_1(s) + \left[J_2 s^2 + K_1 + K_2 \right] \theta_2(s) = 0 \quad \text{——(2)}$$

putting eq (1) and (2) in matrix form

$$\begin{bmatrix} J_1 s^2 + K_1 & -K_1 \\ -K_1 & J_2 s^2 + K_1 + K_2 \end{bmatrix} \begin{bmatrix} \theta_1(s) \\ \theta_2(s) \end{bmatrix} = \begin{bmatrix} T(s) \\ 0 \end{bmatrix}$$

Solving $\theta_1(s)$ using Cramer's rule we get

$$\theta_1(s) = \frac{\begin{bmatrix} T(s) & -K_1 \\ 0 & J_2 s^2 + K_1 + K_2 \end{bmatrix}}{\Delta}$$

$$\theta_1(s) = \frac{T(s) \cdot \left[J_2 s^2 + K_1 + K_2 \right]}{\Delta}$$

$$T.F = \frac{\theta_1(s)}{T(s)} = \frac{J_2 s^2 + K_1 + K_2}{\Delta}$$

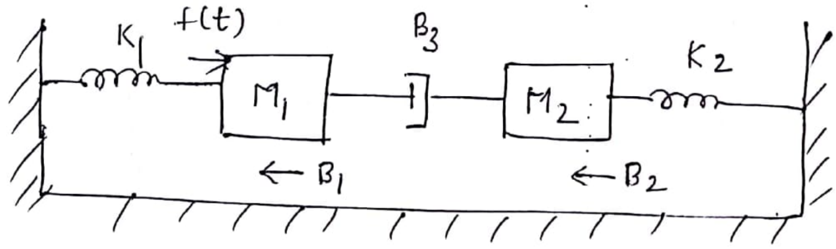
NOTE: No mass be b/w the 2 nodes as due to mass there cannot be change in force as mass cannot store potential energy.

For the Mechanical system given below

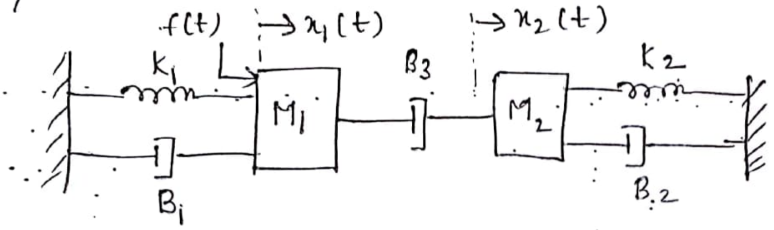
(i) Draw the mechanical network and freebody diagram

(ii) Write equilibrium equation of system

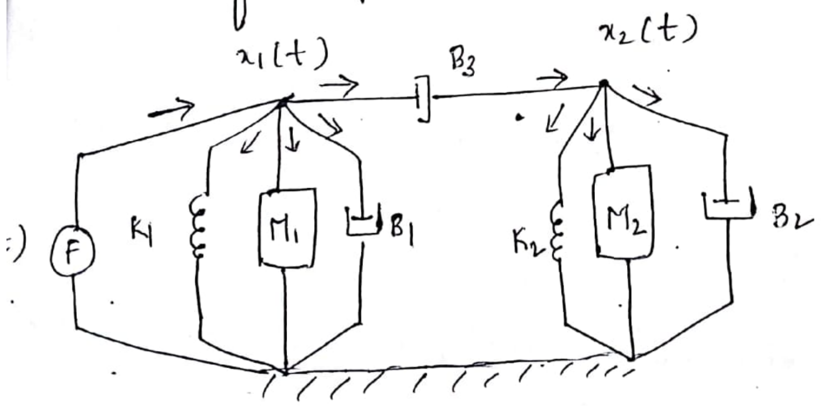
(iii) Draw the FV & FI electrical analogous circuits



The mechanical system is re-drawn as shown below.



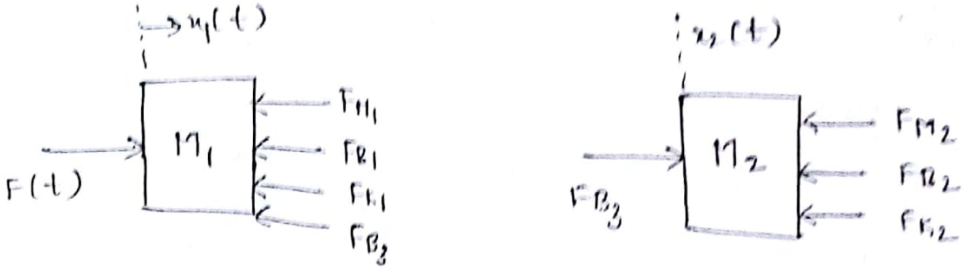
The mechanical n/w is as shown below. The number of junctions in mechanical n/w is equal to number of displacements in mechanical system +1 (reference pt)



The free body diagram is as shown below,

Number of freebody diagrams to be written for given mechanical system is equal to number of displacements in the system.

In each free body diagram we have to Indicate action and reaction forces.



→ At equilibrium, equations of mechanical systems are given by.

<u>At $x_1(t)$</u>,

$$F(t) = F_{M_1} + F_{B_1} + F_{K_1} + F_{B_3}$$

$$F(t) = M_1 \frac{d^2}{dt^2}[x_1(t)] + B_1 \frac{d}{dt} x_1(t) + K_1 x_1(t) + B_3 \frac{d}{dt}(x_1(t) - x_2(t)) \quad \text{—(1)}$$

<u>At $x_2(t)$</u>,

$$F_{B_3} = F_{M_2} + F_{B_2} + F_{K_2}$$

$$B_3 \frac{d}{dt}[x_1(t) - x_2(t)] = M_2 \frac{d^2}{dt^2}[x_2(t)] + B_2 \frac{d}{dt}(x_2(t)) + K_2 x_2(t) \quad \text{—(2)}$$

→ <u>FV Analogy</u>

Substituting electrical analogous based on FV analogy in equations (1) & (2) we get.

from (1)

$$V(t) = L_1 \frac{d^2}{dt^2}[Q_1(t)] + R_1 \frac{d}{dt}(Q_1(t)) + \frac{1}{C_1} Q_1(t) + R_3 \frac{d}{dt}(Q_1(t) - Q_2(t))$$

but $I(t) = \frac{d}{dt} Q(t)$;

$$\frac{di(t)}{dt} = \frac{d^2 Q(t)}{dt^2} \quad ; \quad \int i(t) \cdot dt = Q(t)$$

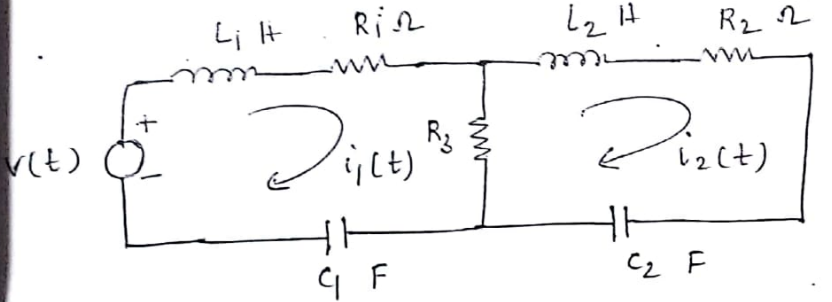
$$V(t) = L_1 \frac{di(t)}{dt} + R_1 i_1(t) + \frac{1}{C_1}\int i_1(t)\cdot dt + R_3\,[i_1(t) - i_2(t)] \quad \text{—(3)}$$

from (2)

$$R_3 \frac{d}{dt}[Q_1(t) - Q_2(t)] = L_2 \frac{d^2}{dt^2}(Q_2(t)) + R_2 \frac{d}{dt} Q_2(t) + \frac{1}{C_2} Q_2(t)$$

$$R_3\,[i_1(t) - i_2(t)] = L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) + \frac{1}{C_2}\int i_2(t)\cdot dt \quad \text{—(4)}$$

The electrical circuit satisfying equations (3) & (4) is as shown.



F-I Analogy

Subsituting electrical analogs based on FI Analogy in equations ① & ②

from ①:

$$I(t) = C_1 \frac{d^2}{dt}\phi_1(t) + G_1 \frac{d}{dt}\phi_1(t) + \frac{1}{L_1}\phi_1(t) + G_3 \frac{d}{dt}(\phi_1(t) - \phi_2(t))$$

but $V(t) = \dfrac{d\phi(t)}{dt}$; $\dfrac{dV(t)}{dt} = \dfrac{d^2\phi(t)}{dt^2}$; $\int V(t)\cdot dt = \phi(t)$

$$\therefore \quad i(t) = C_1 \frac{dV(t)}{dt} + G_1 V_1(t) + \frac{1}{L_1}\int V(t)\cdot dt + G_3\,(V_1(t) - V_2(t)) \quad \text{—(5)}$$

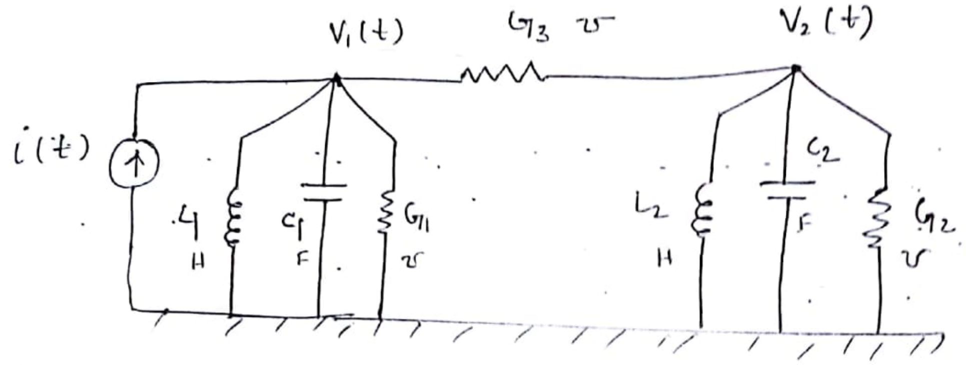
from (2)

$$G_3 \frac{d}{dt}[\phi_1(t) - \phi_2(t)] = C_2 \frac{d^2}{dt^2}\phi_2(t) + G_2 \frac{d}{dt}\phi_2(t) + \frac{1}{L_2}\phi$$

$$G_3 [V_1(t) - V_2(t)] = C_2 \frac{d\, V_2(t)}{dt} + G_2 V_2(t) + \frac{1}{L_2}\int V_2(t)$$

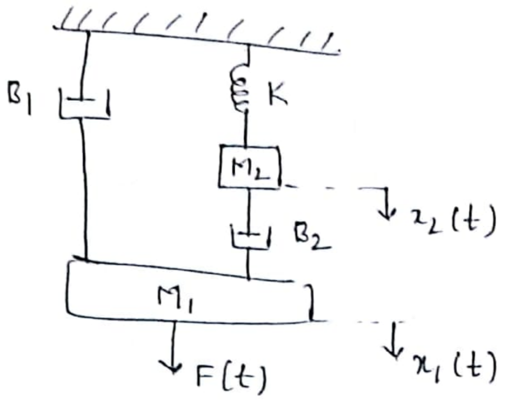
The electrical ckt satisfying eq (5) & (6) is as shown below

Sol

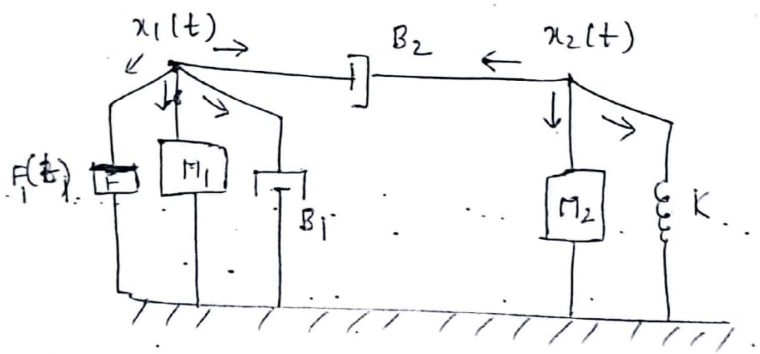


NOTE:
- If the force is directly acting on the mass then the number of displacements in mechanical system is equal to number of masses in system provided there is no direct series connection of 2 springs or 2 dashpots, or dashpot & a spring.
- FV → series circuit
  FI → parallel circuit

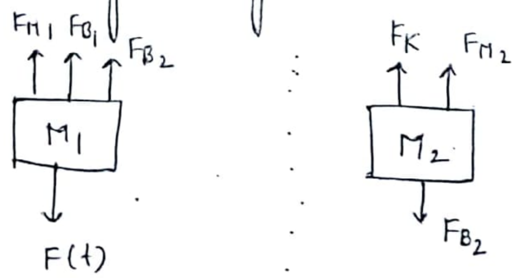
Repeat the above for mech system shown



Sol^n: the mechanical n/w is as shown.



Free body diagram,



At equilibrium, equations are written as

At $x_1(t)$.

$$F(t) = M_1 \frac{d^2}{dt^2} x(t) + B_1 \frac{d}{dt} x(t) + B_2 \frac{d}{dt}\left(x_1(t) - x_2(t)\right)$$

At $x_2(t)$

$$B_2 \frac{d}{dt}\left(x_2(t) - x_1(t)\right) + M_2 \frac{d^2}{dt^2} x_2(t) + K\, x_2(t) = 0$$

— (2)

→ **FV Analogy**

from ①

$$V(t) = L_1 \frac{d^2}{dt^2} q_1(t) + R_1 \frac{d}{dt} q_1(t) + R_2 \frac{d}{dt}\left(q_1(t) - q_2(t)\right)$$

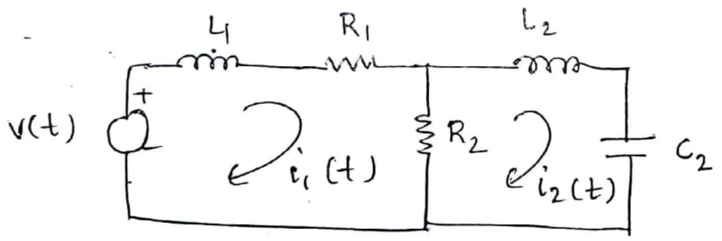
$$V(t) = L_1 \frac{d i_1(t)}{dt} + R_1 i_1(t) + R_2 \left[i_1(t) - i_2(t)\right] \quad —(3)$$

from ②

$$R_2 \frac{d}{dt}\left(q_2(t) - q_1(t)\right) + M_2 \frac{d^2}{dt^2} q_2(t) + \frac{1}{C} \int q_2(t) \cdot dt = 0$$

$$R_2 \cdot \left[i_2(t) - i_1(t)\right] + L_2 \frac{d i_2(t)}{dt} + \frac{1}{C} \int i_2(t)\, dt = 0 \quad —(4)$$

from (3) & (4) equivalent circuit is



→ **FI Analogy**

from ①

$$i(t) = C_1 \frac{d^2}{dt^2} \phi_1(t) + G_1 \frac{d}{dt} \phi_1(t) + G_2 \frac{d}{dt}\left(\phi_1(t) -\right.$$

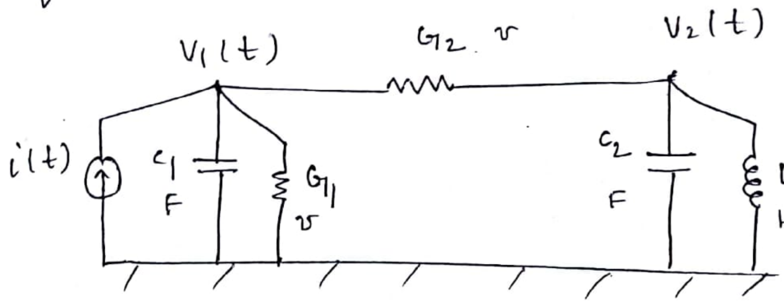
$$i(t) = C_1 \frac{d V_1(t)}{dt} + G_1 V_1(t) + G_2 \left[V_1(t) - V_2(t)\right] \quad —(5)$$

from ②

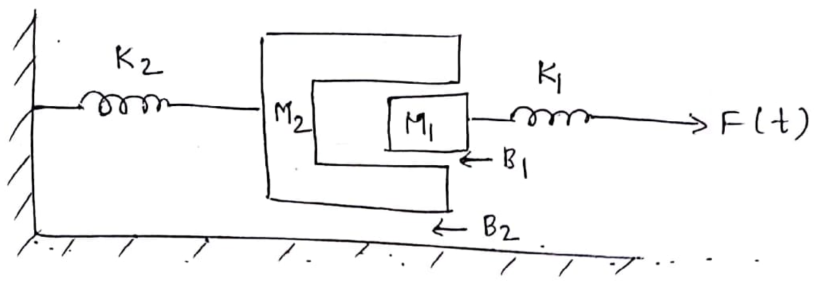
$$G_2 \frac{d}{dt}\left(\phi_2(t) - \phi_1(t)\right) + C_2 \frac{d^2}{dt^2} \phi_2(t) + \frac{1}{L} \phi_2(t) =$$

$$G_2 \left[V_2(t) - V_1(t)\right] + C_2 \frac{d V_2(t)}{dt} + \frac{1}{L} \int V_2(t) \cdot dt =$$

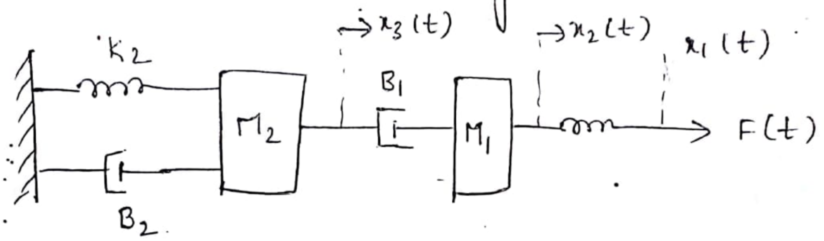
from (5) & (6) electrical n/w is



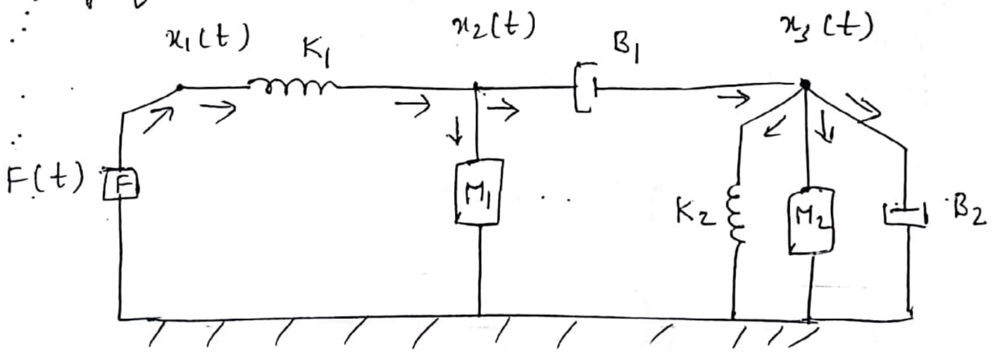
Repeat above problem



∴ Redraw mechanical system.

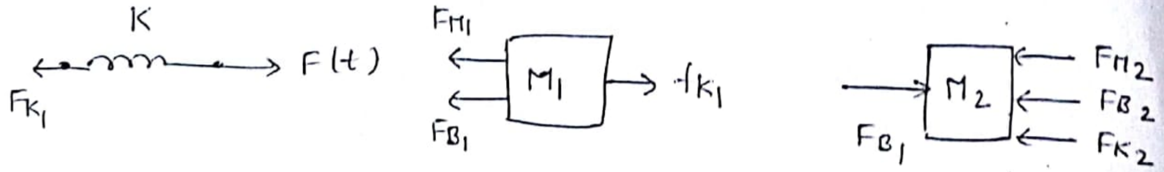


simplified mech n/w



Note: If the force is directly acting on the spring or dashpot then the number of displacement in mechanical system is equal to (number of masses in system + 1) provided there is no direct series connection of 2 springs or 2 dashpots or spring & dashpot.

Free body diagram,



equilibrium equation is given by,

At x_1,

$$F(t) = K_1(x_1(t) - x_2(t)) \quad —(1)$$

At x_2,

$$K_1[x_1(t) - x_2(t)] = M_1 \frac{d^2}{dt^2} x_2(t) + B_1 \cdot \frac{d}{dt}[x_2(t) - x_3(t \quad —(2)$$

At x_3,

$$B_1 \frac{d}{dt}[x_2(t) - x_3(t)] = K_2 x_3(t) + M_2 \frac{d^2}{dt^2} x_3(t) + B_2 \frac{d}{dt} \quad —(3)$$

→ FV Analogy

from (1)

$$V(t) = \frac{1}{C_1}[q_1(t) - q_2(t)]$$

$$V(t) = \frac{1}{C_1}\int (i_1(t) - i_2(t))\, dt \quad —(4)$$

from (2)

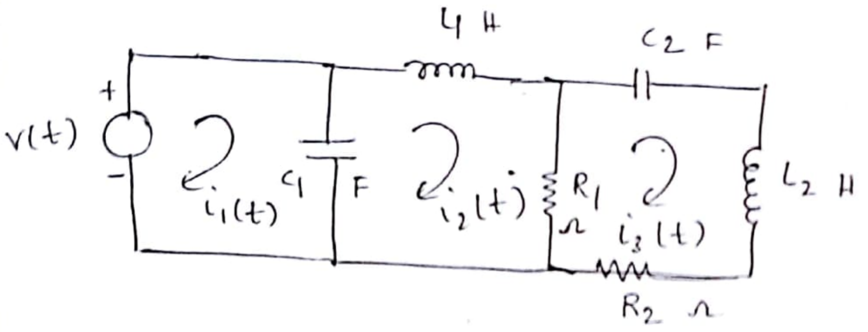
$$\frac{1}{C_1}\int (i_1(t) - i_2(t))dt = L_1 \frac{d^2}{dt^2} q_2(t) + R_1 \frac{d}{dt}(q_2(t) - q$$

$$\frac{1}{C_1}\int (i_1(t) - i_2(t))dt = L_1 \frac{d\, i_2(t)}{dt} + R_1 [i_2(t) - i_3(t)] \quad —(5)$$

from (3)

$$R_1\left[i_2(t) - i_3(t)\right] = \frac{1}{C_2}\int i_3(t)\,dt + L_2\frac{dV_3(t)}{dt} + R_2\, i_3(t) \quad \text{--- (6)}$$

Electrical n/w for eq (4) (5) & (6).



FI Analogy

from (1)

$$i(t) = \frac{1}{L_1}\int \left(V_1(t) - V_2(t)\right) dt \quad \text{--- (7)}$$

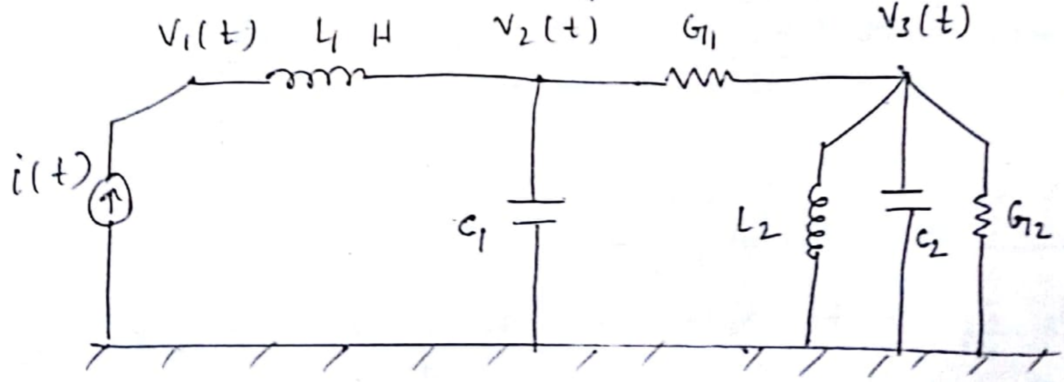
from (2)

$$\frac{1}{L_1}\int \left(V_1(t) - V_2(t)\right) dt = C_1\frac{dV_2(t)}{dt} + G_1\left[V_2(t) - V_3(t)\right] \quad \text{--- (8)}$$

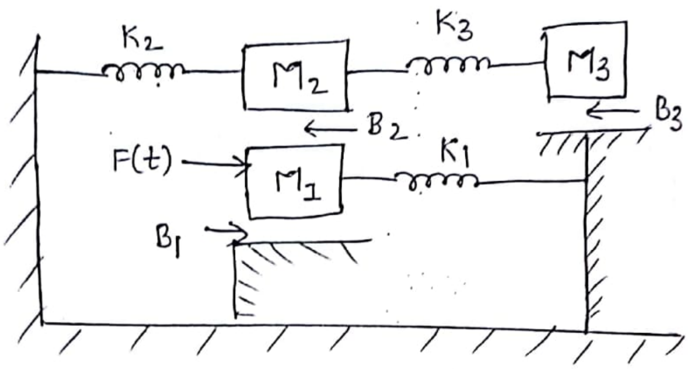
from (3)

$$G_1\left[V_2(t) - V_3(t)\right] = \frac{1}{L_2}\int V_3(t)\,dt + C_2\frac{dV_3(t)}{dt} + G_2 V_3(t) \quad \text{--- (9)}$$

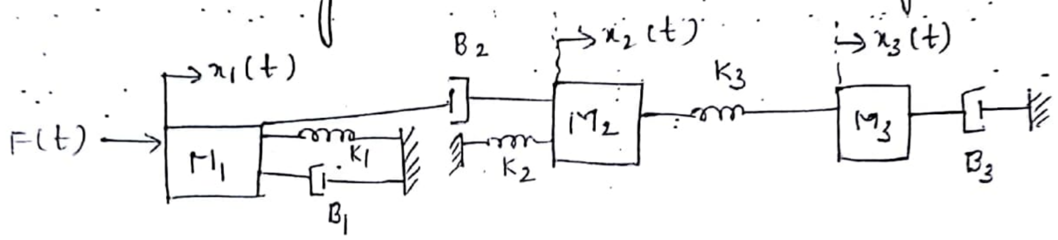
Electrical n/w representing (7), (8) & (9) is.

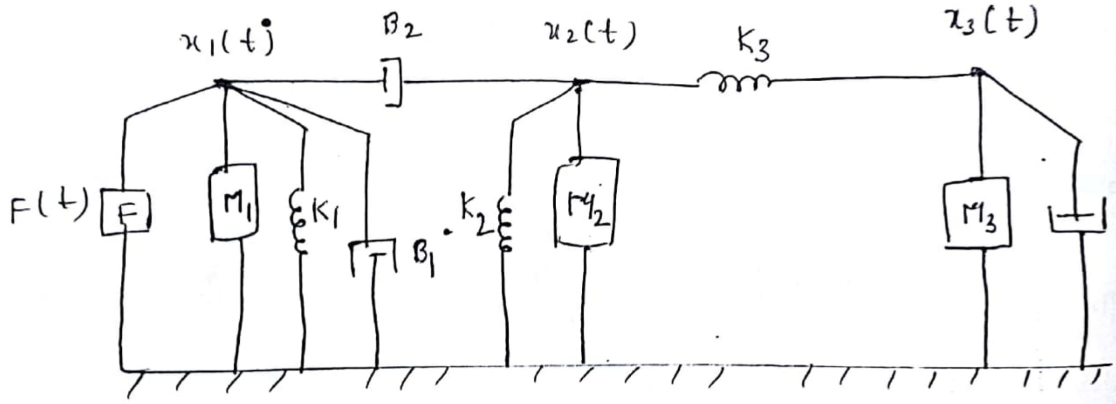


(4) For mechanical system given below draw mech... network and therefore obtain equilibrium equati... of system, also draw electrical FV & FI analogous circuits.



Sol^n: Redrawing the mech n/w, we get:





At x_1

$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{dx_1(t)}{dt} + K_1 x_1(t) + B_2 \frac{d}{dt}(x_1(t$$

At x_2

$$B_2 \frac{d}{dt}(x_1(t) - x_2(t)) = M_2 \frac{d^2}{dt^2} x_2(t) + K_2 x_2(t)$$
$$+ K_3 (x_2(t) - x_3(t$$

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At x_3

$$K_3\,[x_2(t) - x_3(t)] = M_3 \frac{d^2}{dt^2} x_3(t) + B_3 \frac{d}{dt}(x_3(t)) \quad — (3)$$

FV Analogy

from ①

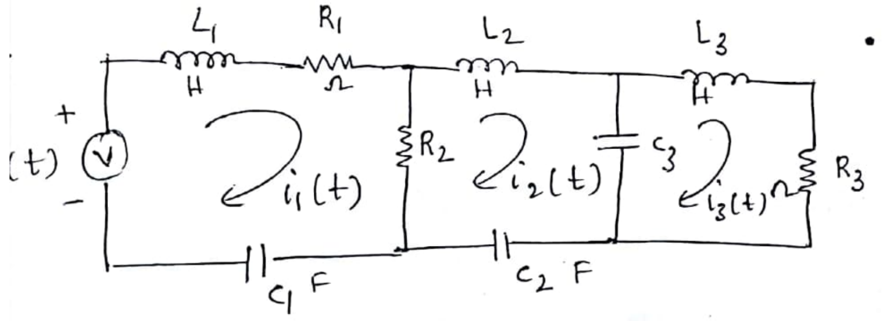
$$v(t) = L_1 \frac{di_1(t)}{dt} + R_1\, i_1(t) + \frac{1}{C_1}\int i_1(t)\cdot dt + R_2\,[i_1(t) - i_2(t)] \quad — (4)$$

from ②

$$R_2\,[i_1(t) - i_2(t)] = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_2}\int i_2(t)\cdot dt + \frac{1}{C_3}\int (i_2(t) - i_3(t))dt \quad — (5)$$

from ③

$$\frac{1}{C_3}\int (i_2(t) - i_3(t))\cdot dt = L_3 \frac{di_3(t)}{dt} + R_3\, i_3(t) \quad — (6)$$



F-I Analogy

from ①

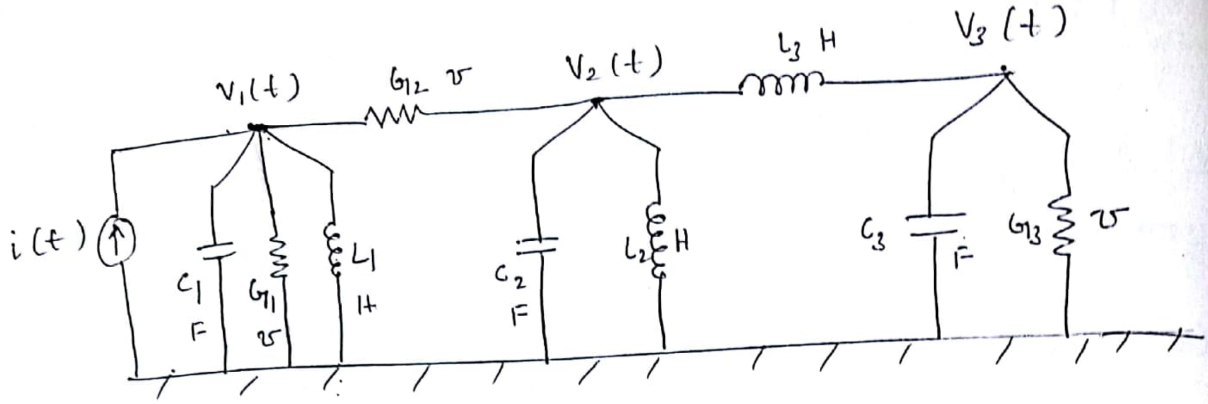
$$i(t) = C_1 \frac{dV_1(t)}{dt} + G_1\, V_1(t) + \frac{1}{L_1}\int V_1(t)\cdot dt + G_2\,[V_1(t) - V_2(t)] \quad — (7)$$

from ②

$$G_2\,[V_1(t) - V_2(t)] = C_2 \frac{dV_2(t)}{dt} + \frac{1}{L_2}\int V_2(t)\cdot dt + \frac{1}{L_3}\int [V_2(t) - V_3(t)]dt \quad — (8)$$

from ③

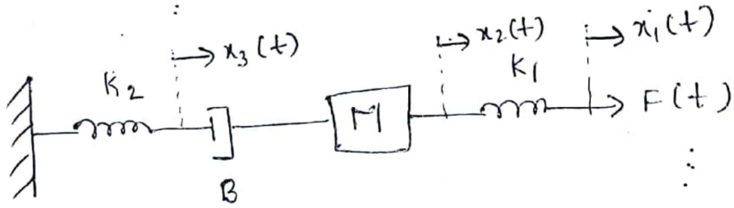
$$\frac{1}{L_3}\int (V_2(t) - V_3(t))\cdot dt = C_3 \frac{dV_3(t)}{dt} + G_3\, V_3(t) \quad — (9)$$

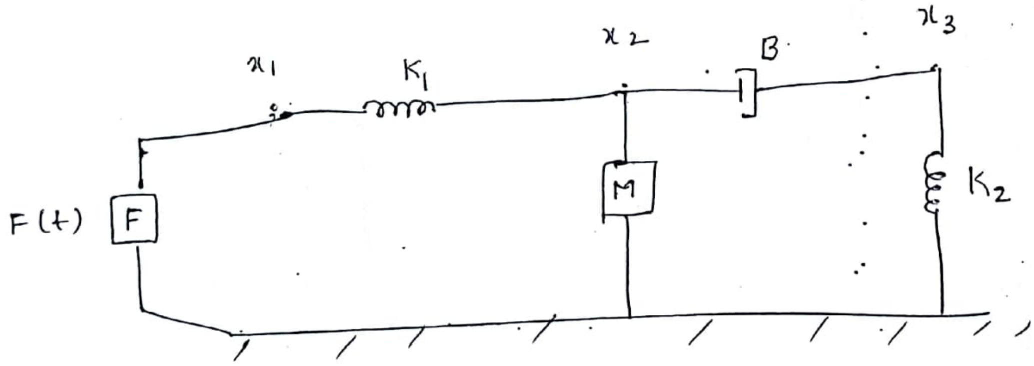


(8) For mech systems given below

(1) Draw mech n/w & therefore obtain equilibrium equation of system

(2) Draw Electrical analogous, based F-V & F-I Elect. n/w





<u>At x_1</u>

$$F(t) = K_1(x_1 - x_2) \quad \text{— (1)}$$

<u>At x_2</u>

$$K_1(x_1 - x_2) = M \cdot \frac{d^2 x_2(t)}{dt^2} + B \frac{d}{dt}\left(x_2(t) - x_3(t)\right)$$

<u>At x_3</u>

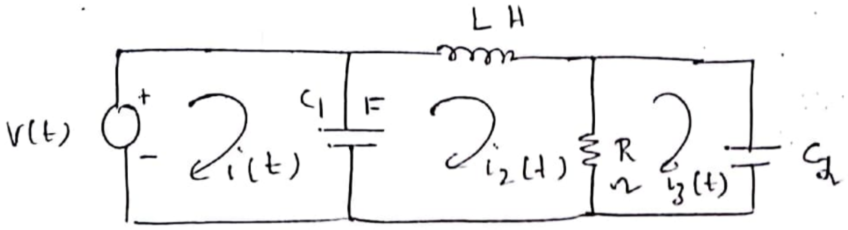
$$B \cdot \frac{d}{dt}\left(x_2(t) - x_3(t)\right) = K_2 x_3(t) \quad \text{— (3)}$$

F-V Analogy

$$V(t) = \frac{1}{C_1}\int [(i_1(t) - i_2(t)] \cdot dt \quad \text{— (4)}$$

$$\frac{1}{C_1}\int (i_1(t) - i_2(t))\, dt = L \cdot \frac{di_2(t)}{dt} + R\,[i_2(t) - i_3(t)] \quad \text{— (5)}$$

$$R\,[i_2(t) - i_3(t)] = \frac{1}{C_2}\int i_3(t) \cdot dt \quad \text{— (6)}$$

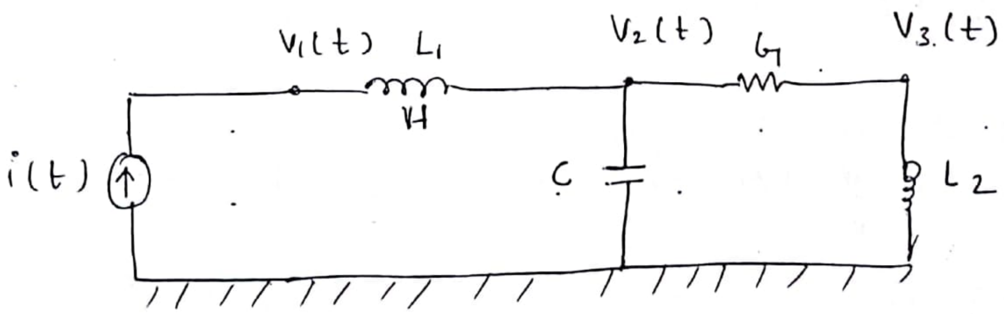


F-I Analogy:

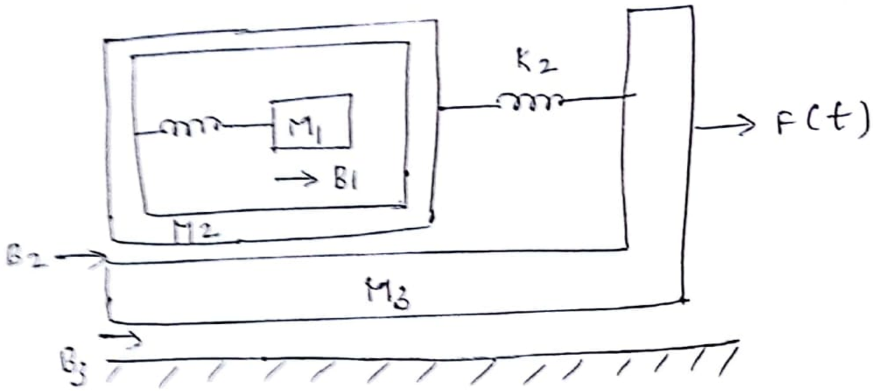
$$i(t) = \frac{1}{L_1}\int (V_1(t) - V_2(t)) \cdot dt \quad \text{— (7)}$$

$$\frac{1}{L_1}\int (V_1(t) - V_2(t))\, dt = C \cdot \frac{dV_2(t)}{dt} + G\,[V_2(t) - V_3(t)] \quad \text{— (8)}$$

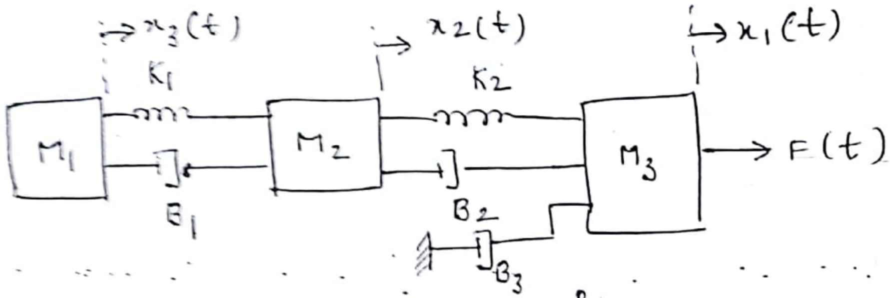
$$G\,[V_2(t) - V_3(t)] = \frac{1}{L_2}\int V_3(t) \cdot dt \quad \text{— (9)}$$

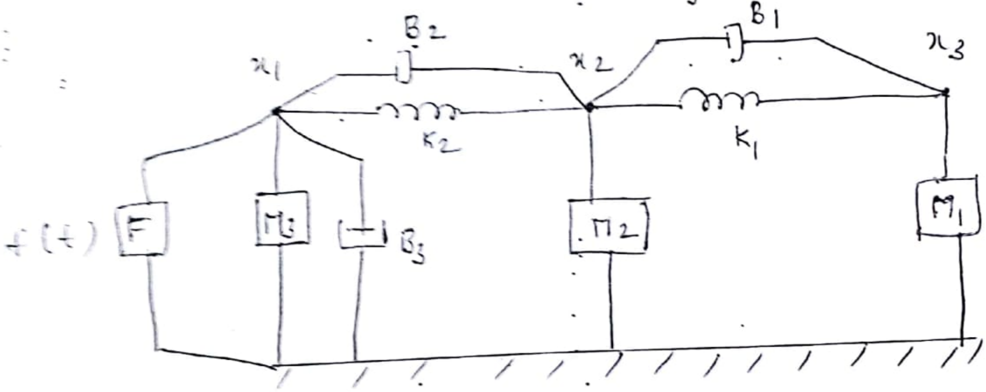


(5)



Soln:





$$f(t) = M_3 \frac{d^2 x_1(t)}{dt^2} + B_3 \frac{dx_1(t)}{dt} + K_2(x_1 - x_2) + B_2 \frac{d}{dt}(x_1 - x_2$$

$$K_2(x_1 - x_2) + B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2(t)}{dt^2} + K_1(x_2 - x_3) + B_1 \frac{d}{dt}(x_2 - x_3)$$

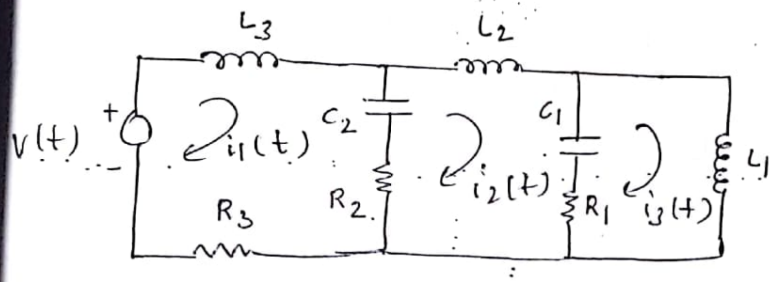
$$K_1(x_2 - x_3) + B_1 \frac{d}{dt}(x_2 - x_3) = M_1 \frac{d^2 x_3(t)}{dt^2} \quad \text{--- (3)}$$

FV Analogy

$$V(t) = L_3 \frac{di_1(t)}{dt} + R_3\, i_1(t) + \frac{1}{C_2}\int (i_1 - i_2)\cdot dt + R_2\,(i_1(t) - i_2(t)) \quad -(4)$$

$$\int (i_1(t) - i_2(t)\cdot dt + R_2\,(i_1(t) - i_2(t)) = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_1}\int (i_2(t) - i_3(t)\cdot dt + R_1\,[i_2(t) - i_3(t)] \quad -(5)$$

$$\int (i_2(t) - i_3(t)\cdot dt + R_2\,(i_2(t) - i_3(t)) = L_1 \frac{di_3(t)}{dt} \quad -(6)$$

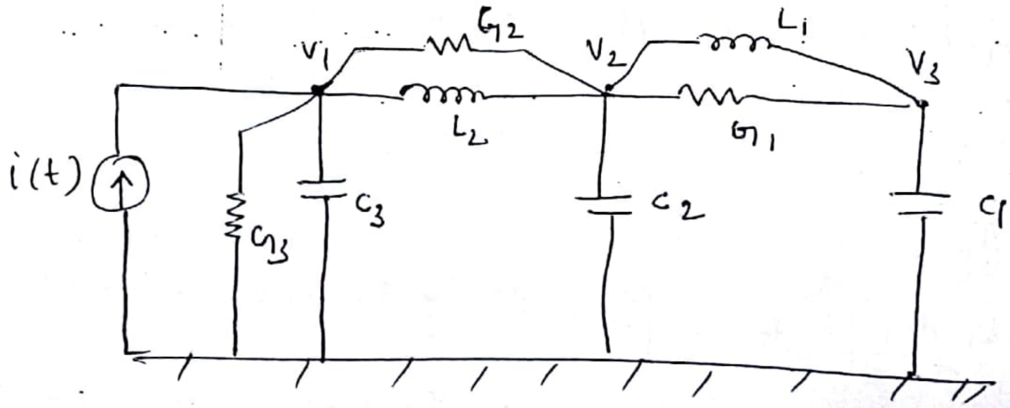


FI Analogy

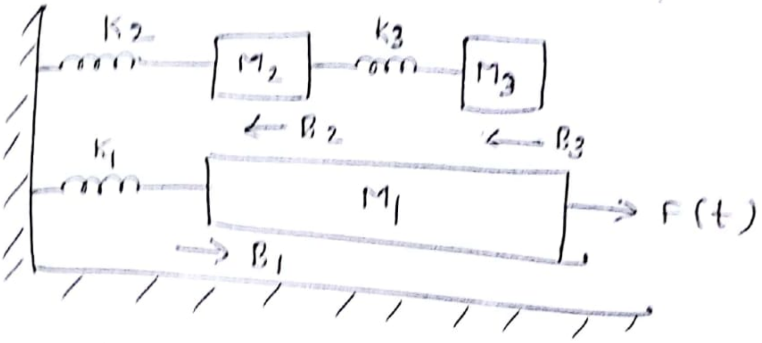
$$i(t) = C_3 \frac{dV_1(t)}{dt} + G_3 V_1(t) + \frac{1}{L_2}\int (V_1(t) - V_2(t)\cdot dt + G_2\,[V_1(t) - V_2(t)] \quad -(7)$$

$$\frac{1}{L_2}\int (V_1(t) - V_2(t)\cdot dt + G_2\,(V_1(t) - V_2(t) = C_2 \frac{dV_2(t)}{dt} + \frac{1}{L_1}\int (V_1(t) - V_2(t)\cdot dt + G_1\,(V_2(t) - V_1(t)) \quad -(8)$$

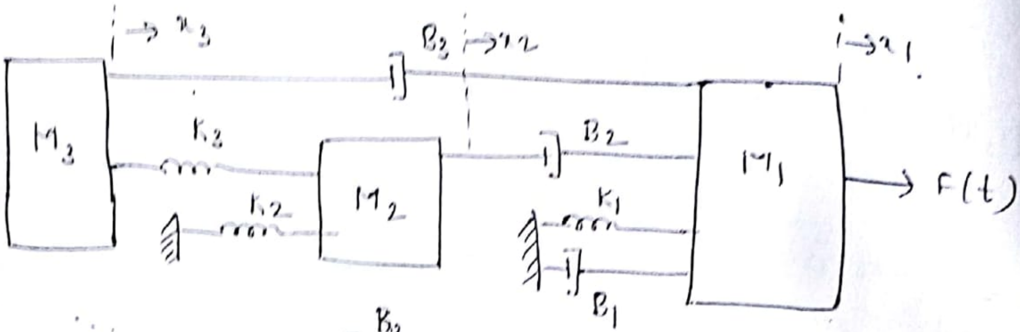
$$\frac{1}{L_1}\int (V_1(t) - V_2(t)\cdot dt + G_2\,(V_2(t) - V_3(t) = C_1 \frac{dV_3(t)}{dt} \quad -(9)$$

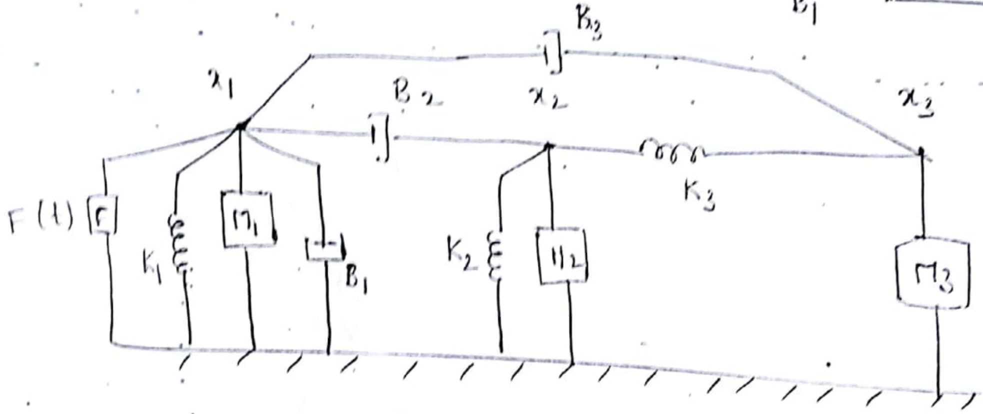


10)



Sol^n :





$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + K_1 x_1 + B_1 \frac{d x_1(t)}{dt} + B_3 \frac{d}{dt}\left(x_1(t) - x_3(t)\right) + B_2 \frac{d}{dt}\left(x_1(t) - x_2(t)\right) \quad — (1)$$

$$B_2 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2(t)}{dt^2} + K_2 x_2(t) + K_3 (x_2 - x_3) \quad — (2)$$

$$B_3 \frac{d}{dt}(x_1 - x_3) + K_3 (x_2 - x_3) = M_3 \cdot \frac{d^2 x_3(t)}{dt^2} \quad — (3)$$

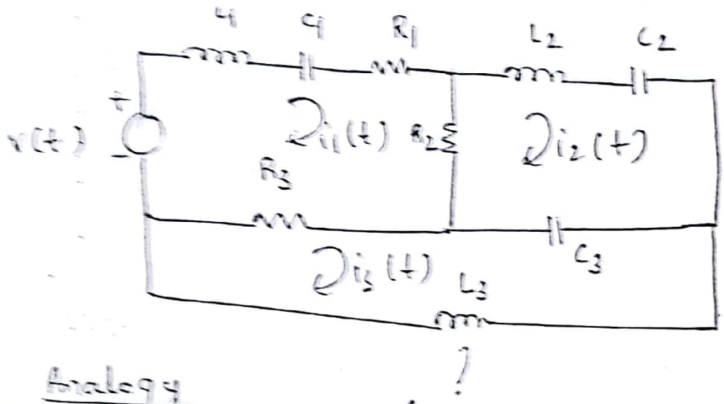
FV Analogy

$$V(t) = L_1 \frac{d i_1(t)}{dt} + \frac{1}{C_1}\int i_1(t)\, dt + R_1 i_1(t) + R_2 \left(i_1(t) - i_2\right.$$

$$+ R_3 \left(i_1(t) - i_3(t)\right) \quad — (4)$$

$$R_2(i_1 - i_2) = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\, dt + \frac{1}{C_3}\int (i_2 - i_3)\, dt \quad -(5)$$

$$R_3(i_1 - i_3) + \frac{1}{C_3}\int (i_2 - i_3)\cdot dt = L_3 \frac{di_3}{dt} \quad -(6)$$

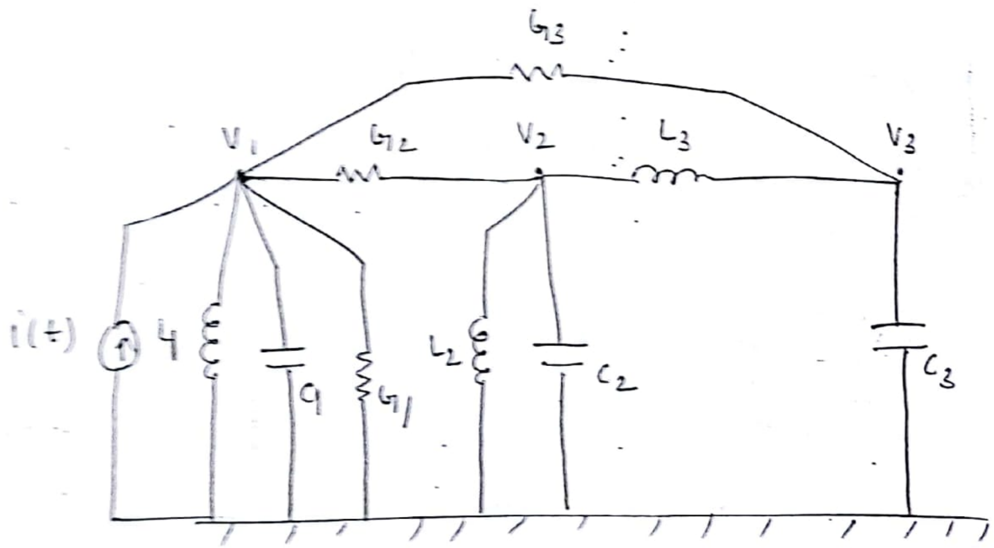


I Analogy

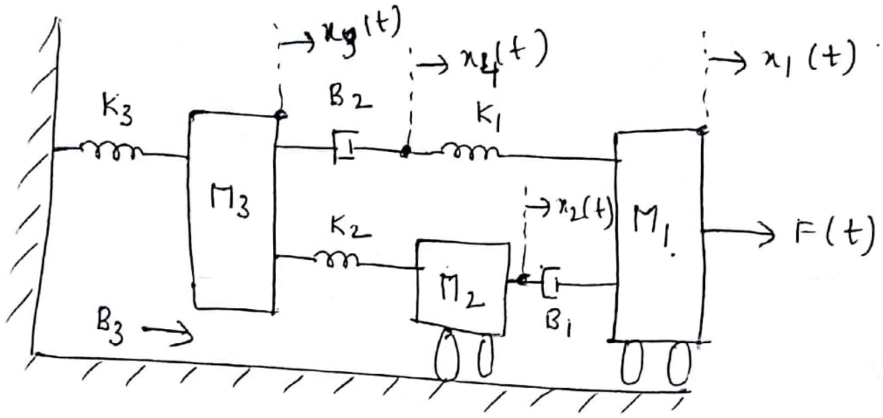
$$i(t) = C_1 \frac{dV_1(t)}{dt} + \frac{1}{L_1}\int V_1(t)\cdot dt + G_1 V_1 + G_2\cdot(V_1 - V_2) + G_3\cdot(V_2 - V_3) \quad -(7)$$

$$G_2(V_1 - V_2) = C_2 \frac{dV_2(t)}{dt} + \frac{1}{L_2}\int V_2(t)\cdot dt + \frac{1}{L_3}\int (V_2 - V_3)\cdot dt \quad -(8)$$

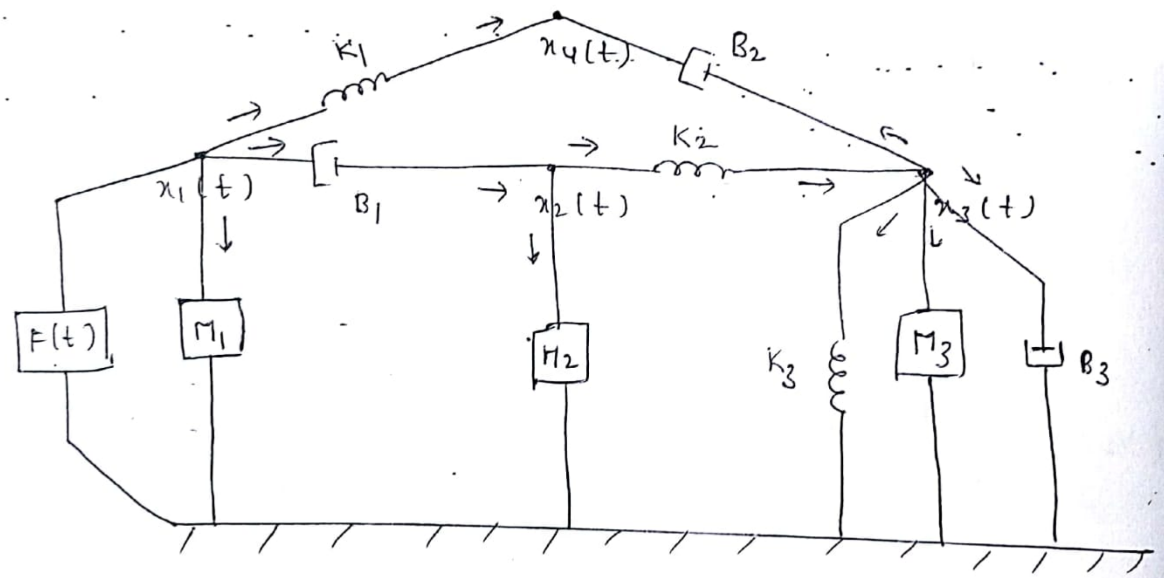
$$G_3(V_2 - V_3) + \frac{1}{L_3}\int (V_2 - V_3)\cdot dt = C_3 \frac{dV_3}{dt} \quad -(9)$$



11) For the mechanical system in fig. draw mech n/w & equilibrium equations.



Sol^n: The mechanical n/w is as shown below



The equilibrium equations are given by.

at $x_1(t)$,

$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + B_1 \frac{d}{dt}(x_1(t) - x_2(t)) + K_1(x_1 - x_4) \quad —(1)$$

At $x_2(t)$,

$$B_1 \frac{d}{dt}(x_1 - x_2) = M_2 \frac{d^2 x_2(t)}{dt^2} + K_2(x_2 - x_3) \quad —(2)$$

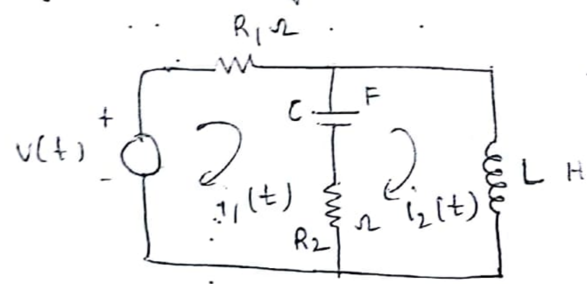
At x_3,

$$K_2(x_2 - x_3) = M_3 \frac{d^2 x_3(t)}{dt^2} + B_3 \frac{d\, x_3(t)}{dt} + K_3 x_3 + B_2 \frac{d}{dt}(x_3 - x_4) \quad — (3)$$

At x_4

$$K_1(x_1 - x_4) + B_2 \frac{d}{dt}(x_3 - x_4) = 0 \quad — (4)$$

Draw the FV analogous mechanical system for the electrical ckt shown in fig. writing the loop equations for electrical ckt then transforming to mechanical ~~system~~ analogous.



Sol[n]: Applying KVL for i_1 and i_2

$$V(t) = R_1 i_1 + \frac{1}{C}\int (i_1 - i_2)\,dt + R_2(i_1 - i_2) \quad — (1)$$

$$\frac{1}{C}\int (i_1 - i_2)\,dt + R_2(i_1 - i_2) = L \cdot \frac{di_2(t)}{dt} \quad — (2)$$

changing to mechanical analogs

$V(t) \rightarrow F(t)$

$L \rightarrow M$

$C \rightarrow 1/K$

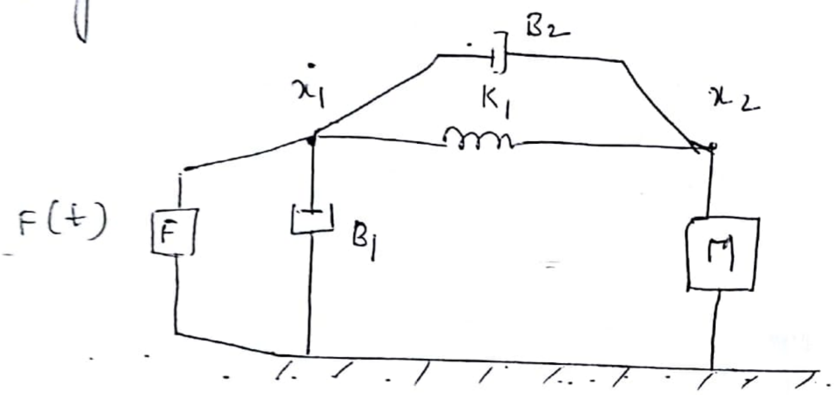
$R \rightarrow B$

$i(t) \rightarrow v(t) \rightarrow \frac{dx}{dt}$

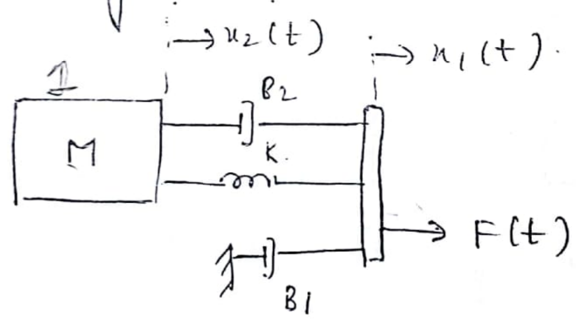
$$F(t) = B_1 \frac{dx_1(t)}{dt} + K(x_1 - x_2) + B_2(x_1 - x_2) \quad \text{— (3)}$$

$$K(x_1 - x_2) + B_2(x_1 - x_2) = M \cdot \frac{d^2 x_2(t)}{dt^2} \quad \text{— (4)}$$

using (3) & (4) Mechanical n/w is as shown belo

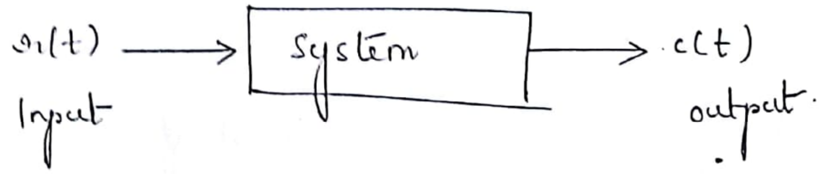


Mechanical system can be written as.



Transfer function (T.F)

It is defined as the ratio of laplace transform of the output to laplace transform of input with zero initial conditions.



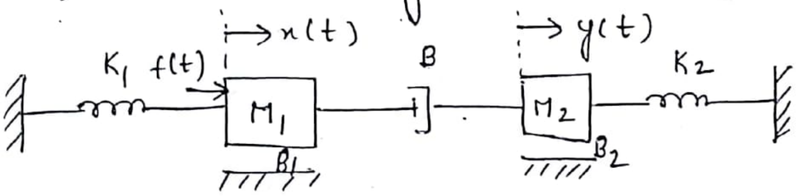
$$\text{Transfer function} = \frac{L\, c(t)}{L\, r(t)} = \frac{C(s)}{R(s)}$$

$L\, f(t) = F(s)$

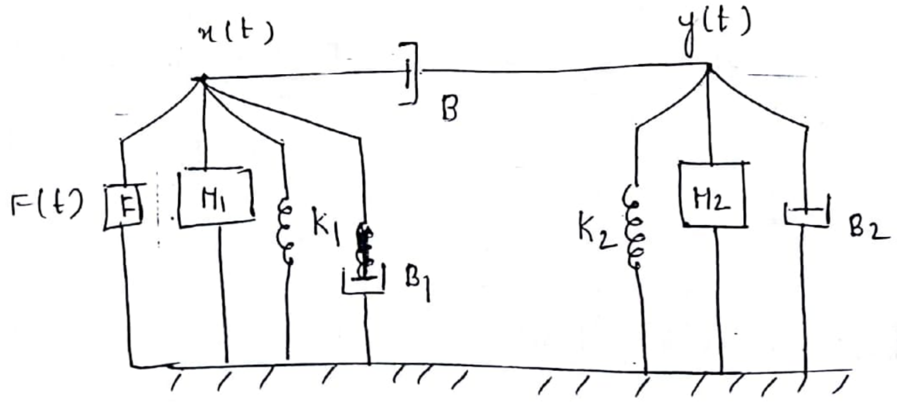
$L\, \frac{d}{dt} f(t) = s\, F(s)$

$L\, \frac{d^2 f(t)}{dt^2} = s^2\, F(s)$

derive the transfer function $\frac{Y(s)}{F(s)}$ of mechanical system shown in fig.



4. The mechanical n/w is as shown below,



at $x_1(t)$,

$$f(t) = M_1 \frac{d^2 x(t)}{dt^2} + B_1 \frac{dx(t)}{dt} + K_1 x(t) + B \frac{d}{dt} (x(t)$$

at $y(t)$,

$$B \frac{d}{dt} (x(t) - y(t)) = M_2 \frac{d^2 y(t)}{dt} + K_2 y(t) + B_2 \frac{d}{dt} y$$

Applying laplace transform to both sides, assuming zero initial conditions,

$$F(s) = M_1 s^2 X(s) + B_1 s X(s) + K_1 X(s) + B s. [X(s) - Y(s$$

$$B s. [X(s) - Y(s)] = M_2 s^2 X(s) + K_2 Y(s) + B_2 . s \ Y(s)$$

from (3)

$$F(s) = [M_1 s^2 + B_1 s + BS + K_1] X(s) - B.s \ Y(s) \quad \text{--- (5)}$$

from (4)

$$B.s \ X(s) = [B.s + M_2 s^2 + K_2 + B_2 . s] Y(s)$$

$$\therefore X(s) = \frac{M_2 s^2 + Bs + B_2 s + K_2}{B.s} . Y(s) \quad \text{--- (6)}$$

subs (6) in (5)

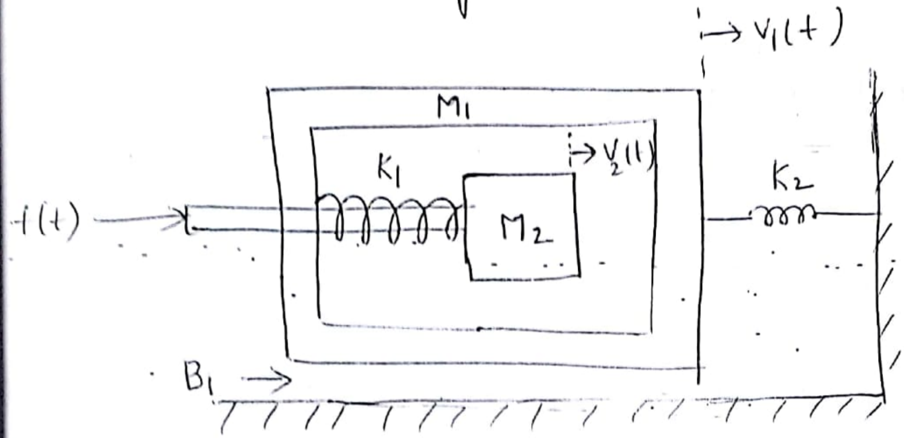
$$F(s) = (M_1 s^2 + B_1 s + Bs + K_1) \left(\frac{M_2 s^2 + Bs + B_2 s + K_2}{B.s} \right) Y(s$$

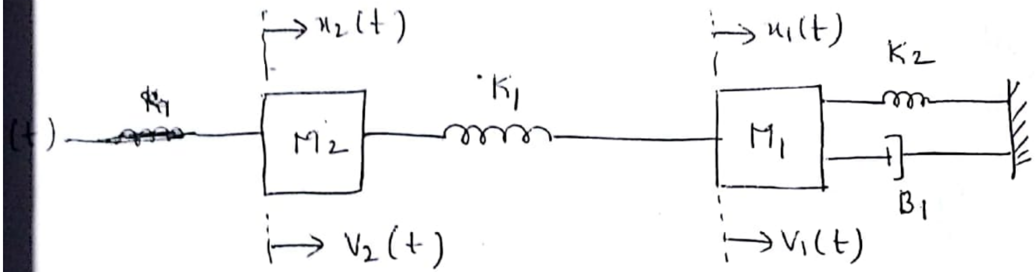
B.s

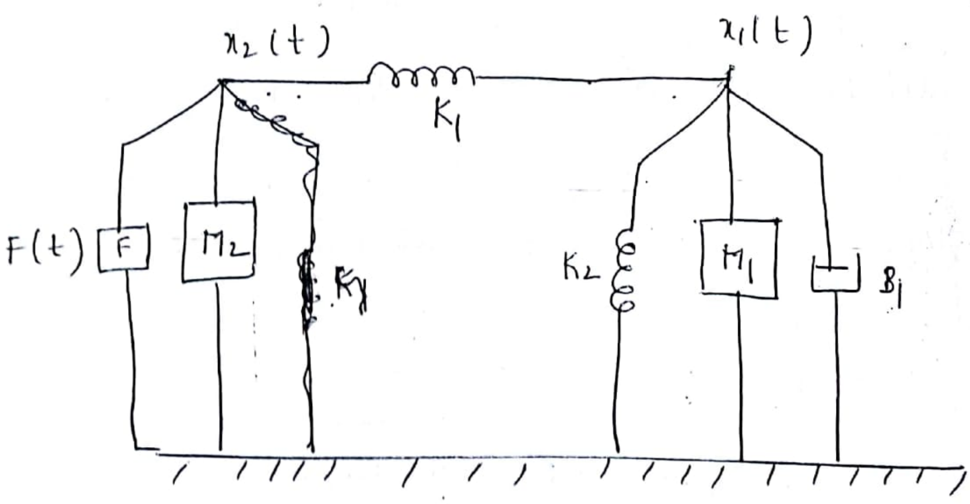
$$F(s) = Y(s) \left[\frac{(M_1 s^2 + B_1 s + Bs + K_1)(M_2 s^2 + Bs + B_2 s + K_2)}{B.s} \right.$$

$$T.F = \frac{Y(s)}{F(s)} = \frac{B \cdot s}{[M_1 s^2 + B_1 s + Bs + K_1][M_2 s^2 + B_2 s + Bs + K_2] - (Bs)^2}$$

derive the T.F $\frac{V_2(s)}{F(s)}$ for mechanical system shown in fig. $V_1(t)$ and $V_2(t)$ are velocities of mass M_1 & M_2 respectively.







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equilibrium equations,

$$F(t) = M_2 \frac{d^2 x_2(t)}{dt^2} + K_1 [x_2(t) - x_1(t)] \quad \text{--- (1)}$$

$$K_1 [x_2(t) - x_1(t)] = M_1 \frac{d^2 x_1(t)}{dt^2} + K_2 x_1(t) + B_1 \frac{d}{dt} x_1($$

Interms of Velocity, $V(t) = \frac{d x(t)}{dt}$, $\frac{d V(t)}{dt} = \frac{d^2 x(t)}{dt^2}$

$$\int V(t) \cdot dt = x(t)$$

$$F(t) = M_2 \frac{d V_2(t)}{dt} + K_1 \int (V_2(t) - V_1(t)) \cdot dt$$

$$K_1 \int (V_2(t) - V_1(t)) \cdot dt = M_1 \frac{d V_1(t)}{dt} + K_2 \int V_1(t) \cdot dt + B_1 V$$

taking laplace on both sides.

$$F(s) = M_2 \cdot s^2 X_2(s) + K_1 [X_2(s) - X_1(s)]$$

$$F(s) = (M_2 s^2 + K_1) X_2(s) - K_1 X_1(s) \quad \text{--- (3)}$$

$$K_1 [X_2(s) - X_1(s)] = M_1 s^2 X_1(s) + K_2 X_1(s) + K_2 X_2(s).$$

$$K_1 X_2(s) = [M_1 s^2 + K_2 + B_1 s + K_1] X_1(s)$$

$$X_1(s) = \frac{K_1 X_2(s)}{[M_1 s^2 + B_1 s + K_1 + K_2]} \quad \text{--- (4)}$$

Subs (4) in (3)

$$F(s) = (M_2 s^2 + K_1)\, X_2(s) - \frac{K_1^2 \, X_2(s)}{M_1 s^2 + Bs + K_1 + K_2}$$

$$F(s) = X_2(s) \left[\frac{(M_2 s^2 + K_1)(M_1 s^2 + Bs + K_1 + K_2) - K_1^2}{M_1 s^2 + Bs + K_1 + K_2} \right]$$

$$\frac{X_2(s)}{F(s)} = \frac{M_1 s^2 + Bs + K_1 + K_2}{(M_1 s^2 + K_1)(M_1 s^2 + Bs + K_1 + K_2) - K_1^2}$$

$V_2(s) = s \cdot X_2(s)$

Multiply by s

$$T.F = \frac{s \cdot X_2(s)}{F(s)} = \frac{V_2(s)}{F(s)} = \frac{s(M_1 s^2 + Bs + K_1 + K_2)}{(M_1 s^2 + K_1)(M_1 s^2 + Bs + K_1 + K_2) - K_1^2}$$

Derive the transfer functions of system shown in fig ① & ② and hence show that, they are analogous to each other.

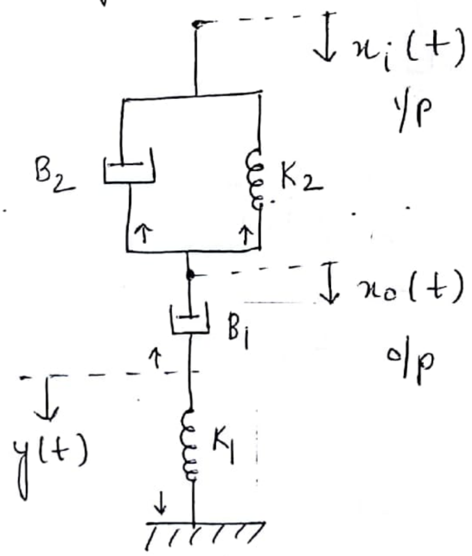


fig (1)

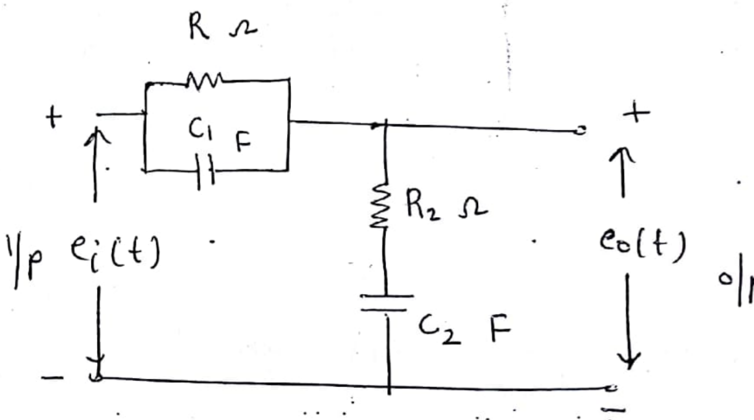


fig (2)

Equilibrium equations is given by,

at $x_o(t)$,

$$B_2 \frac{d}{dt}(x_o(t) - x_i(t)) + K_2(x_o(t) - x_i(t)) + B_1 \frac{d}{dt}(x_o(t) - y(t$$

at $y(t)$,

$$B_1 \frac{d}{dt}(x_o(t) - y(t)) = K_1 y(t) \quad \text{--- (2)}$$

Taking LT, Zero Initial Condition

from (1)

$$B_2 \cdot s(X_o(s) - X_i(s)) + K_2(X_o(s) - X_i(s)) + B_1 s[X_o(s) - Y(s$$

$$[B_2 s + K_2 + B_1 s] X_o(s) - [B_2 s + K_2] X_i(s) = B_1 s \, Y(s) \quad \text{--- (3)}$$

from (2)

$$B_1 s [X_o(s) - Y(s)] = K_1 Y(s)$$

$$B_1 s \, X_o(s) = (K_1 + B_1 s) Y(s)$$

$$\frac{1}{Y(s)} = \frac{K_1 + B_1 s}{B_1 s \, X_o(s)}$$

$$Y(s) = \frac{B_1 s \, X_o(s)}{K_1 + B_1 s} \quad \text{--- (4)}$$

subs (4) in (3)

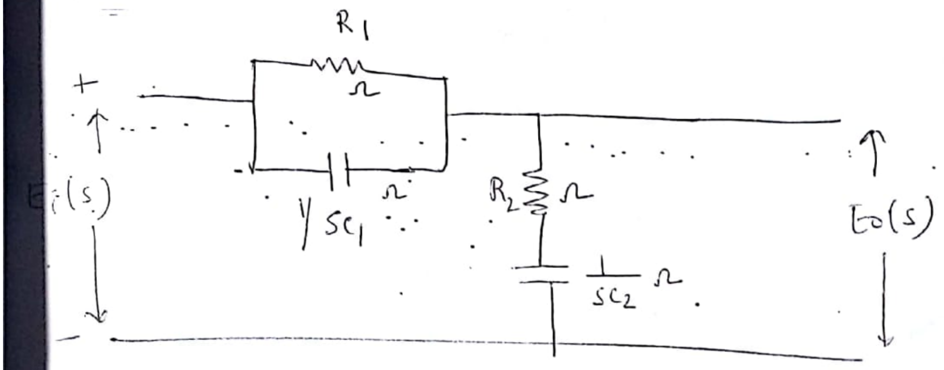
$$(B_2 s + K_2 + B_1 s) X_o(s) - [B_2 s + K_2] X_i(s) = \frac{B_1^2 s^2 X_o(s)}{K_1 + B_1 s}$$

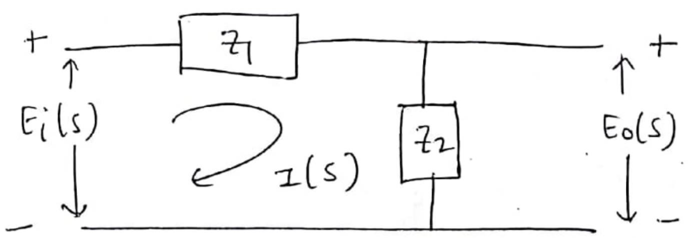
$$\left[\frac{(B_2 s + K_2 + B_1 s)(K_1 + B_1 s) - B_1^2 s^2}{K_1 + B_1 s}\right] X_o(s) = (B_2 s + K_2) X_i(s$$

$$\frac{X_o(s)}{X_i(s)} = \frac{(B_2 s + K_2)(B_1 s + K_1)}{\cancel{B_1^2 s^2} + B_1 s K_1 + B_1 B_2 s^2 + B_2 s K_1 + K_2 B_1 s + K_1 K_2 - \cancel{B_1^2 s^2}}$$

$$\therefore \frac{X_o(s)}{X_i(s)} = \frac{(B_1 s + K_1)(B_2 s + K_2)}{B_1 B_2 s^2 + (B_1 K_1 + B_2 K_1 + B_1 K_2)s + K_1 K_2} \quad \text{--- (5)}$$

laplace transform n/w of electrical ckt is as shown below,





$$Z_1 = \frac{\cancel{\quad}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{sC_1R_1 + 1}$$

$$Z_2 = R_2 + \frac{1}{sC_2} = \frac{sC_2R_2 + 1}{sC_2}$$

$$I(s) = \frac{Ei(s)}{Z_1 + Z_2}$$

$$Eo(s) = Z_2\, I(s) = \frac{Z_2 \cdot Ei(s)}{Z_1 + Z_2}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2}{Z_1 + Z_2}$$

$$= \frac{\dfrac{sC_2R_2+1}{sC_2}}{\dfrac{R_1}{sC_1R_1+1} + \dfrac{sC_2R_2+1}{sC_2}}$$

$$= \frac{\dfrac{sC_2R_2+1}{s\cancel{C_2}}}{\dfrac{R_1sC_2 + (sC_2R_2+1)(sC_1R_1+1)}{(sC_1R_1+1)(s\cancel{C_2})}}$$

$$= \frac{(sC_2R_2+1)(sC_1R_1+1)}{R_1sC_2 + (sC_2R_2+1)(sC_1R_1+1)}$$

$$= \frac{C_2(sR_2 + 1/C_2)\; C_1(sR_1 + 1/C_1)}{sC_2R_1 + s^2C_1C_2R_1R_2 + sC_2R_1 + sC_1R_1 + 1}$$

$$= \frac{\cancel{C_1C_2}[R_2s + 1/C_2][R_1s + 1/C_1]}{\cancel{C_1C_2}\left[R_1R_2s^2 + \dfrac{sR_1}{C_1} + \dfrac{sR_1}{C_2} + \dfrac{sR_2}{C_1} + \dfrac{1}{C_1C_2}\right]}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{(R_2s + 1/C_2)(R_1s + 1/C_1)}{R_1R_2s^2 + s\left(\dfrac{R_1}{C_1} + \dfrac{R_1}{C_2} + \dfrac{R_2}{C_1}\right)s + \dfrac{1}{C_1C_2}} \quad \text{—(6)}$$

eq (5) & (6) are mathematically identical.

$B_1 = R_1$, $B_2 = R_2$, $K_1 = 1/C_1$ & $K_2 = 1/C_2$

If above conditions are satisfied then eq (5) & (6) are mathematically identical & hence two systems are analogous to each other.

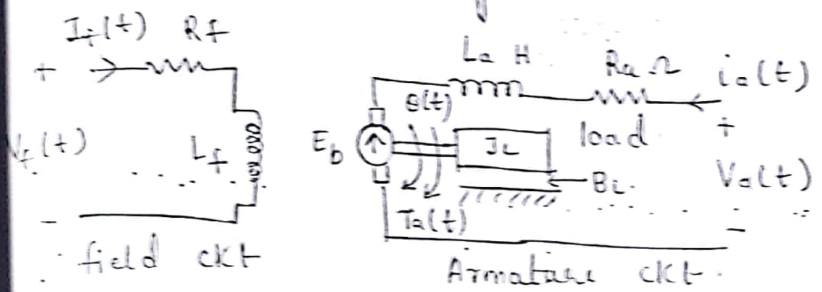
DC Servomotor

The speed of a DC motor can be controlled by 2 methods.

(1) Armature Voltage Controll
(2) Field Controll.

Armature Voltage Controlled DC servomotor

Consider a separately excited DC motor as shown in fig.



R_a & L_a are the resistance and Inductance of armature winding circuit, R_f & L_f are the resistance and Inductance of field winding.

E_b is the back emf or opposing emf developed in the armature, for given dc machine

$$E_b \propto N\phi$$

$$\text{or} \quad N \propto \frac{E_b}{\phi}$$

In armature Voltage controll, the magnetic flux produced by each pole is kept constant.

$$\therefore N \propto E_b$$

$$\text{but} \quad \omega = \frac{d\theta}{dt}$$

$$\omega = \frac{2\pi}{60} \cdot N \quad r/s$$

$N \propto \omega$ $\omega \rightarrow$ shaft ve

$N \propto \frac{d\theta}{dt}$

$E_b \propto \frac{d\theta}{dt}$ or $E_b = K_1 \frac{d\theta}{dt}$.

Taking LT,

$E_b(s) = K_1 . s . \theta(s)$ —— (1)

Assumptions

(1) flux is directly proportional to current through field winding

$\phi m = K_f . I_f =$ constant.

(2) Torque produced is proportional to product of flux and armature current

$T = K'_a \phi I_a$

$T = K'_a K_f I_f I_a$

(3) Back emf is directly proportional to shaft velocity ωm, as flux ϕ is constant.

$\omega m = \frac{d\theta(t)}{dt}$

$E_b = K_b . \omega_m(s)$

$E_b = K_b . s . \theta_m(s)$

Applying KVL to armature,

$Va(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + E_b$

Taking L.T.

$V_a(s) = R_a I_a(s) + L_a s . I_a(s) + E_b(s)$

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subs for $E_b(s)$

$$V_a(s) = (R_a + s \cdot L_a)\, I_a(s) + K_1 s\, \theta(s) \quad \text{—— (2)}$$

For a given DC motor, torque developed in armature

$$T_a(t) \propto \phi\, i_a(t)$$

The armature voltage controll method field flux is kept constant,

$$T_a(t) \propto i_a(t)$$

$$T_a(t) = K_2\, i_a(t) \quad \text{—— (2)}$$

equilibrium equation of mechanical system is given by

$$T_a(t) = J_L \frac{d^2\theta(t)}{dt^2} + B_L \frac{d\theta(t)}{dt} \quad \text{—— (3)}$$

sub (2) in (3)

$$K_2\, i_a(t) = J_L \cdot \frac{d^2\theta(t)}{dt^2} + B_L \cdot \frac{d\theta(t)}{dt}$$

Taking LT, zero Initial Condition

$$K_2\, I_a(s) = J_L \cdot s^2\, \theta(s) + B_L \cdot s \cdot \theta(s)$$

$$I_a(s) = (J_L \cdot s^2 + B_L \cdot s)\, \frac{\theta(s)}{K_2} \quad \text{—— (4)}$$

sub (4) in (2)

$$V_a(s) = \left[(R_a + s_1 L_a) \left[\frac{J_L s^2 + B_L \cdot s}{K_2} \right] + K_1 s \cdot \theta(s) \right]$$

$$TF = \frac{\theta(s)}{V_a(s)} = \frac{K_2}{(R_a + s_1 L_a)\,(J_L s^2 + B_L \cdot s) + K_1 K_2 s}$$

Input to system is armature supply e_a & output of system is angular displacement θ

(2) Field controlled DC servomotor

In this method voltage applied to armature ckt is kept constant, by varying voltage applied to field ckt, speed of dc motor is varied.

→ Apply KVL to field ckt.

$$V_f(t) = R_f I_f(t) + L_f \cdot \frac{d I_f(t)}{dt}.$$

Taking L.T,

$$V_f(s) = R_f \cdot I_f(s) + L_f \cdot s \cdot I_f(s)$$

$$V_f(s) = (R_f + L_f \cdot s) I_f(s) \quad \text{——— (1)}$$

Assumptions

(1) constant armature current is fed into motor

(2) $\phi_f \propto I_f$, flux produced is proportional to field current.

$$\phi_f = K_f I_f.$$

(3) Torque proportional to product of flux and armature current.

$$T_a(t) \propto \phi \, i_a(t)$$

$i_a(t)$ constant.

$$\therefore T_a(t) \propto \phi$$

$$T_a(t) \propto K_f \cdot I_f. \quad \text{——— (2)}$$

equilibrium equ of mechanical system.

$$T_a(t) = J_L \frac{d^2\theta(t)}{dt^2} + B_L \cdot \frac{d\theta(t)}{dt} \quad \text{---} (3)$$

(2) = (3)

$$K_f I_f = J_L \cdot \frac{d^2\theta(t)}{dt^2} + B_L \cdot \frac{d\theta(t)}{dt}$$

Taking LT,

$$K_f I_f(s) = (J_L \cdot s^2 + B_L s)\,\theta(s)$$

$$I_f(s) = \left(\frac{J_L s^2 + B_L s}{K_f}\right)\theta(s) \quad \text{---} (4)$$

subs (4) in eq (1)

$$V_f(s) = (R_f + sL_f)\left(\frac{J_L s^2 + B_L s}{K}\right)\cdot\theta(s)$$

$$T.F = \frac{\theta(s)}{V_f(s)} = \frac{K}{(R_f + sL_f)(J_L s^2 + B_L s)}$$

<u>Applications of DC servomotor</u>

(1) Air craft controll systems

(2) Electromechanical actuators

(3) process controllers.

(4) Robotics.

(5) Machine tools

→ The work done by one gear is same as other.

$$T_1\theta_1 = T_2\theta_2 \qquad \frac{T_1}{T_2} = \frac{\theta_2}{\theta_1} = \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

distance $= r\theta$

(1) The number of teeth N are proportional to radius r of a gear.

(2) The distance travelled on each gear is same

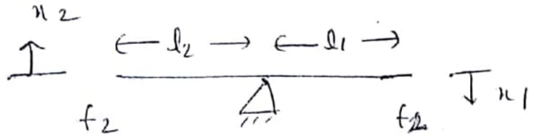
(3) Work done = $T\theta$ by each gear is same

Armature controlled	field controlled
(1) field current is kept const	(1) Armature current kept
(2) controll voltage is applied to the armature	(2) controll voltage is applied to the field.
(3) ~~open~~ closed - loop system	(3) ~~closed~~ open loop system
(4) Better efficiency	(4) poor efficiency.

Servomotors

These motor are used to convert electrical sign applied, into the angular velocity or movement of

levers



By law of moment,

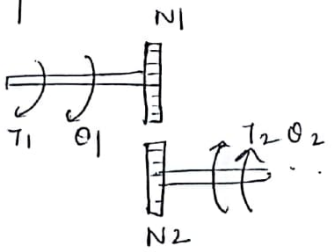
$$f_1 l_1 = f_2 l_2$$

by work done,

$$f_1 x_1 = f_2 x_2$$

$$\frac{f_1}{f_2} = \frac{l_2}{l_1} = \frac{x_2}{x_1}.$$

Gear trains



A gear train is a mechanical devi that transmits energy from one pa of system to another in such way that force, torque, speed and displacement may be altered.

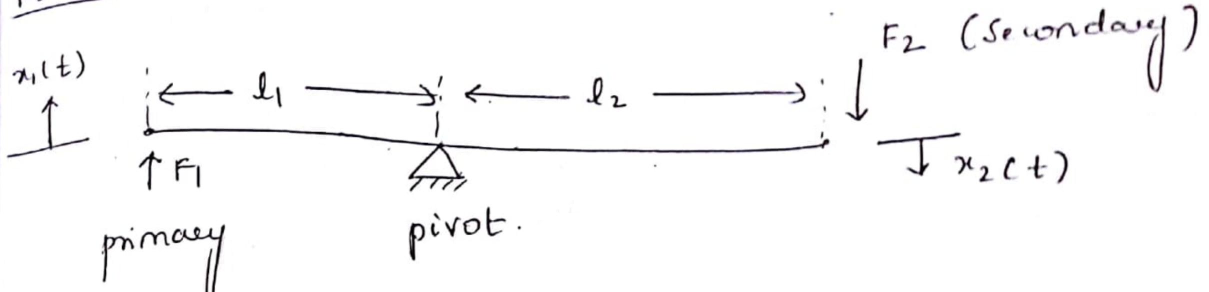
→ The number of teeth on the surface of gears is proportional to radii r_1 & r_2 of gears.

$$\boxed{r_1 N_2 = r_2 N_1}$$

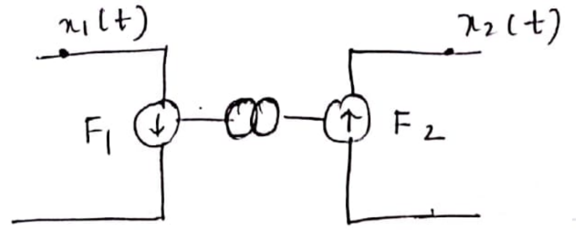
distance travelled along the surface of each gear is same

$$\theta_1 r_1 = \theta_2 r_2$$

Lever



Mechanical n/w for lever



Displacement is proportional to length of the arm

i.e. $x_1(t) \propto l_1$

$x_2(t) \propto l_2$

$$\frac{x_1(t)}{x_2(t)} = \frac{l_1}{l_2}$$

$$u_1(t) = \frac{dx_1(t)}{dt}$$

$$u_2(t) = \frac{dx_2(t)}{dt}.$$

i.e. $$\frac{u_1(t)}{u_2(t)} = \frac{l_1}{l_2}$$

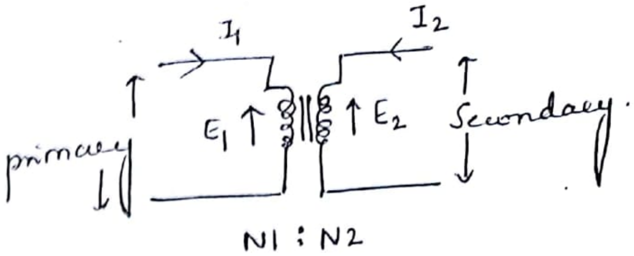
At equilibrium $F_1 l_1 = F_2 l_2$

$$\frac{F_2}{F_1} = \frac{l_1}{l_2}$$

$$\therefore \frac{F_2}{F_1} = \frac{l_1}{l_2} = \frac{x_1}{x_2} = \frac{u_1}{u_2} \quad \text{——} \; (1)$$

$\therefore$ F_1 and F_2 are the Induced forces at primary and Secondary side of lever respectively.

The Electrical analog for lever is transformer.



$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$E_1 I_1 = E_2 I_2$$

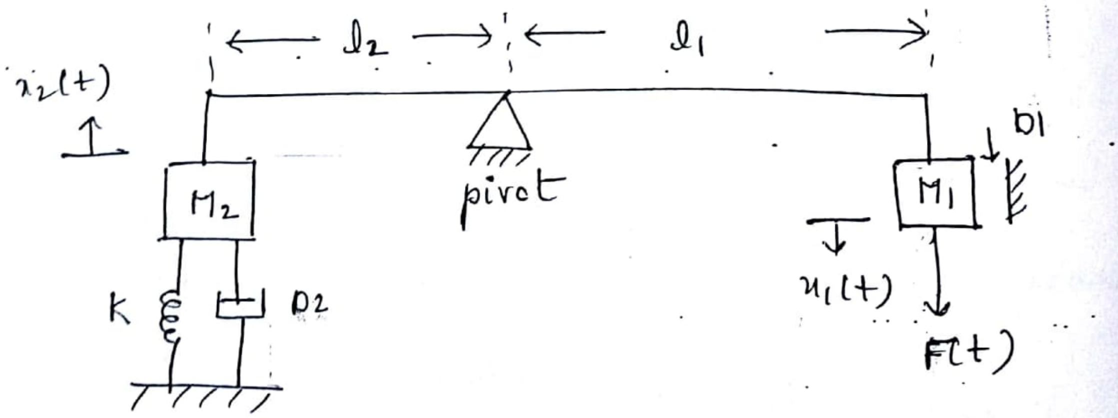
$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = \frac{q_1}{q_2} \quad \text{——(2)}$$

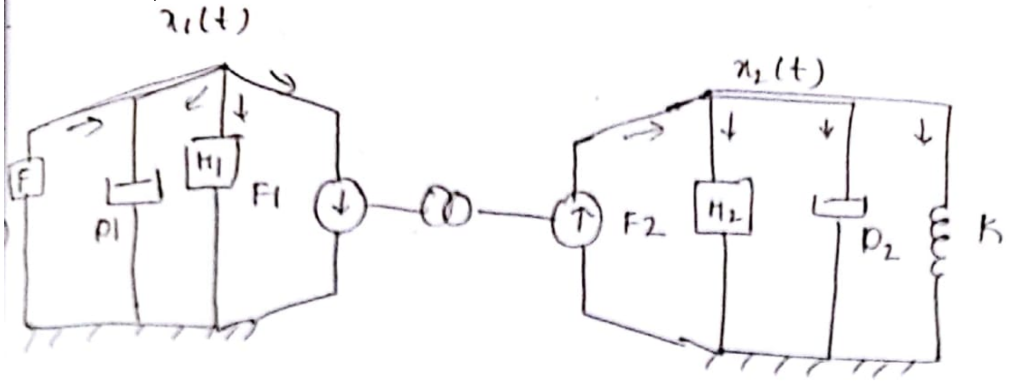
for FV analogy eq (1) is compared with equ (2).

FI →
$$\frac{I_2}{I_1} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{\phi_1}{\phi_2}$$

for FI analogy eq (1) is compared with eq (3)

(1) for mech system shown in fig. find the T.F $\frac{X_2(s)}{F(s)}$ also draw FV & FI electrical analogous.





The mechanical n/w is as shown above

$$F(t) = M_1 \frac{d^2 x_1(t)}{dt^2} + D_1 \frac{d x_1(t)}{dt} + F_1 \quad \text{———} (1)$$

$$F_2 = M_2 \frac{d^2 x_2(t)}{dt^2} + D_2 \frac{d x_2(t)}{dt} + K \cdot x_2(t) \quad \text{———} (2)$$

for the lever,

$$\frac{F_2}{F_1} = \frac{l_1}{l_2} = \frac{x_1}{x_2} \; ; \quad \frac{F_2(t)}{F_1(t)} = \frac{l_1}{l_2}$$

$$F_2(t) = \frac{l_1}{l_2} F_1(t).$$

&

$$x_1(t) = \frac{l_1}{l_2} x_2(t).$$

Taking laplace transform assuming zero Initial condition from (1)

$$F(s) = M_1 s^2 X_1(s) + D_1 s X_1(s) + F_1(s)$$

$$F(s) = (M_1 s^2 + D_1 s) X_1(s) + F_1(s) \quad \text{———} (3)$$

from (2)

$$F_2(s) = M_2 \cdot s^2 X_2(s) + D_2 \cdot s X_2(s) + K X_2(s)$$

$$F_2(s) = (M_2 s^2 + D_2 s + K) X_2(s) \quad \text{———} (4)$$

WKT,

$$F_1(s) = \frac{l_2}{l_1} F_2(s)$$

$$X_1(s) = \frac{l_1}{l_2} X_2(s)$$

subs this in eq (3)

$$F(s) = (M_1 s^2 + D_1 s) \frac{l_1}{l_2} X_2(s) + \frac{l_2}{l_1} F_2(s)$$

$$F(s) - (M_1 s^2 + D_1 s) \frac{l_1}{l_2} X_2(s) = \frac{l_2}{l_1} F_2(s)$$

$$\frac{l_1}{l_2}\left[F(s) - (M_1 s^2 + D_1 s) \frac{l_1}{l_2} X_2(s)\right] = F_2(s) \quad \text{—— (5)}$$

sub (4) in (5)

$$\frac{l_1}{l_2}\left[F(s) - (M_1 s^2 + D_1 s) \frac{l_1}{l_2} X_2(s)\right] = (M_2 s^2 + D_2 s + K) X_2(s)$$

$$\therefore F(s) = \left[\frac{l_2}{l_1} (M_2 s^2 + D_2 s + K) + \frac{l_1}{l_2} (M_1 s^2 + D_1 s)\right] X_2$$

$$\therefore T.F = \frac{X_2(s)}{F(s)} = \frac{1}{(M_2 s^2 + D_2 s + K)\frac{l_2}{l_1} + \frac{l_1}{l_2}(M_1 s^2 + D_1 s)}$$

→ <u>FV</u> <u>analogy</u>

Subs analogs in eq (1) and (2)

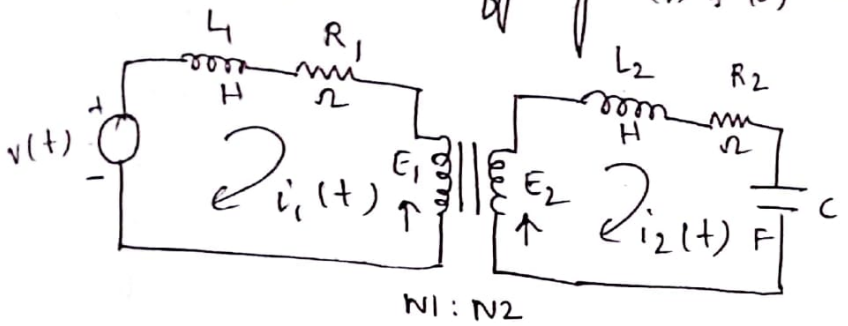
from (1)

$$v(t) = L_1 \frac{d\, i_1(t)}{dt} + R_1 i_1(t) + E_1 \quad \text{—— (7)}$$

from (2).

$$E_2 = L_2 \frac{di_2(t)}{dt} + R_2 i_2(t) + \frac{1}{C}\int i_2(t)\, dt \quad \text{—— (8)}$$

Electrical ckt satisfying (7) & (8) is as shown



$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2} = \frac{I_1}{I_2} \Leftrightarrow \frac{F_2}{F_1} = \frac{l_2}{l_1} = \frac{x_1}{x_2} = \frac{u_1}{u_2}$$

FI Analogy

from (1)

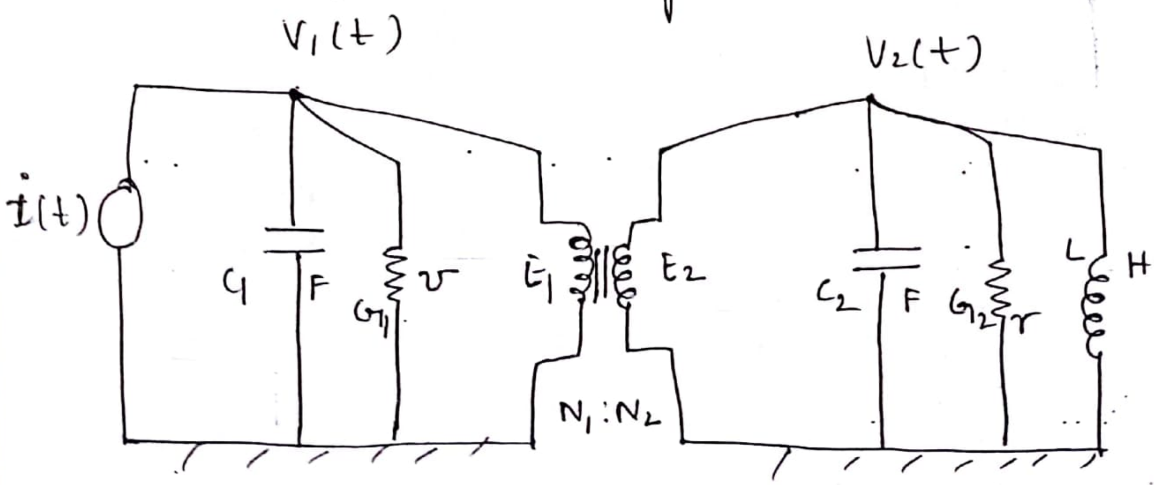
$$i(t) = C_1 \frac{dv_1(t)}{dt} + G_1 v_1(t) + i_1 \quad \text{--- (9)}$$

from (2)

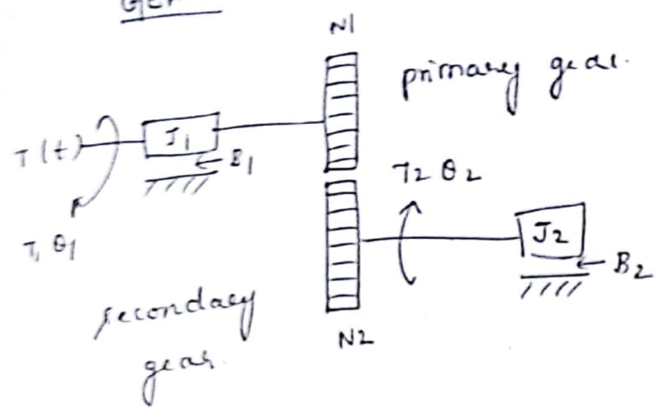
$$i_2 = C_2 \frac{dV_2(t)}{dt} + G_1 V_2(t) + \frac{1}{L}\int V_2(t)\, dt \quad \text{--- (10)}$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{\phi_1}{\phi_2} \Leftrightarrow \frac{F_2}{F_1} = \frac{l_2}{l_1} = \frac{x_1}{x_2} = \cancel{\frac{\theta_1}{\theta_2}}\ \frac{u_1}{u_2}$$

Electrical circuit satisfying (9) & (10)



GEAR TRAIN



T → Applied torque

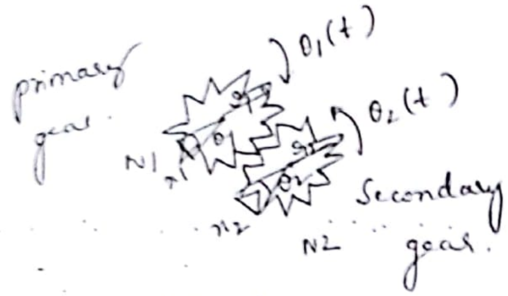
N_1, N_2 → Number of teeth

$\theta_1\ \theta_2$ → Angular displa

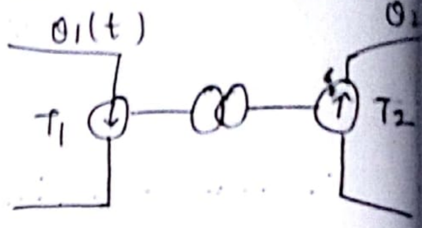
J_1, J_2 → Inertia of gear

B_1, B_2 → Friction coeff

T_1, T_2 → Torque transmitted gears.



Mechanical n/w for gea



r_1 & r_2 are the radii of primary & secondary gear. linear distance travel by both gear wheels is

i.e $x_1 = x_2$

$$r_1\,\theta_1(t) = r_2\,\theta_2(t).$$

diff w.r.t t.

$$r_1 \frac{d\theta_1(t)}{dt} = r_2 \frac{d\theta_2(t)}{dt}.$$

$$\boxed{r_1\omega_1 = r_2\omega_2}$$

$$\omega(t) = \frac{d\theta(t)}{dt}$$

diff w.r.t 't'

$$r_1 \frac{d\omega_1}{dt} = r_2 \frac{d\omega_2}{dt}.$$

$$r_1\alpha_1 = r_2\,\alpha_2$$

α_1 and α_2 are the angular acceleration of primary and secondary gears respectively.

In an Ideal gear train, power in primary gear is equal to power in secondary gear.

$$\boxed{P = T_1 \omega_1 = T_2 \omega_2}$$

No of teeth on a gear wheel is proportional to radius of gear wheel.

$$N_1 \propto r_1$$

$$N_2 \propto r_2$$

$$\boxed{\frac{N_2}{N_1} = \frac{r_2}{r_1}}$$

$$\frac{N_2}{N_1} = \frac{r_2}{r_1} = \frac{\theta_1}{\theta_2} = \frac{\omega_1}{\omega_2} = \frac{\alpha_1}{\alpha_2} = \frac{T_2}{T_1}$$

Electrical analog for a pair of gear wheels is transformer, for

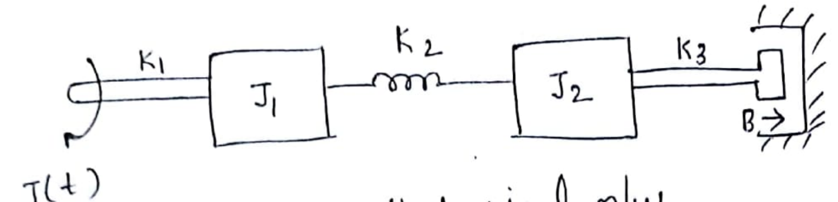
T-V Analogy

$$\frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{\theta_1}{\theta_2} \quad \text{is compared with} \quad \frac{E_2}{E_1} = \frac{I_1}{I_2} = \frac{N_{2T}}{N_{2T}} = \frac{q_1(t)}{q_2(t)}$$

T-I Analogy

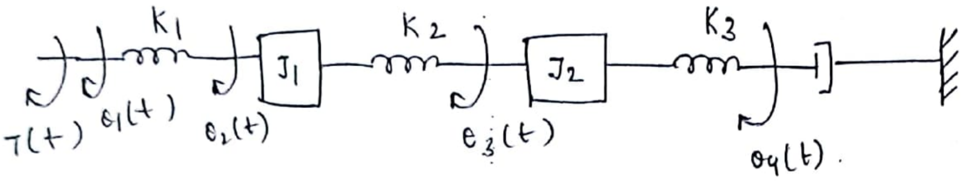
$$\frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{\theta_1}{\theta_2} \quad \text{is compared with} \quad \frac{I_2}{I_1} = \frac{E_1}{E_2} = \frac{N_{2T}}{N_{2T}} = \frac{\phi_1(t)}{\phi_2(t)}$$

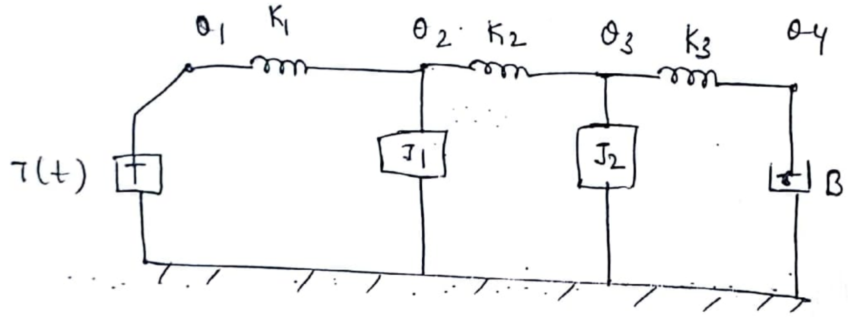
(1)



T–V
T-I

Mechanical n/w





T – V
J/M – L
B – R
K – 1/C

At $\theta_1(t)$,

$$T(t) = K_1(\theta_1 - \theta_2) \quad \text{——} (1)$$

At $\theta_2(t)$

$$K_1(\theta_1 - \theta_2) = J_1 \frac{d^2\theta_2(t)}{dt^2} + K_2(\theta_2 - \theta_3) \quad \text{——} (2)$$

At $\theta_3(t)$

$$K_2(\theta_2 - \theta_3) = J_2 \frac{d^2\theta_3(t)}{dt^2} + K_3(\theta_3 - \theta_4) \quad \text{——} (3)$$

At $\theta_4(t)$

$$K_3(\theta_3 - \theta_4) = B \frac{d\theta_4(t)}{dt} \quad \text{——} (4)$$

F – T
M – J
K – K
B – B
$\frac{dx}{dt}$ – $d\theta/dt$
$\frac{d^2x}{dt^2}$ – $\frac{d^2\theta}{dt^2}$

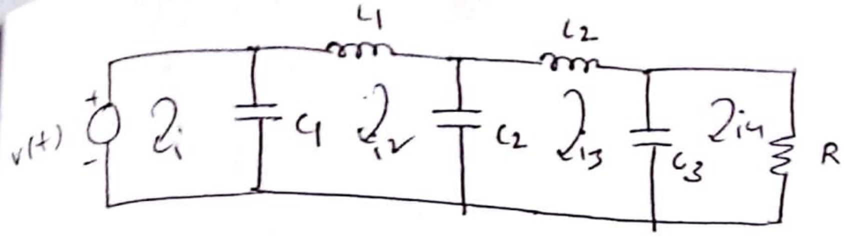
→ T–V analogy

from (1) $$v(t) = \frac{1}{C_1}\int (i_1 - i_2)\, dt \quad \text{—} (5)$$

(2) $$\frac{1}{C_1}\int (i_1 - i_2)\, dt = L_1 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_3)\, dt \quad \text{—} (6)$$

(3) $$\frac{1}{C_2}\int (i_2 - i_3)\, dt = L_2 \frac{di_3}{dt} + \frac{1}{C_3}\int (i_3 - i_4)\, dt \quad \text{—} (7)$$

(4) $$\frac{1}{C_3}\int (i_3 - i_4)\, dt = R\, i_4(t) \quad \text{—} (8)$$



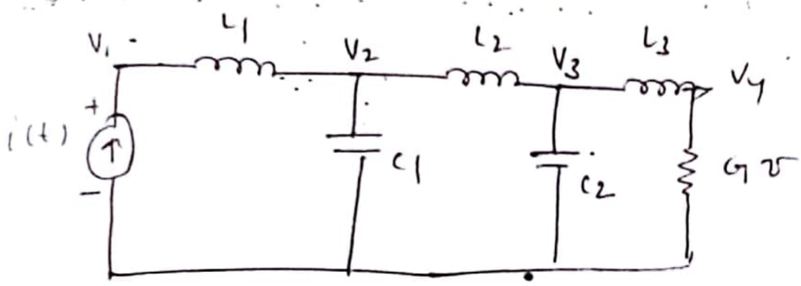
T-I

$$i(t) = \frac{1}{L_1}\int (V_1 - V_2)\cdot dt \quad \text{—— (9)}$$

$$\frac{1}{L_1}\int (V_1 - V_2)\cdot dt = C_1 \frac{dV_2}{dt} + \frac{1}{L_2}\int (V_2 - V_3)\cdot dt \quad \text{—— (10)}$$

$$\frac{1}{L_2}\int (V_2 - V_3)\cdot dt = C_2 \frac{dV_2}{dt} + \frac{1}{L_3}\int (V_3 - V_4)\cdot dt \quad \text{—— (11)}$$

$$\frac{1}{L_3}\int (V_3 - V_4)\cdot dt = G_1 \cdot V_4 \quad \text{—— (12)}$$



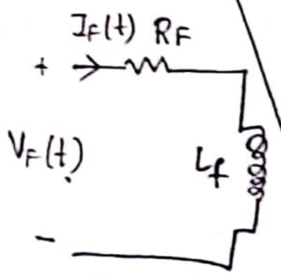
DC Servomotor

The speed of a DC motor can be controlled by 2 methods.

(1) Armature Voltage controll
(2) Field controll

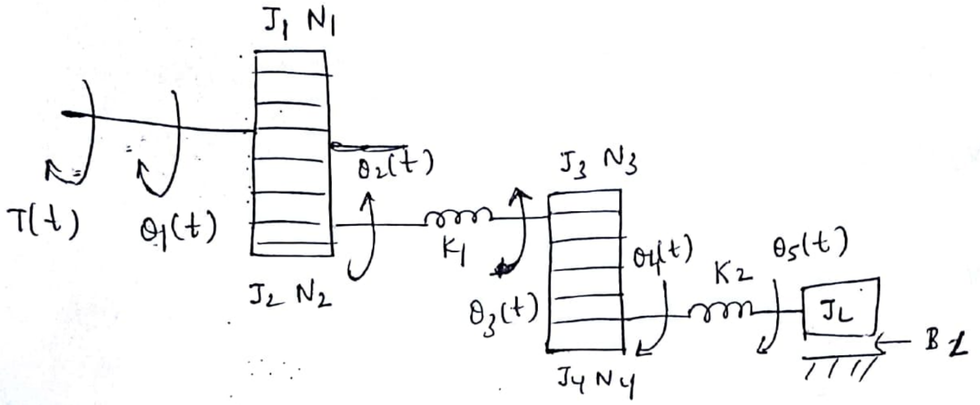
(1) Armature Voltage controlled DC servomotor

Consider a separately excited DC motor as shown below.

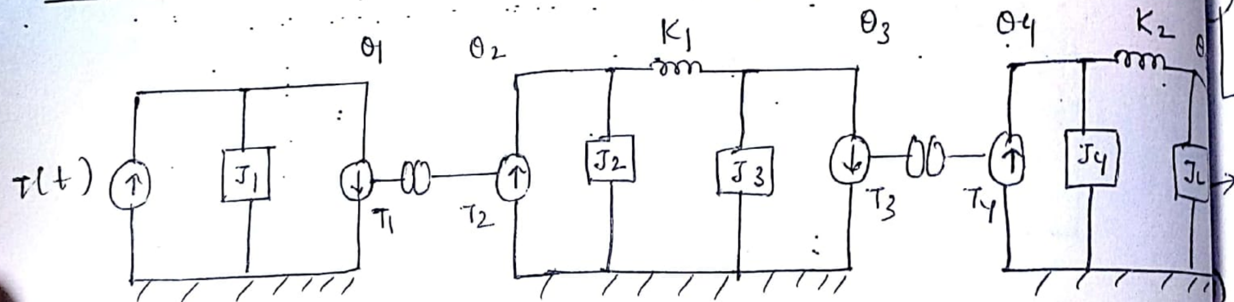


field ckt.

1) For the mechanical system shown below. obtain equilibrium equations of the system. Also draw torque-V & T-I electrical analogous



Sol^n: The mechanical n/w is as shown



at $\theta_1(t)$,

$$T(t) = J_1 \frac{d^2\theta_1(t)}{dt^2} + T_1 \quad \text{--- (1)}$$

at $\theta_2(t)$.

$$T_2 = J_2 \frac{d^2\theta_2(t)}{dt^2} + k_1(\theta_2 - \theta_3) \quad \text{--- (2)}$$

at $\theta_3(t)$.

$$k_1(\theta_2 - \theta_3) = J_3 \frac{d^2\theta_3(t)}{dt^2} + T_3 \quad \text{--- (3)}$$

at $\theta_4(t)$.

$$T_4 = J_4 \frac{d^2\theta_4(t)}{dt^2} + k_2(\theta_4 - \theta_5) \quad \text{--- (4)}$$

at $\theta_5(t)$.

$$k_2(\theta_4 - \theta_5) = J_L \frac{d^2\theta_5(t)}{dt^2} + B_L \frac{d\theta_5(t)}{dt} \quad \text{--- (5)}$$

→ TV Analogy (T-V) $\left[\frac{T_2}{T_1} = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2} \Rightarrow \frac{E_2}{E_1} = \frac{N_{2T}}{N_{1T}} = \frac{I_1}{I_2} \right]$ (E)

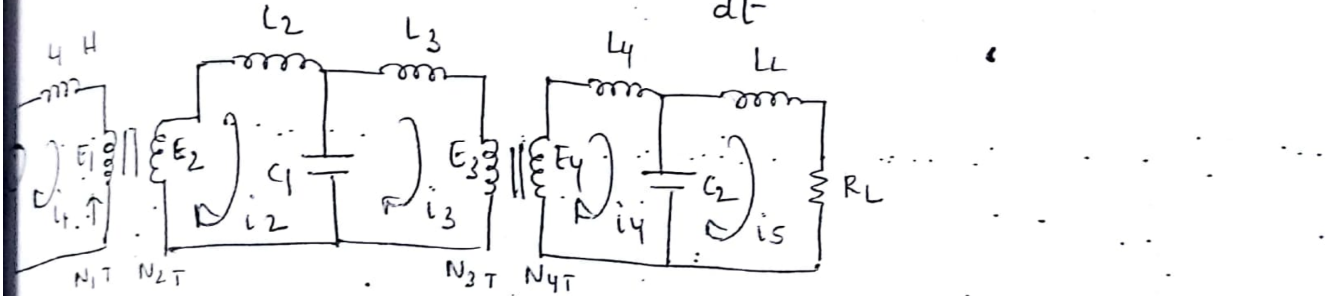
from ①, $$V(t) = L_1 \frac{di_1(t)}{dt} + E_1 \quad \text{— (6)}$$

from ②, $$E_2 = L_2 \frac{di_2(t)}{dt} + \frac{1}{C_1}\int (i_2 - i_3)\, dt \quad \text{— (7)}$$

from ③, $$\frac{1}{C_1}\int (i_2 - i_3)\cdot dt = L_3 \frac{di_3(t)}{dt} + E_3 \quad \text{— (8)}$$

from ④, $$E_4 = L_4 \frac{di_4(t)}{dt} + \frac{1}{C_2}\int (i_4 - i_5)\cdot dt \quad \text{— (9)}$$

from ⑤, $$\frac{1}{C_2}\int (i_4 - i_5)\cdot dt = L_L \frac{di_5(t)}{dt} + R_L\, i_5(t) \quad \text{— (10)}$$



T-I Analogy $\left[\frac{T_2}{T_1} = \frac{N_2}{N_1} = \frac{\omega_1}{\omega_2} \Rightarrow \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{E_1}{E_2} \right]$

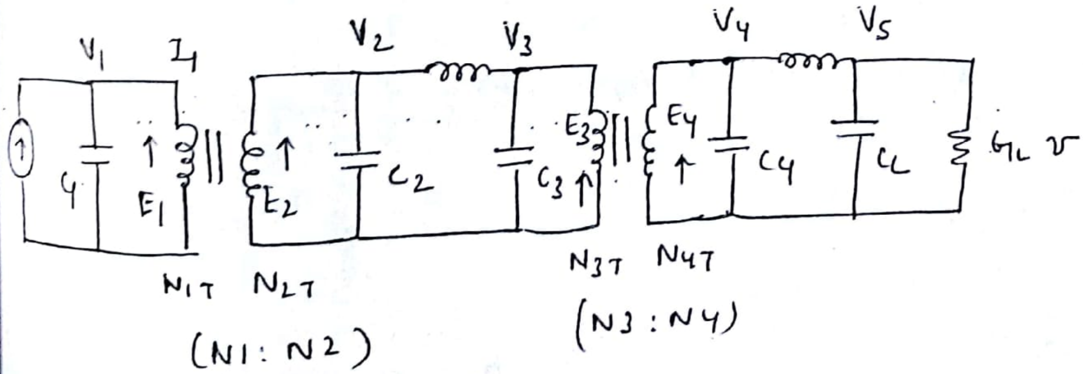
$$\Rightarrow \quad i(t) = C_1 \frac{dV_1}{dt} + i_1(t) \quad \text{— (11)}$$

$$\Rightarrow \quad i_2(t) = C_2 \frac{dV_2}{dt} + \frac{1}{L_1}\int (V_2 - V_3)\cdot dt \quad \text{— (12)}$$

$$\Rightarrow \quad \frac{1}{L_1}\int (V_2 - V_3)\, dt = C_3 \frac{dV_3}{dt} + i_3(t) \quad \text{— (13)}$$

$$\Rightarrow \quad i_4(t) = C_4 \frac{dV_4}{dt} + \frac{1}{L_2}\int (V_4 - V_5)\cdot dt \quad \text{— (14)}$$

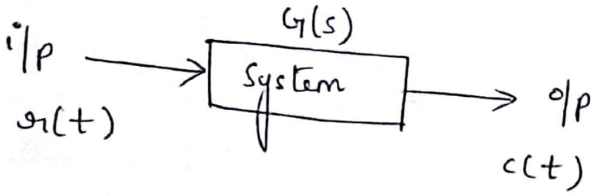
$$\Rightarrow \quad \frac{1}{L_2}\int (V_4 - V_5)\cdot dt = C_L \frac{dV_5}{dt} + G_L V_5 \quad \text{— (15)}$$



Unit 2 : Block diagram Reduction Techniques

Transfer function

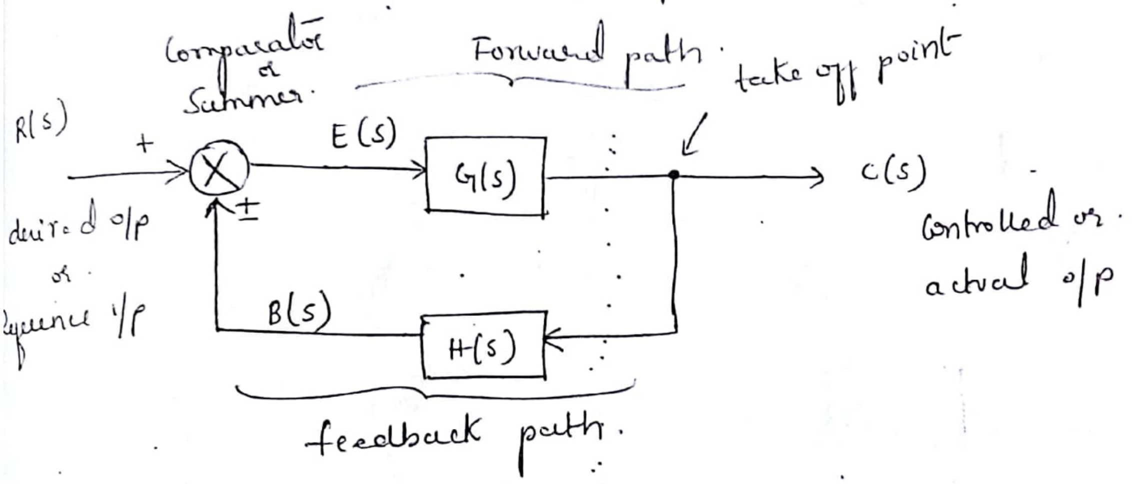
It is defined as the ratio of laplace transform (LT) of the output to the LT of Input with zero initial conditions.



T.F $\boxed{G(s) = \frac{c(s)}{r(s)}}$ $G(s) \to$ Gain of System

Rules of block diagram

Feedback CS or closed loop CS



$E(s) \to$ error signal

$B(s) \to$ primary feedback signal

Error signal $E(s) = R(s) \pm B(s)$ —— (1)

from defin of T.F

$$\frac{c(s)}{E(s)} = G(s) \quad \text{or} \quad E(s) = \frac{c(s)}{G(s)} \quad —— (2)$$

$$\frac{B(s)}{C(s)} = H(s) \quad \text{or} \quad B(s) = C(s) \cdot H(s)$$

Subs ② and ③ in ①

$$E(s) = R(s) \pm B(s).$$

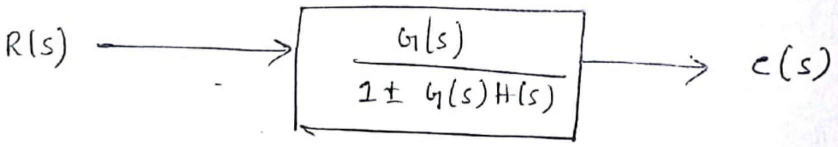
$$\frac{C(s)}{G(s)} = R(s) \pm C(s)\, H(s).$$

$$C(s) = G(s)\, R(s) \pm G(s)\, H(s)\, C(s)$$

$$C(s)\left[1 \pm G(s)\, H(s)\right] = G(s)\, R(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)\, H(s)}$$

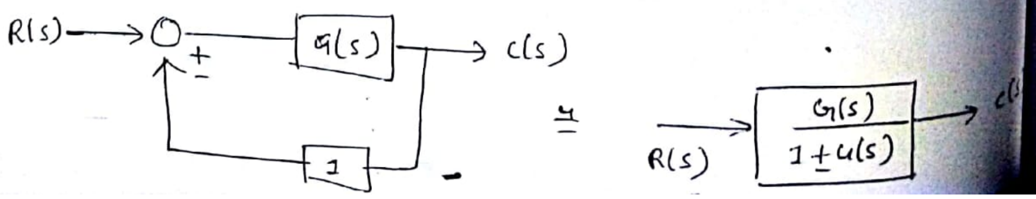
$B(s) = H(s)\, C(s)$



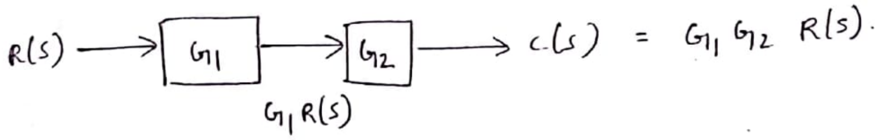
$\frac{C(s)}{R(s)}$ is known as overall transfer function.

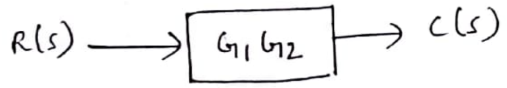
- G(s), H(s) is known as loop transfer function. If H(s) = 1, then its said to be unity feedback.
- $1 \pm G(s)\, H(s)$ is known as characteristic equation of system.
- G(s) is known as open loop transfer function

$\boxed{\text{If } H(s) = 1}$,



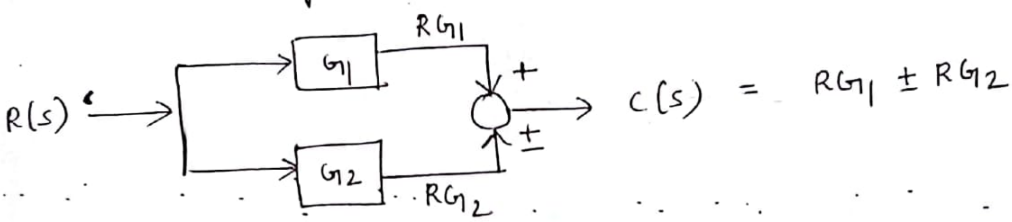
Blocks in Cascade (series)

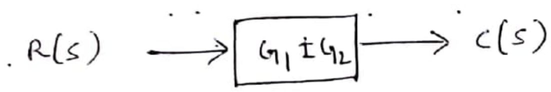




$$\therefore \frac{C(s)}{R(s)} = G_1 G_2$$

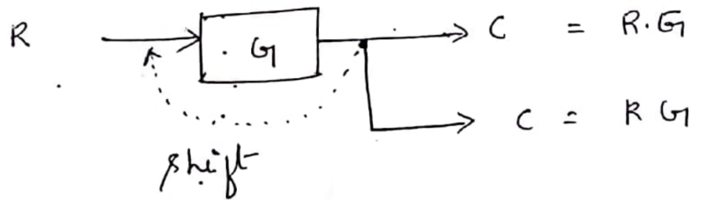
Blocks in parallel





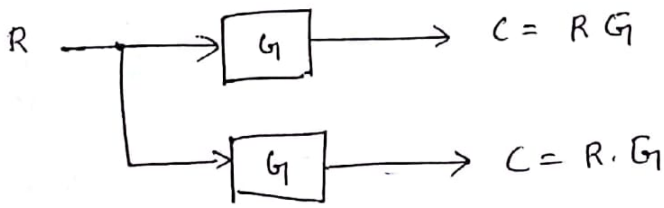
$$\therefore \frac{C(s)}{R(s)} = G_1 \pm G_2$$

Shifting of take off point ahead of block.



	Before	After
T	G	1/G
S	1/G	G

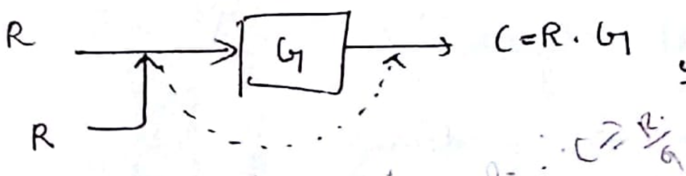
||



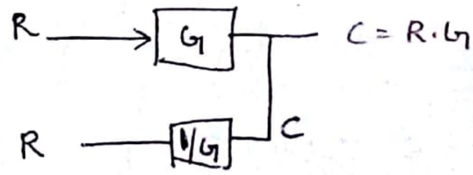
$g = \frac{C}{R}$

$1/g = \frac{R}{C}$

Shifting a takeoff point after the block.



||

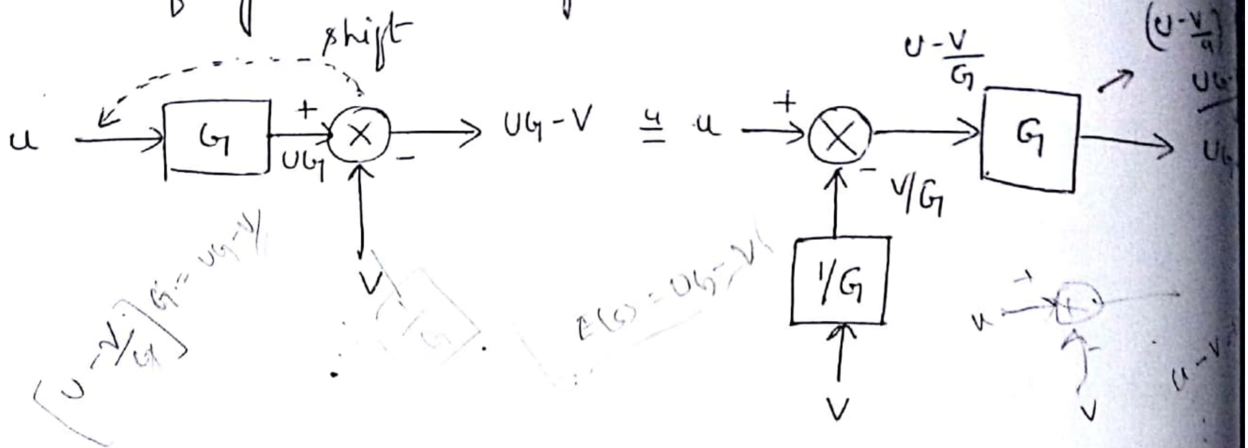


$\frac{C}{R} = G \quad G = \frac{R}{C} \quad C = \frac{R}{G}$

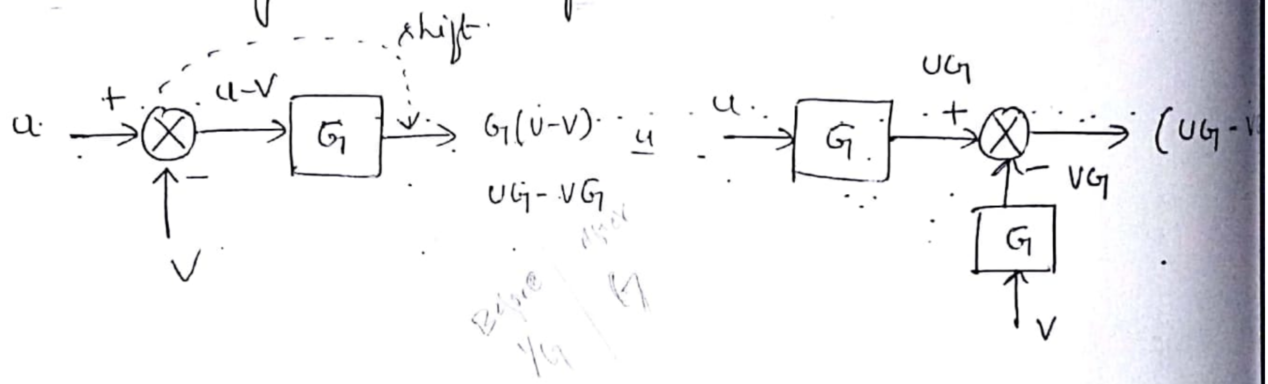
$R/C = 1/G$

$C = R \cdot G$

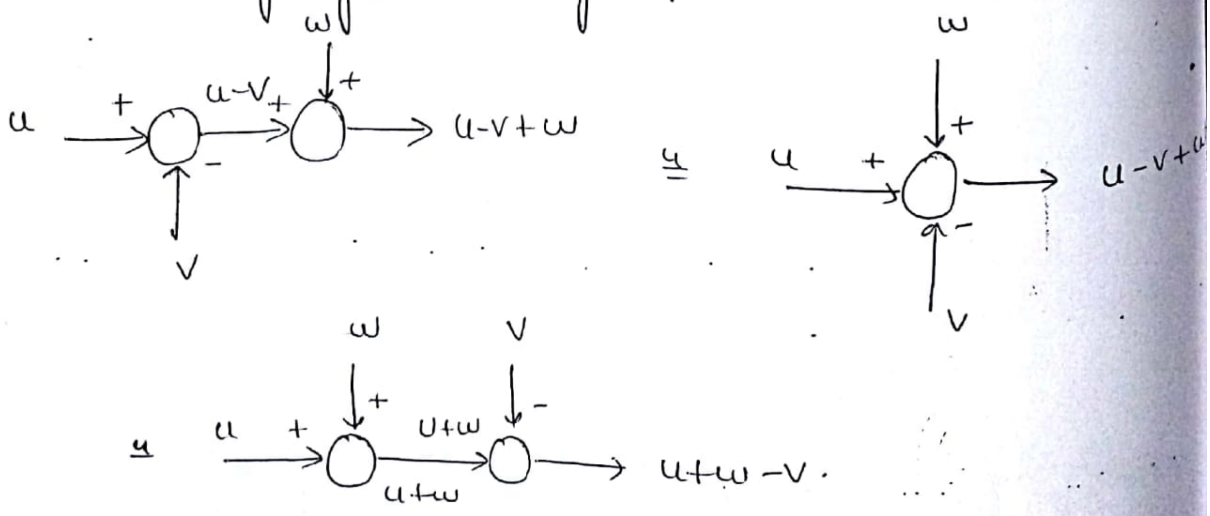
(6) Shifting a summing point ahead of block



(7) Shifting a summing point after the block.



(8) Re-arranging summing points



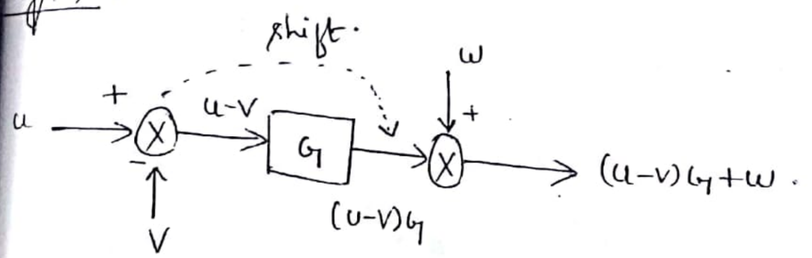
+ sign for negative feedback

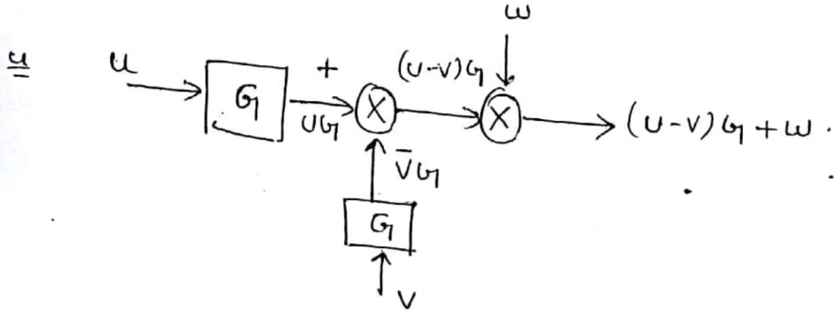
- sign for positive feedback.

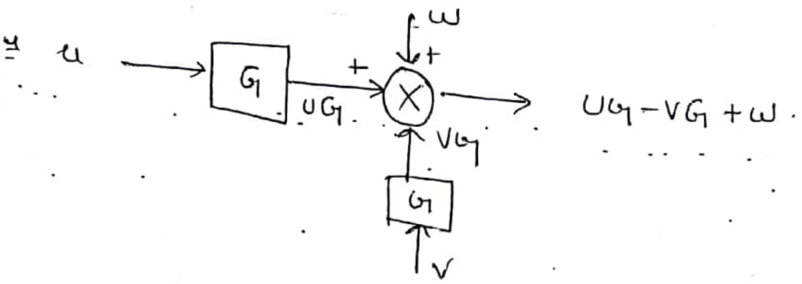
+ve -ve

-ve +ve

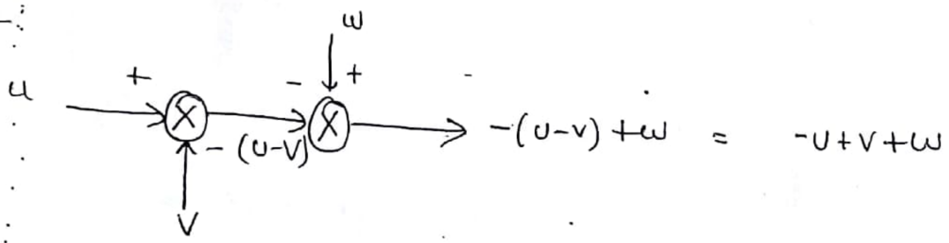
eg (1) :

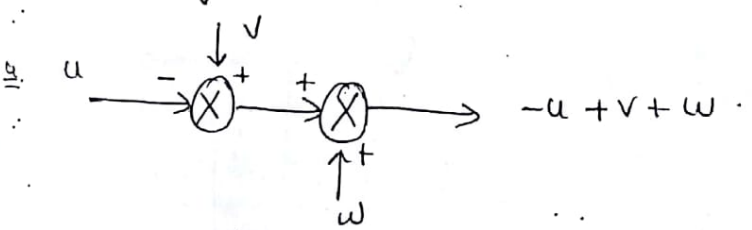

shift
u
+
−
V
u−V
G
(u−V)G
W
+
(u−v)G+W

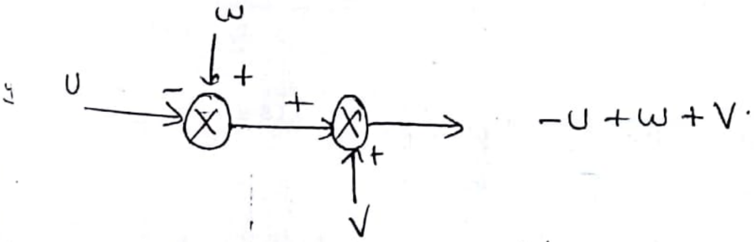

u
G
+
UG
(u−v)G
W
VG
G
V
(u−v)G+W

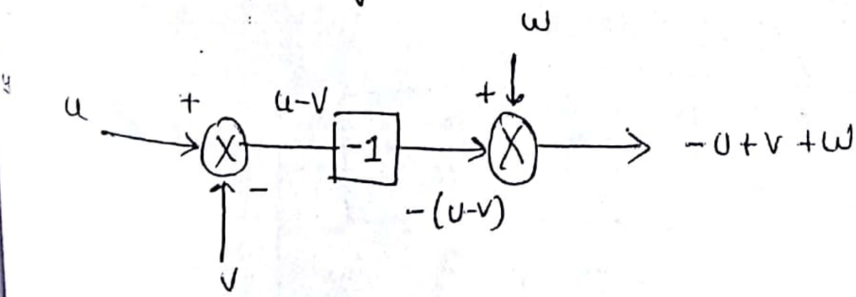

u
G
+
UG
W
+
VG
G
V
UG−VG+W

(2) :

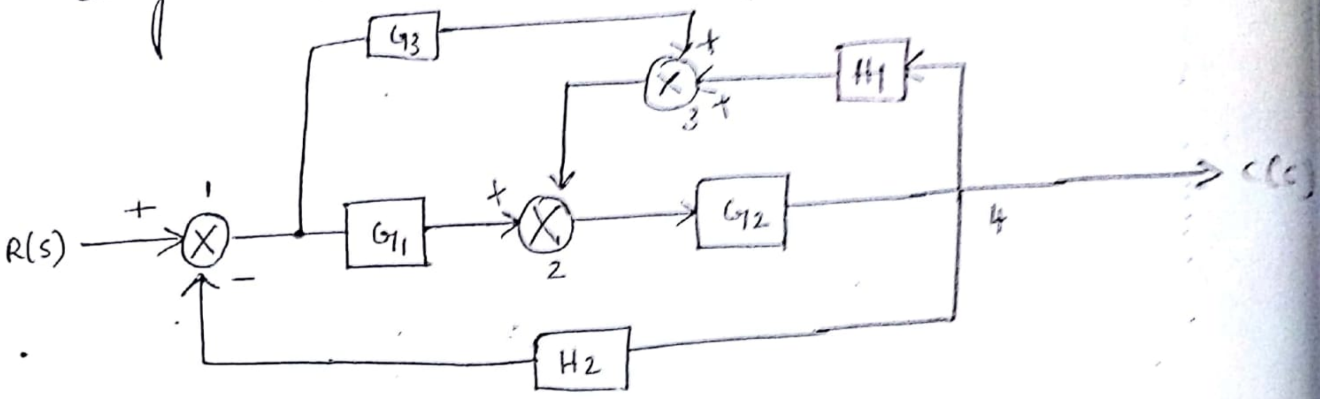

u
+
−
V
−(u−V)
W
+
−(u−v)+W = −U+V+W


u
V
−
+
+
+
W
−u+V+W

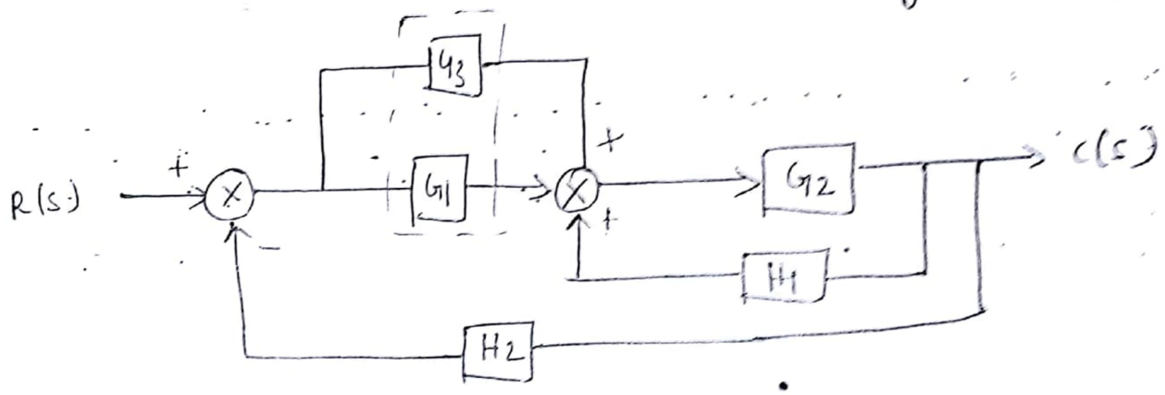

U
W
−
+
+
+
V
−U+W+V


u
+
−
V
u−V
−1
−(u−v)
W
+
−U+V+W

(1) Obtain the overall transfer function of the block diagram shown in fig by reduction technique.

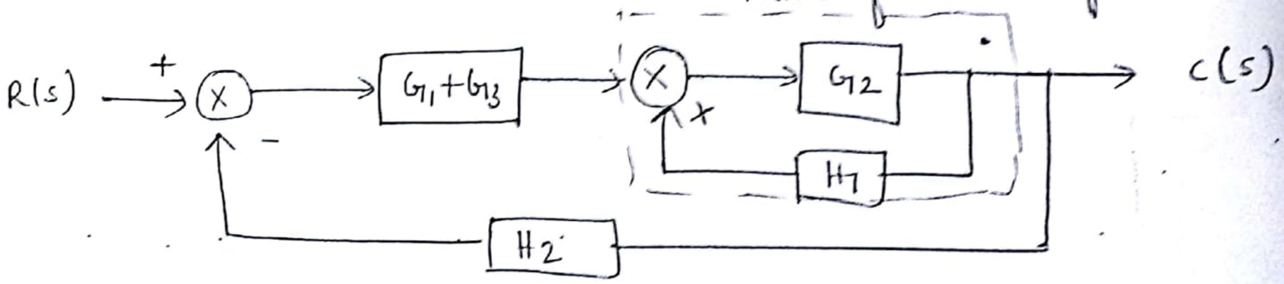


Sol^n : step 1 : Combine the 2 summing points (2) & (3)

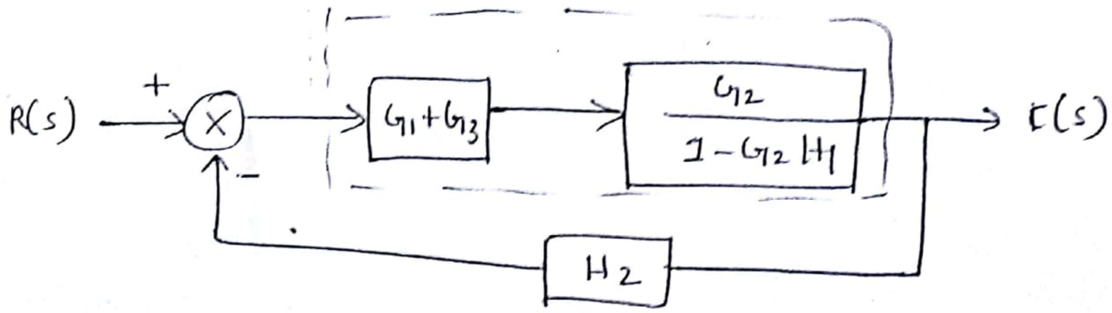


step 2 : Combine parallel blocks G_3 & G_1 = $G_1 + G_3$

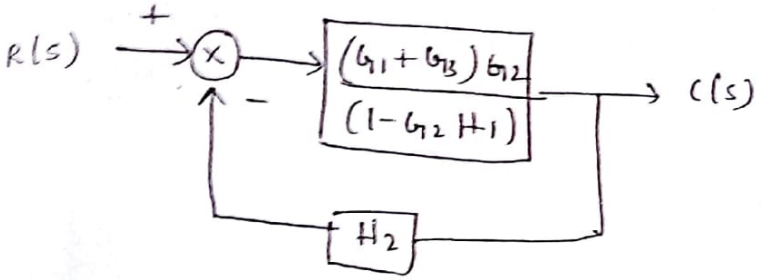
Minor feedback loop



step 3 : Eliminate the minor feedback loop $= \dfrac{G_2}{1-G_2H_1}$



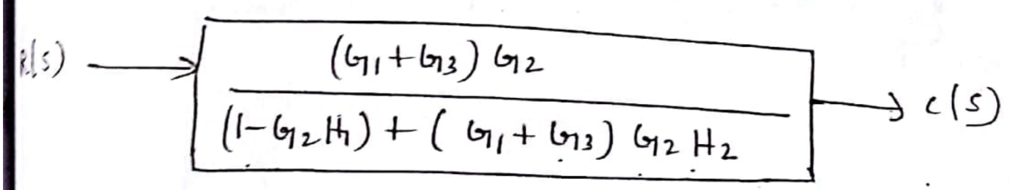
step 4 Combine the cascade blocks $= \dfrac{(G_1+G_3)G_2}{(1-G_2H_1)}$



step 5: Eliminate the minor feedback loop

$$= \frac{\dfrac{(G_1+G_3)G_2}{1-G_2H_1}}{1+\left(\dfrac{(G_1+G_3)\cdot G_2}{1-G_2H_1}\right)\cdot H_2}$$

$$= \frac{\dfrac{(G_1+G_3)G_2}{\cancel{1-G_2H_1}}}{\dfrac{(1-G_2H_1)+(G_1+G_3)G_2H_2}{\cancel{1-G_2H_1}}} = \frac{(G_1+G_3)G_2}{(1-G_2H_1)+(G_1+G_3)G_2H_2}$$



∴ overall transfer function is.

$$T.F = \frac{C(s)}{R(s)} = \frac{G_1G_2+G_2G_3}{(1-G_2H_1)+H_2G_2G_1+G_2G_3H_2}$$

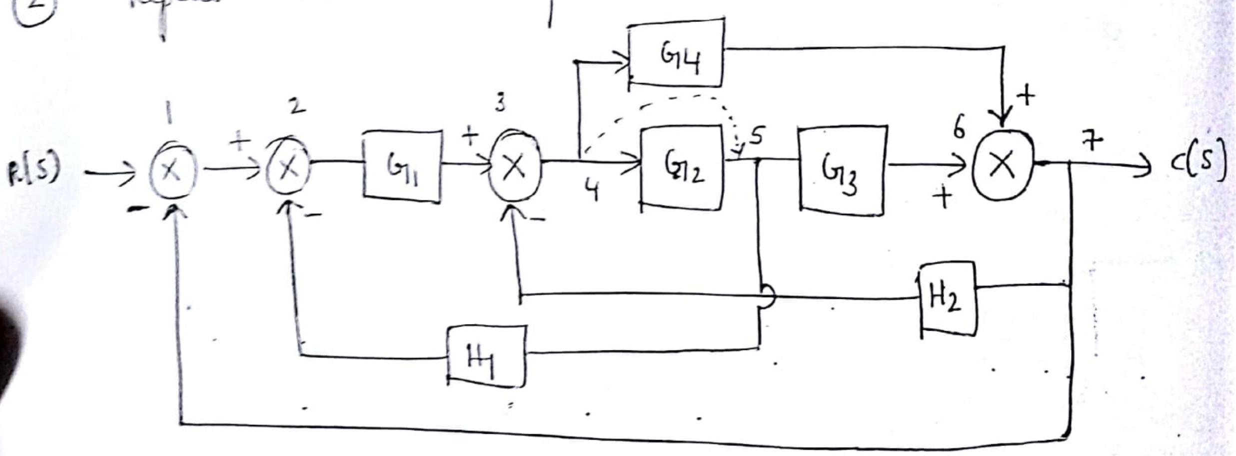
(2) → <u>General procedure</u> of reduction of block diagram.

Steps Involved in reduction of block diagram.

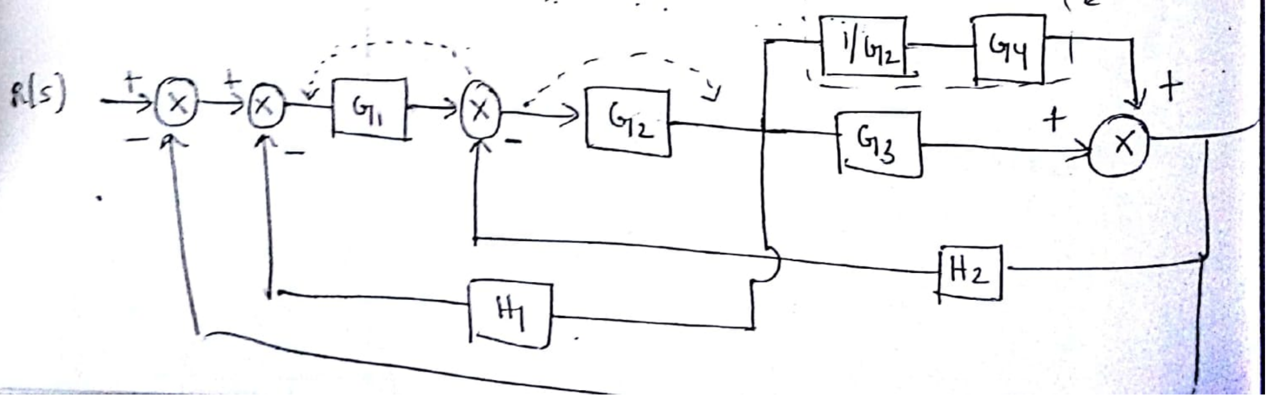
(1) Combine all the blocks in Cascade
(2) Combine all the blocks in parallel
(3) Eliminate all minor feedback loops
(4) Shift Summing points and take off points if needed
(5) Repeat step 1 to 4 until Canonical form has been obtained.
(6) Using standard transfer function of simple closed loop system, obtain $T.F = \frac{C(s)}{R(s)}$ of overall system.

<u>HINT</u> : As far as possible try to shift take off points towards right and Summing points to left.

(2) Repeat the above problem.

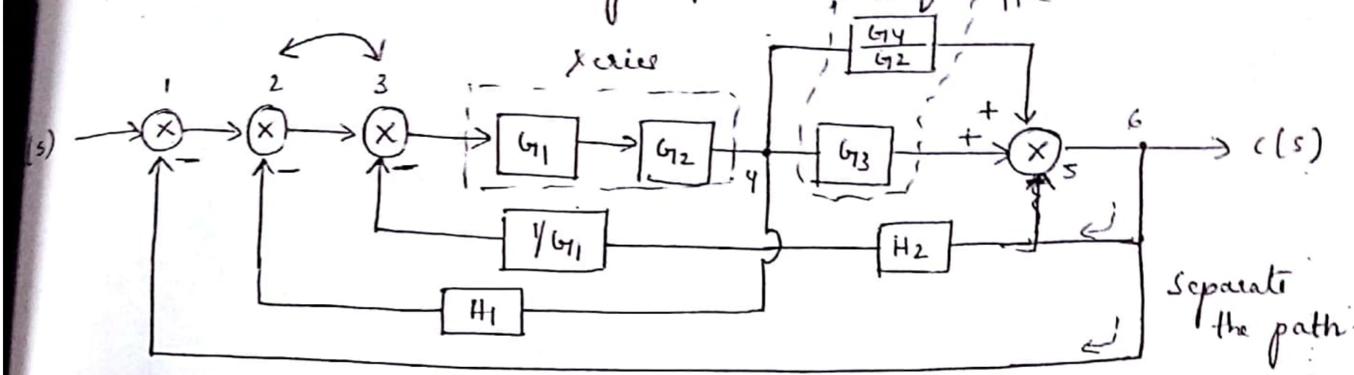


<u>Sol^n</u> : Step 1 Move the take off points (4) after the block series



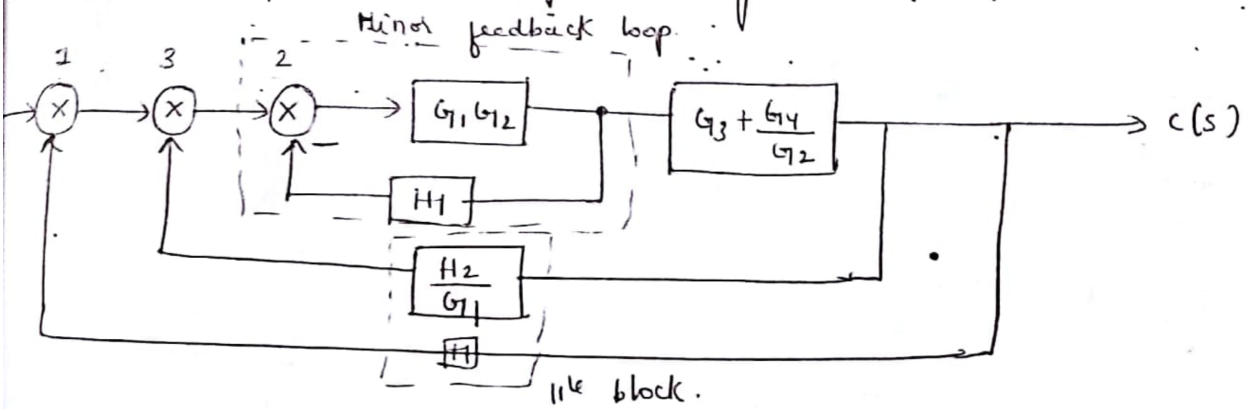
(5

Step 2: Move the summing point (3) before the block G_1



Step 3: Interchange the summing point 2 & 3

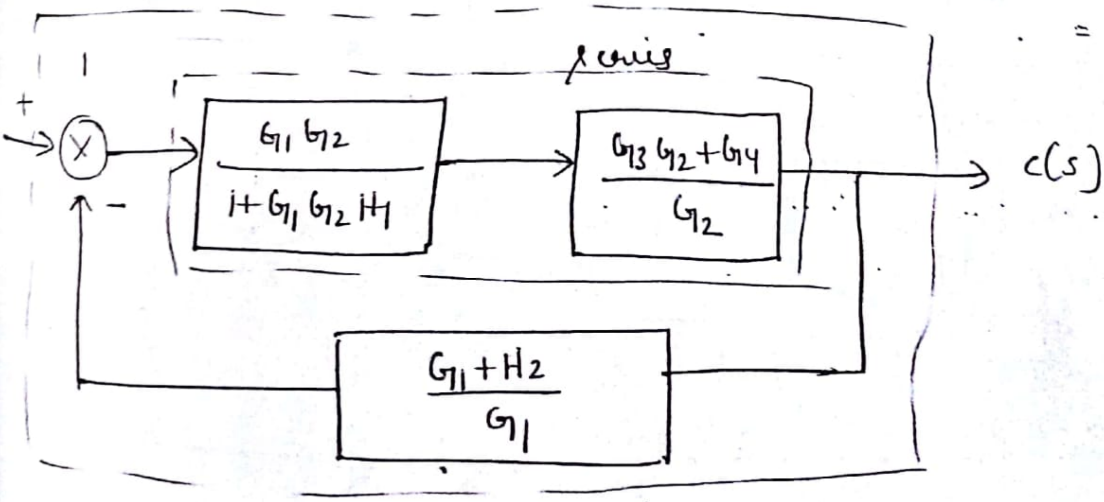
- Combine the parallel blocks $\frac{G_4}{G_2}$ & G_3,
- Combine the series blocks $1/G_1$ & H_2, G_1 & G_2
- Separate the path along the dotted lines



Step 4: Eliminate minor feedback $= \dfrac{G_1G_2}{1+G_1G_2H_1}$

Combine the parallel blocks of 1 & $\frac{H_2}{G_1}$ $= 1+\frac{H_2}{G_1}$

$= G_1+H_2/G_1$



Minor feed back loop.

step 5: • Combine the Cascade blocks.

$$\frac{G_1 \cancel{G_2}(G_2G_3+G_4)}{(1+G_1G_2H_1)\cancel{G_2}} = \frac{G_1(G_2G_3+G_4)}{(1+G_1G_2H_1)}$$

• Eliminating minor feedback loop

$$\frac{\dfrac{G_1(G_2G_3+G_4)}{(1+G_1G_2H_1)}}{1+\left[\dfrac{\cancel{G_1}(G_2G_3+G_4)}{(1+G_1G_2H_1)}\right]\times\left[\dfrac{G_1+H_2}{\cancel{G_1}}\right]} = \frac{\dfrac{G_1(G_2G_3+G_4)}{\cancel{(1+G_1G_2H_1)}}}{\dfrac{(1+G_1G_2H_1)+G_4(G_2G_3+G_4)(G_1+H_2)}{\cancel{(1+G_1G_2H_1)}}}$$

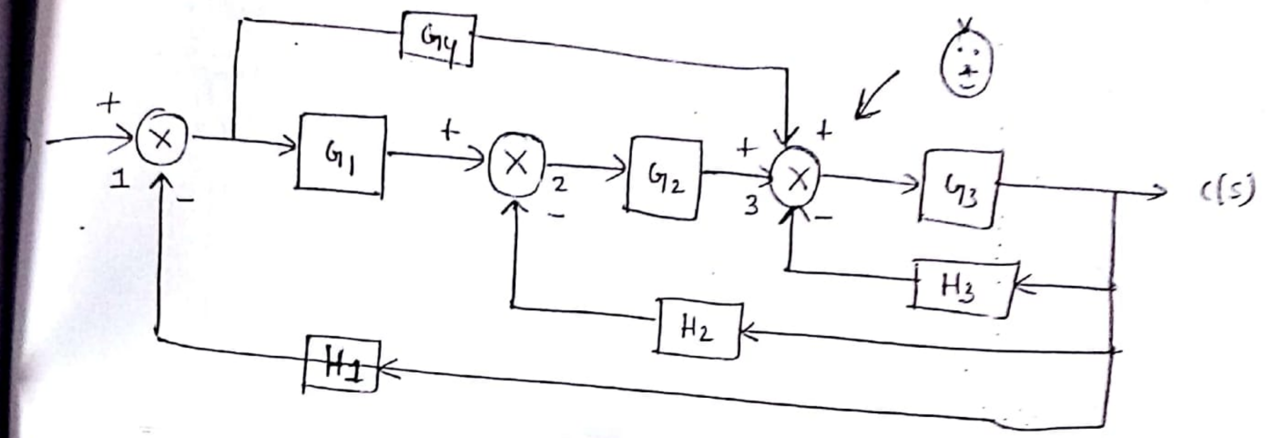
Thus

$$R(s) \rightarrow \boxed{\frac{G_1(G_2G_3+G_4)}{(1+G_1G_2H_1)+(G_2G_3+G_4)(G_1+H_2)}} \rightarrow C(s)$$

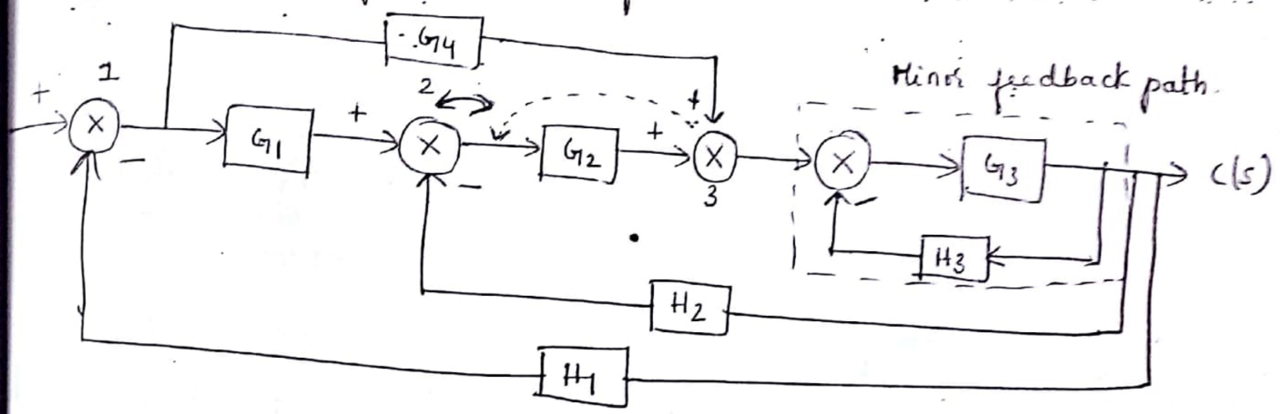
∴ closed loop transfer function given by

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3+G_1G_4}{(1+G_1G_2H_1)+(G_1G_2G_3+G_1G_4+G_2G_3H_2+G_4H_2)}$$

Reduce the given block diagram shown in fig and then obtain the transfer function of the system if $G_1 = G_2 = 1$, $G_3 = G_4 = 2$, $H_1 = H_2 = 1$, $H_3 = 2$

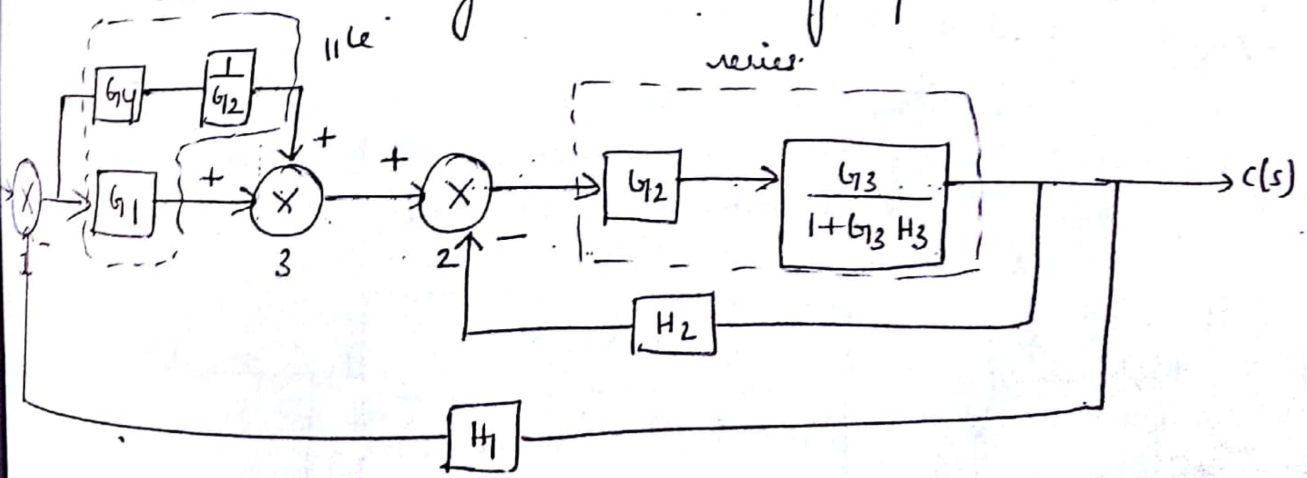


Sol[n] : step 1: Split the summing points (3) and Separate the paths



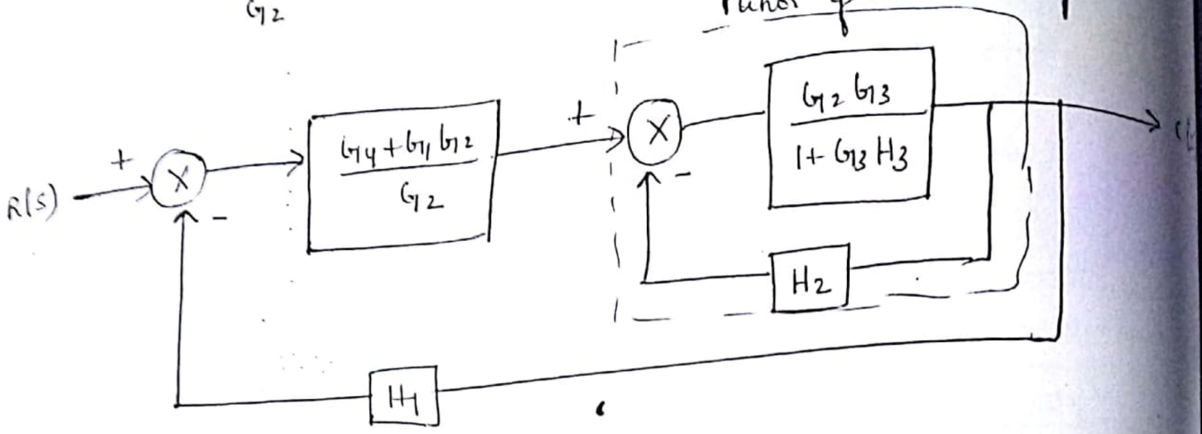
step 2: • Eliminate the minor feedback loop $= \frac{G_3}{1+G_3H_3}$

• Move the summing point (3) before the block G_2 and Interchange the summing points (2) & (3)



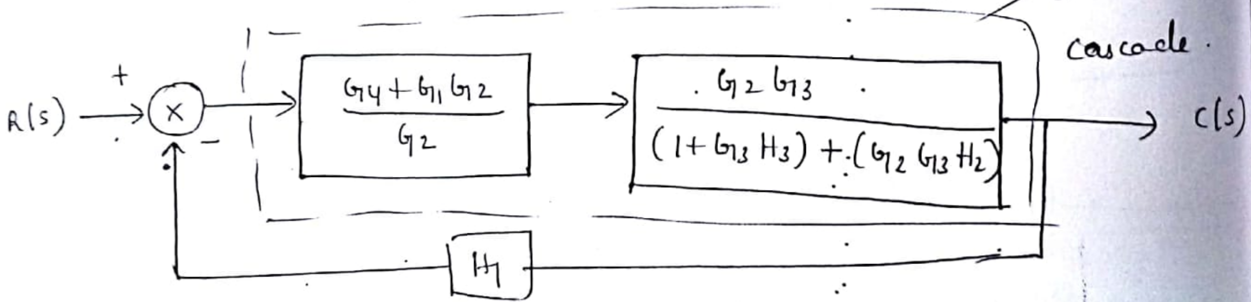
Step 3 : Combine the blocks in parallel and cascade

$$\frac{G_4}{G_2}+G_1=\frac{G_4+G_1G_2}{G_2} \quad \text{and} \quad \frac{G_2G_3}{1+G_3H_3}$$



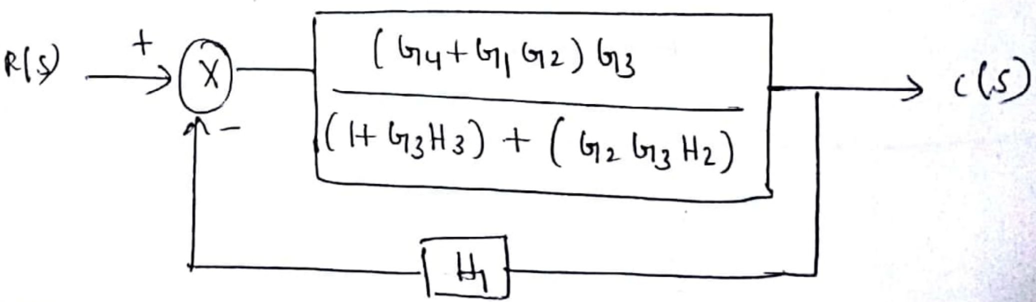
Step 4 : Eliminate the minor feedback loop

$$=\frac{\dfrac{G_2G_3}{1+G_3H_3}}{1+\left(\dfrac{G_2G_3}{1+G_3H_3}\right)(H_2)}=\frac{\dfrac{G_2G_3}{1+\cancel{G_3H_3}}}{\dfrac{(1+G_3H_3)+(G_2G_3H_2)}{1+\cancel{G_3H_3}}}$$



Step 5: Combine the cascade blocks.

$$\frac{(G_4+G_1G_2)\cancel{G_2}G_3}{\cancel{G_2}(1+G_3H_3)+(G_2G_3H_2)}$$



step 6: Eliminate the minor feed back loop

$$\frac{\dfrac{(G_4+G_1G_2)G_3}{1+G_3H_3+G_2G_3H_2}}{1+\left[\dfrac{(G_4+G_1G_2)G_3}{1+G_3H_3+G_2G_3H_2}\right]\times H_1} = \frac{\dfrac{G_4G_3+G_1G_2G_3}{\cancel{1+G_3H_3+G_2G_3H_2}}}{\dfrac{(1+G_3H_3+G_2G_3H_2)+H_1(G_4G_3+G_1G_2G_3)}{\cancel{1+G_3H_3+G_2G_3H_2}}}$$

$$R(s) \longrightarrow \boxed{\dfrac{(G_4G_3+G_1G_2G_3)}{1+G_3H_3+G_2G_3H_2+G_3G_4H_1+G_1G_2G_3H_1}} \longrightarrow C(s)$$

$\therefore$ Transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{G_4G_3+G_1G_2G_3}{1+G_3H_3+G_2G_3H_2+G_3G_4H_1+G_1G_2G_3H_1}$$

If

$G_1 = G_2 = 1$

$G_3 = G_4 = 2$

$H_1 = H_2 = 1$

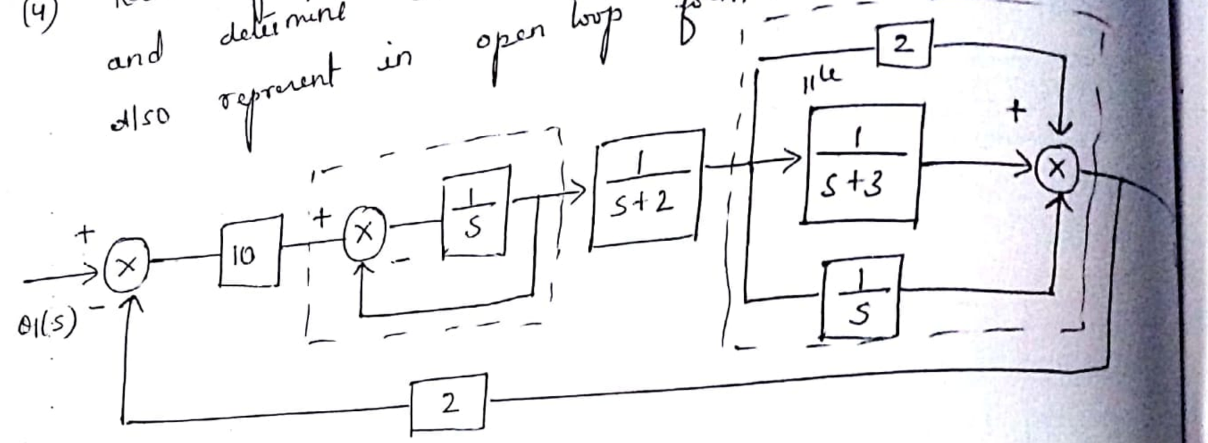
$H_3 = 2$

$$\frac{C(s)}{R(s)} = \frac{(2\times 2)+(1)(2)}{1+(2)(2)+(1)(2)(1)+(2)(2)(1)+(1)(1)(2)(1)}$$

$$= \frac{4+2}{5+2+4+2}$$

$$\frac{C(s)}{R(s)} = \frac{6}{13}$$

(4) Reduce given block diagram into Canonical form and determine closed loop transfer function. and also represent in open loop form.



Sol^n: Step 1

(a) Eliminating unity feedback path.

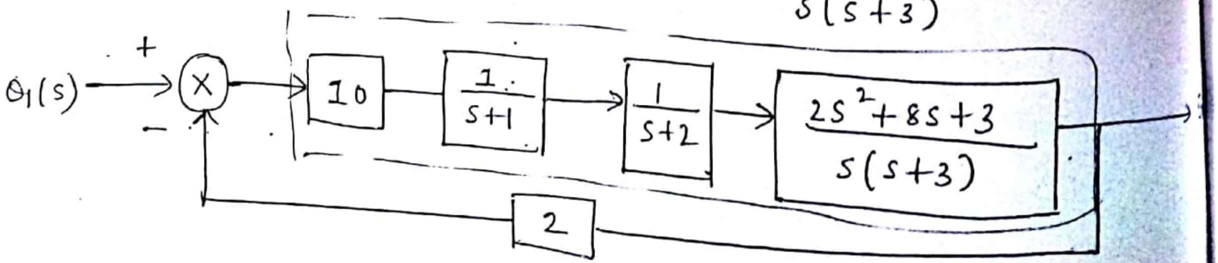
$$\frac{\frac{1}{s}}{1+(1/s)(1)} = \frac{1}{s+1}$$

(b) Combining 3 blocks in parallel.

$$2+\frac{1}{s+3}+\frac{1}{s} = \frac{2(s)(s+3)+s+s+3}{s(s+3)}$$

$$= \frac{2s^2+6s+2s+3}{s(s+3)}$$

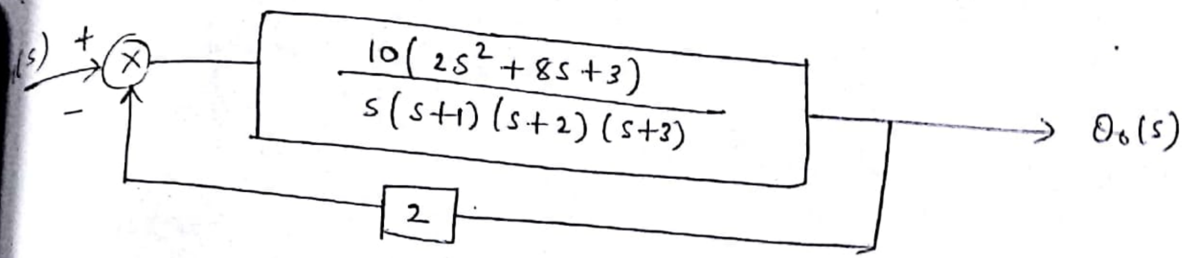
$$= \frac{2s^2+8s+3}{s(s+3)}$$



Step 2 (a) Combining blocks in Cascade (series)

$$(10)\left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)\left(\frac{2s^2+8s+3}{s(s+3)}\right)$$

$$= \frac{10(2s^2+8s+3)}{s(s+1)(s+2)(s+3)}$$



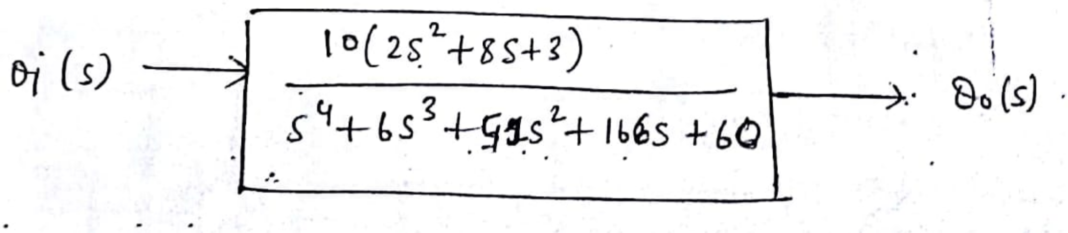
step 3 : Eliminating feedback path:

$$= \frac{\dfrac{10(2s^2+8s+3)}{s(s+1)(s+2)(s+3)}}{1 + \left(\dfrac{10(2s^2+8s+3)}{s(s+1)(s+2)(s+3)}\right)(2)}$$

$$= \frac{\dfrac{10(2s^2+8s+3)}{\cancel{s(s+1)(s+2)(s+3)}}}{\dfrac{s(s+1)(s+2)(s+3)\ 20(2s^2+8s+3)}{\cancel{s(s+1)(s+2)(s+3)}}}$$

$$= \frac{10(2s^2+8s+3)}{s^4+6s^3+51s^2+166s+60}$$

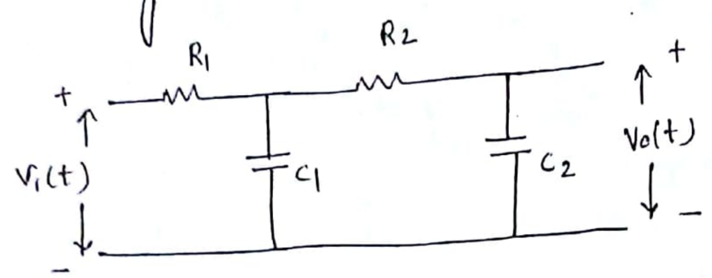
∴ open-loop form



$$T.F = \frac{\theta_o(s)}{\theta_i(s)} = \frac{20s^2+80s+30}{s^4+6s^3+49s^2+166s+60}$$

51

(5) For the electrical system shown below draw the block diagram determine its T.F using B.R.T



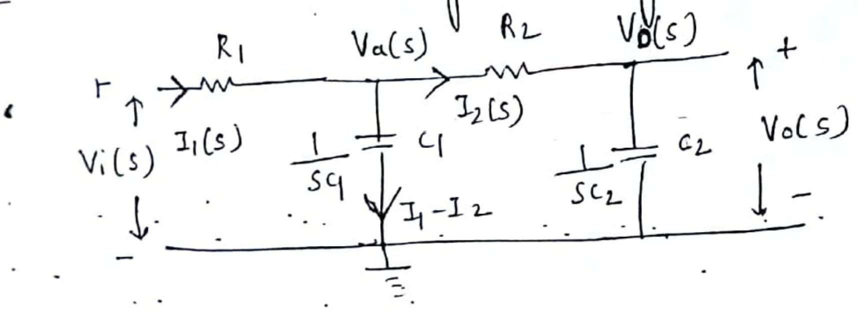
$R_1 = 100k\Omega$

$R_2 = 1 M\Omega$

$C_1 = 10\mu F$

$C_2 = 1\mu F$

Sol^n : LT ckt is given by.

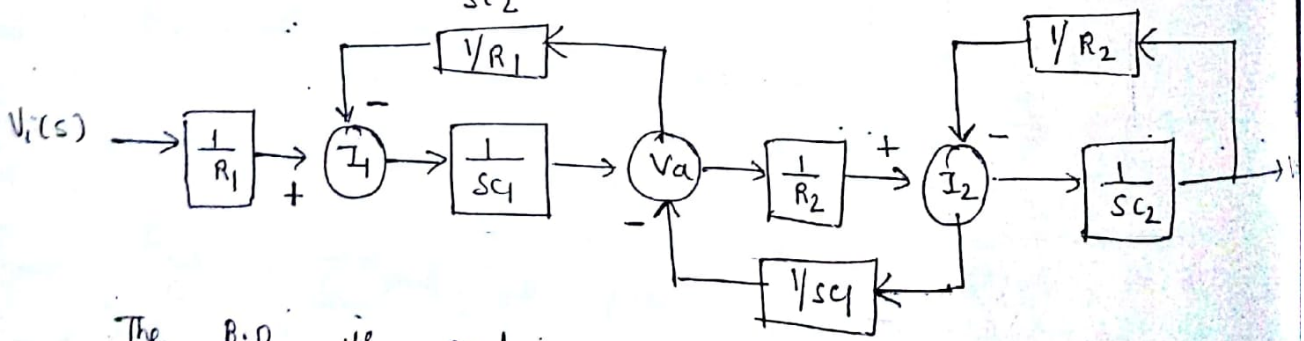


$$I_1(s) = \frac{V_i(s) - V_a(s)}{R_1} = \frac{1}{R_1} V_i(s) - \frac{1}{R_1} V_a(s) \quad \text{---(1)}$$

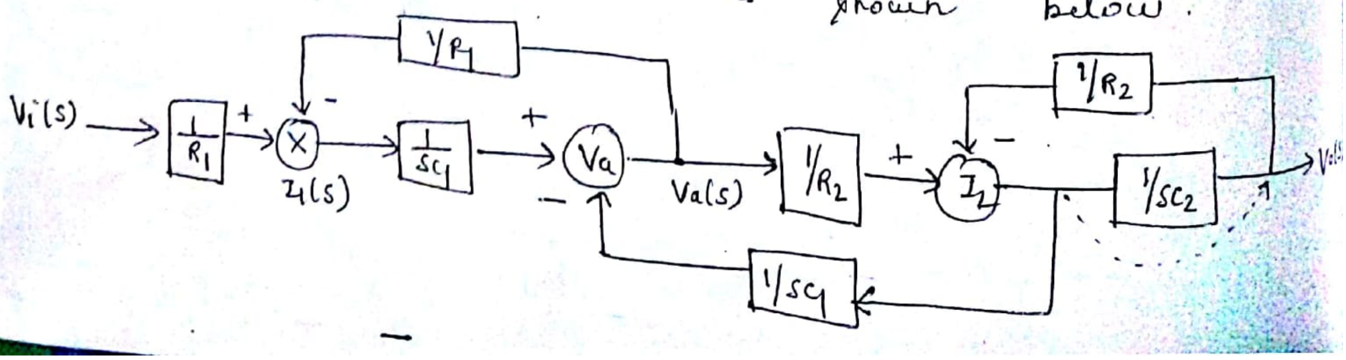
$$I_2(s) = \frac{V_a(s)}{R_2} - \frac{V_o(s)}{R_2} \quad \text{---(2)}$$

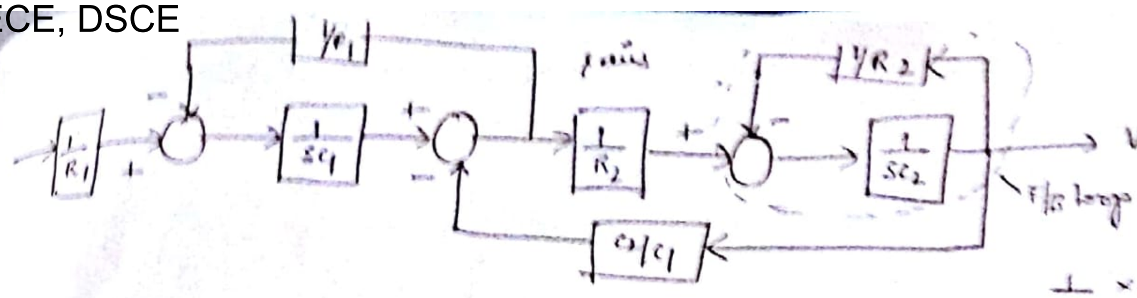
$$V_a(s) = \frac{1}{sC_1} \left(I_1(s) - I_2(s) \right) \quad \text{---(3)}$$

$$V_o(s) = \frac{1}{sC_2} I_2(s) \quad \text{---(4)}$$

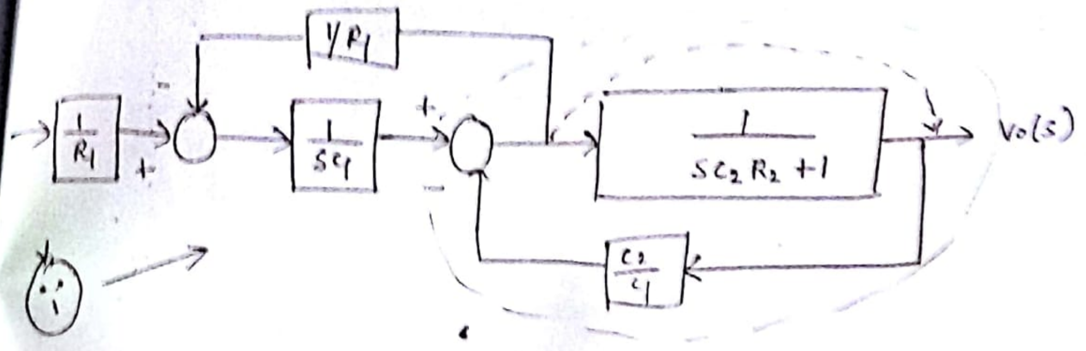


The B.D is redrawn as shown below.





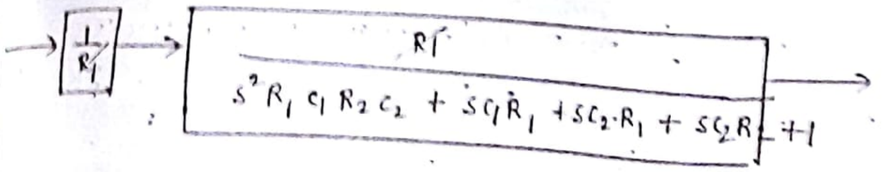
$$\frac{\frac{1}{sC_1}}{\frac{1/sC_2}{1}} = C_2/C_1$$



$$\frac{1/sC_2}{1 + 1/sC_2 R_2}$$

$$\frac{1/sC_2}{\frac{sC_2R_2+1}{sC_2R_2}}$$

$$\frac{1}{R_2} \times \frac{R_2}{sC_2R_2+1} = \frac{1}{sC_2R_2+1}$$



$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2R_1C_1R_2C_2 + sC_1R_1 + sC_2R_2 + sC_2R_2 + 1}$$

$R_1C_1 = 1$

$R_2C_2 = 1$

$R_1C_2 = 0.1$

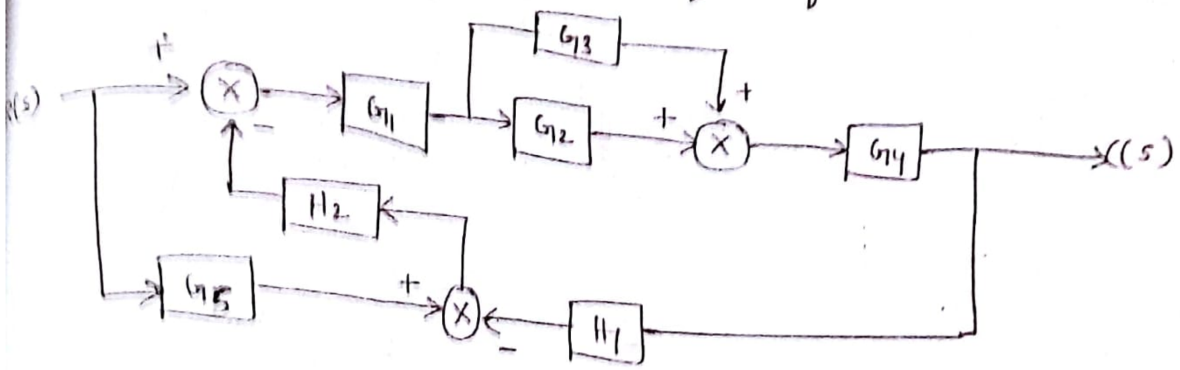
100 x K

10×10^{-6}

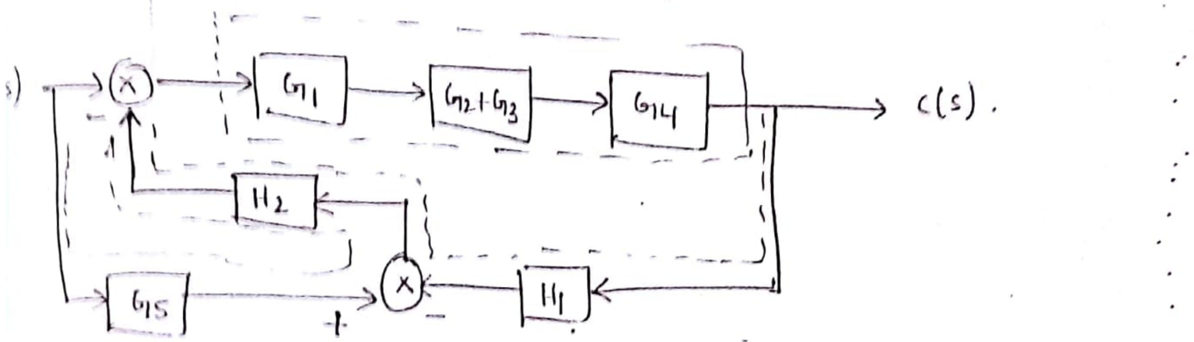
$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + s + 0.1s + s + 1}$$

$$T.F = \frac{V_o(s)}{V_i(s)} = \frac{1}{s^2 + 2.1s + 1}$$

and find the overall transfer function of $\frac{C}{R}$ –

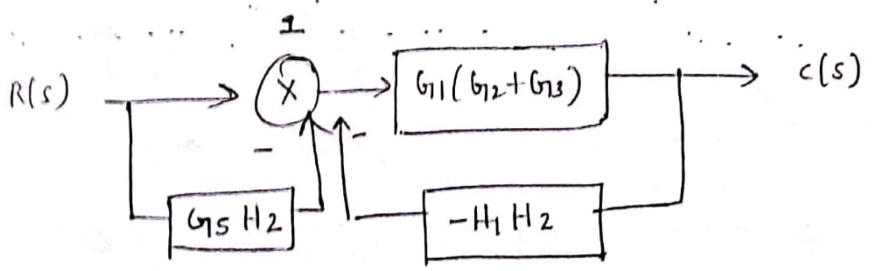


Sol^n: Step 1 Combine the parallel blocks G_2 & G_3 = $G_2 + G_3$

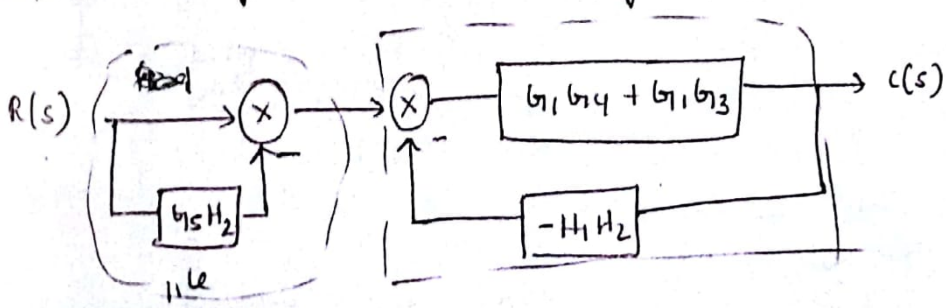


Step 2 : Combine the cascade blocks = $G_1(G_2+G_3)G_4$.

Separate the paths along the dotted lines



Step 3: split the summing points.



Step 4: Combine the parallel block = $-G_5H_2 + 1 = 1 - G_5H_2$

Eliminate the minor feedback loop.

$$\frac{G_1G_4(G_2+G_3)}{1+G_1G_4(G_2+G_3)(-H_1H_2)} = \frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1H_2(G_2+G_3)}$$

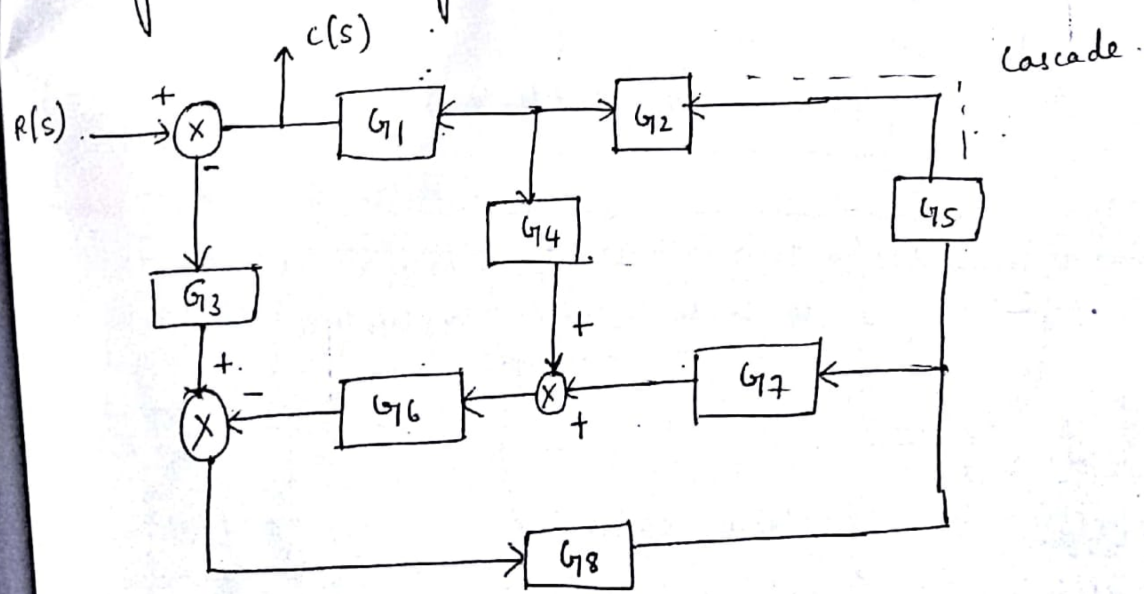
$$R(s) \longrightarrow \boxed{1-G_5H_2} \longrightarrow \boxed{\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1H_2(G_2+G_3)}} \longrightarrow C(s)$$

Step 5: Combine the cascade block.

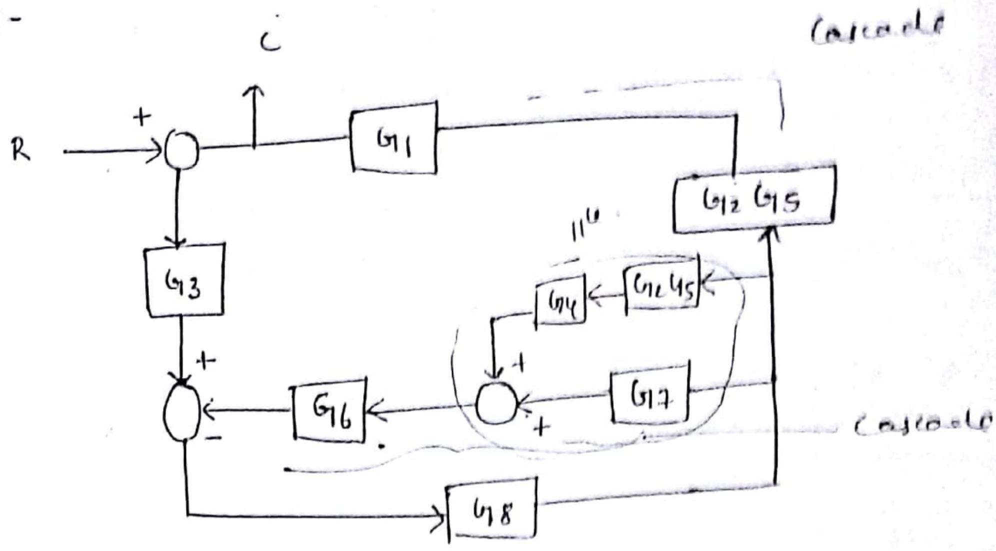
$$R(s) \longrightarrow \boxed{\frac{(1-G_5H_2)\cdot(G_1G_4(G_2+G_3))}{1-G_1G_4H_1H_2(G_2+G_3)}} \longrightarrow C(s)$$

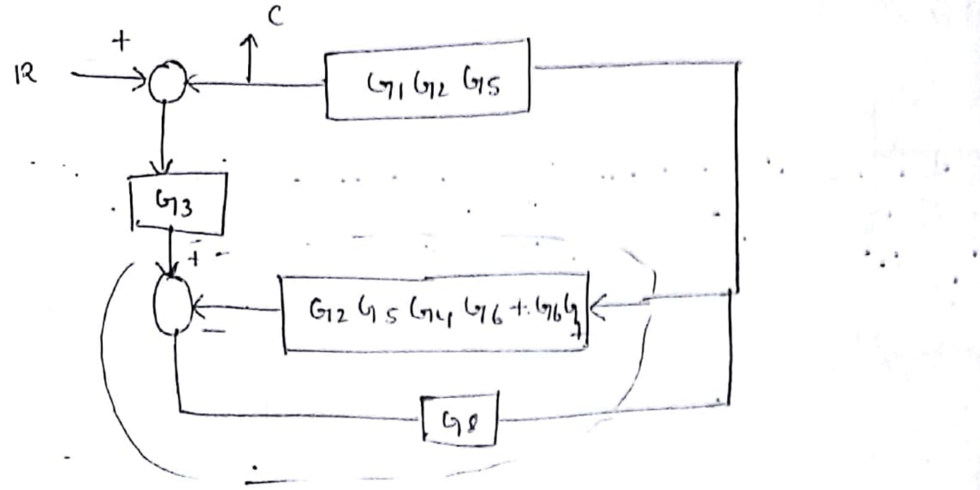
$$\therefore \text{ overall T.F} = \frac{C(s)}{R(s)} = \frac{(1-G_5H_2)(G_1G_4)(G_2+G_3)}{1-G_1G_4H_1H_2(G_2+G_3)}$$

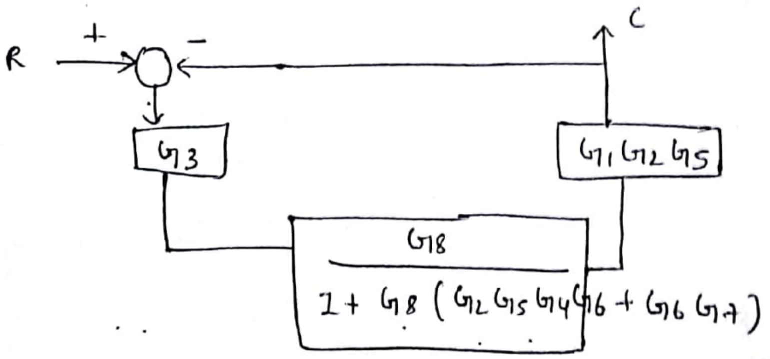
Obtain $\frac{C(s)}{R(s)}$ for block diagram shown in fig using block diagram reduction technique.

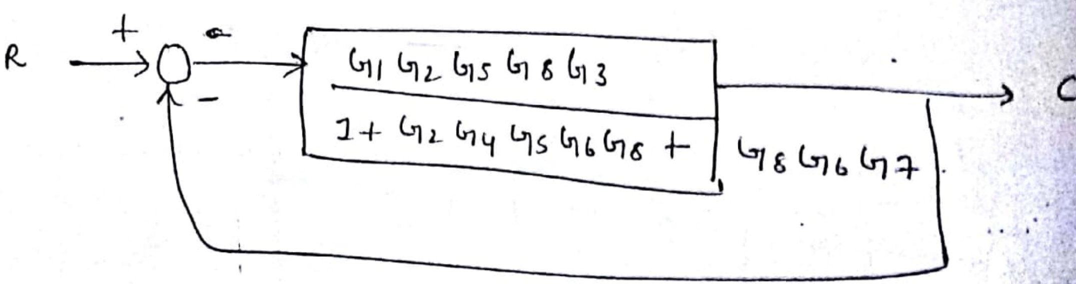


Sol^n :-



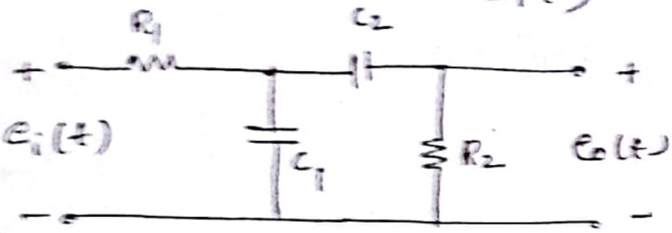






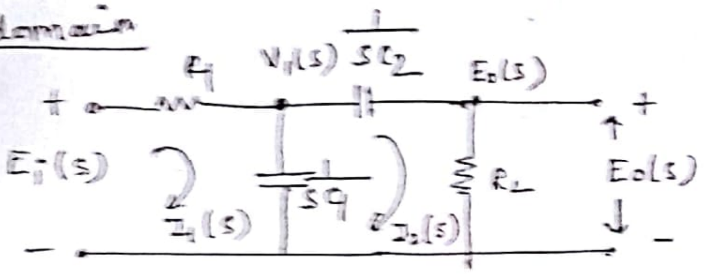
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_5 G_8}{1 + G_2 G_4 G_5 G_6 G_8 + G_6 G_7 G_8}$$

Draw a block diagram for electric circuit shown and hence find $T.F = \dfrac{E_o(s)}{E_i(s)}$



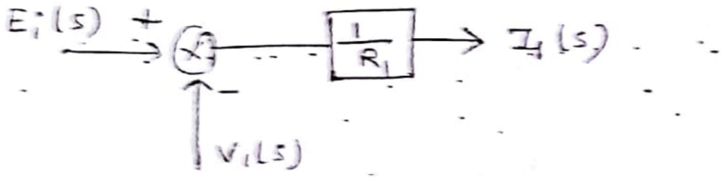
let $R_1 = 100k\Omega$, $R_2 = 1M\Omega$, $C_1 = 10\mu F$, $C_2 = 1\mu F$

s-domain



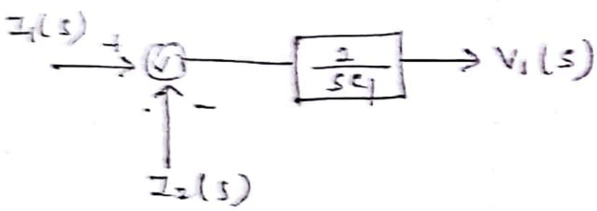
from branch R_1,

$$I_1(s) = \frac{E_i(s) - V_1(s)}{R_1}$$



from branch $\frac{1}{sC_1}$

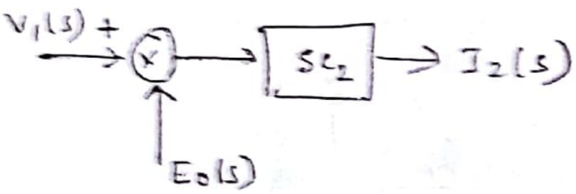
$$V_1(s) = [I_1(s) - I_2(s)] \times \frac{1}{sC_1}$$



from $\frac{1}{sC_2}$ branch.

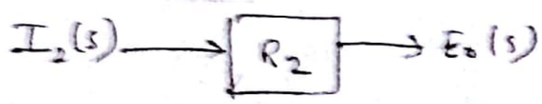
$$I_2(s) = \frac{V_1(s) - E_o(s)}{(1/sC_2)}$$

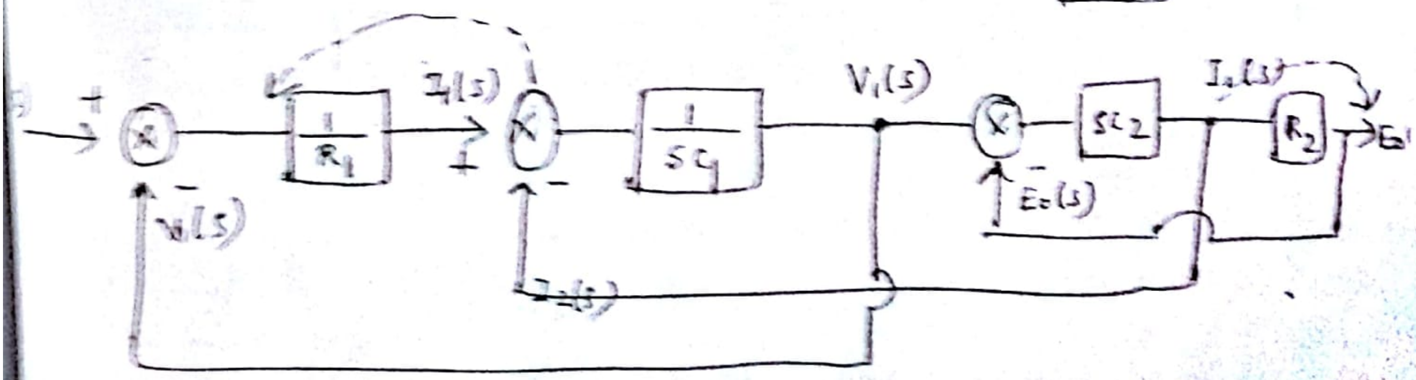
$$= sC_2 [V_1(s) - E_o(s)]$$



from R_2 branch,

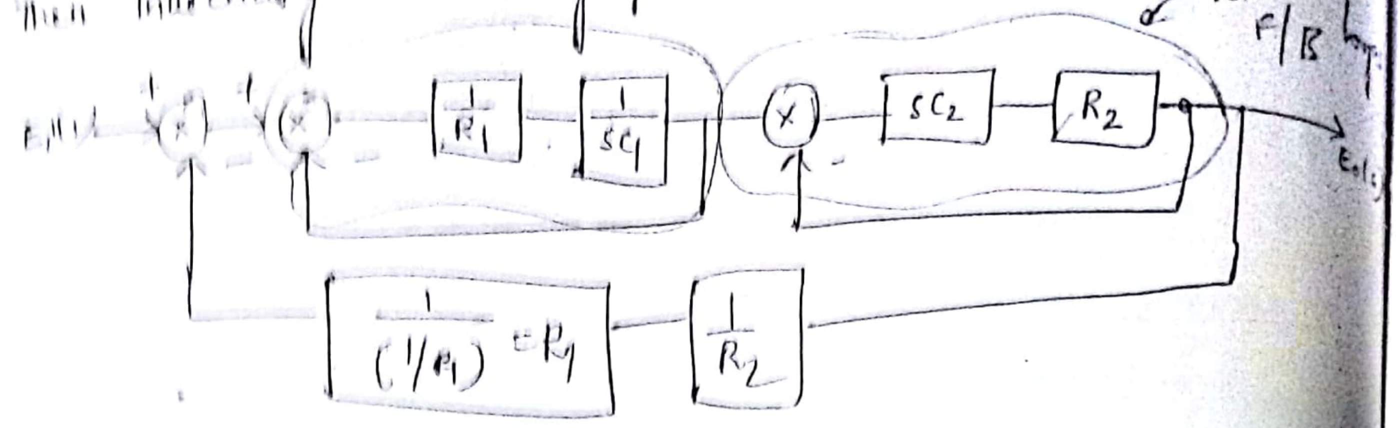
$$E_o(s) = I_2(s) R_2$$

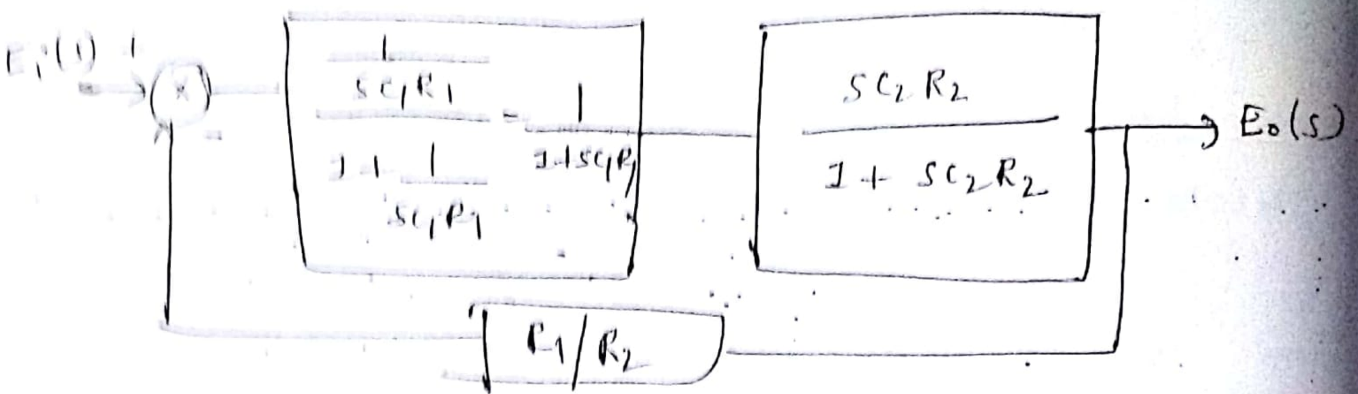


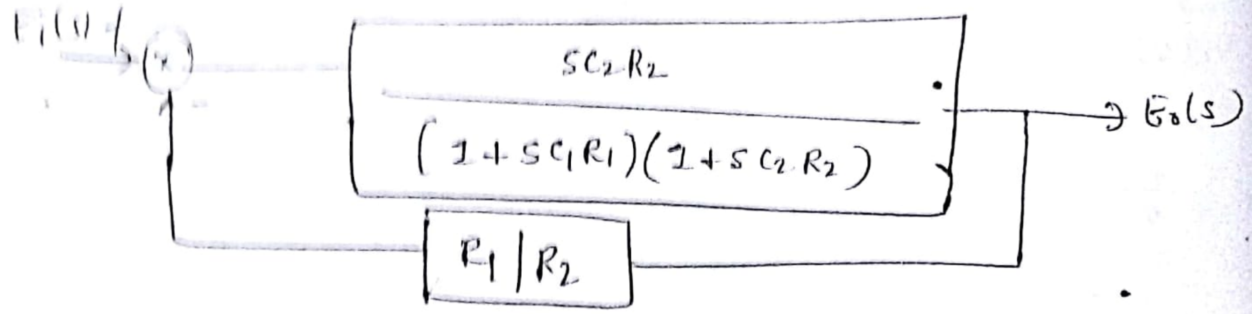


Shift summing pt before $1/R_1$ & takeoff pt after R_2

then interchange summing pt near $E_i(s)$ & minor F/B lp.







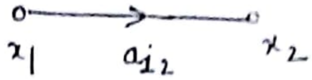
$$\frac{E_o(s)}{E_i(s)} = \frac{sC_2R_2}{1 + sC_1R_1 + sC_2R_2 + s^2R_1C_1R_2C_2 + sC_2R_1}$$

$$\frac{E_o(s)}{E_i(s)} = \frac{s}{s^2 + 2.1s + 1}$$

Signal flow graph (SFG)

It is graphical representation of a set of simultaneous equations representing the linear controll system is called signal flow graph (SFG)

For Instance, consider that a linear system is represented by simple algebraic equation



$\rightarrow$ Indicate flow of signal.

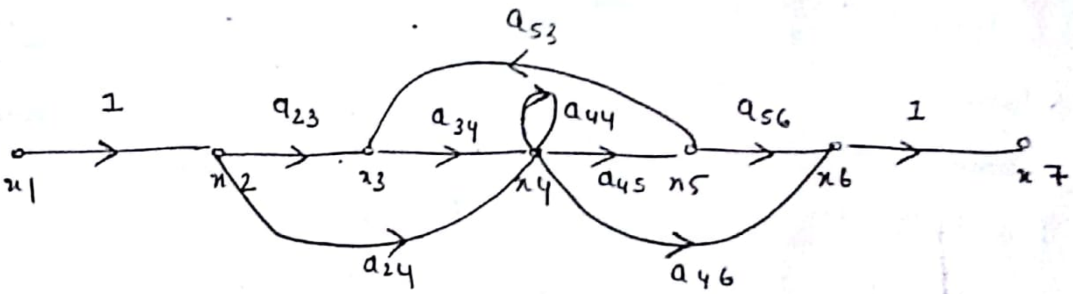
$$x_2 = a_{12}\, x_1$$

where $x_1 \rightarrow$ Input

$x_2 \rightarrow$ output

(T.F) . $a_{12} \rightarrow$ gain b/w the 2 Variables.

→ Terms used in SFG



(1) Node : Nodes are the Variable of the System represented by small circles $(x_1 \rightarrow x_7)$

(2) Input Node : The Node that has only outgoing branches is known as Input or source Node.

ex: x_1 is Input Node

(3) output Node : The Node that has only Incoming branches is known as output or Sink node

ex: x_7 is output Node.

(4) Mixed Node : The node that having both Incoming and outgoing branches is known as mixed or chain node.

ex: x_2, x_3, x_4, x_5 & x_6

(5) Branch : directed line segment joining 2 Nodes is known as branch.

(6) path : It is traversal from one node to another node in direction of the branch arrow, such that no node traverse more than once.

(7) Branch gain : The gain b/w nodes is known as branch gain & transmittance. such gain are expressed interms of transfer functions.

(8) Forward path : The path that starts from an Input node and ends at an output Node and along which no node is traversed more than once is known as forward path.

ex: $x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7$

$x_1 - x_2 - x_4 - x_6 - x_7$

$x_1 - x_2 - x_4 - x_5 - x_6 - x_7$

$x_1 - x_2 - x_3 - x_4 - x_6 - x_7$.

(9) path gain : The product of the branch gains encountered while going through the a forward path is known as path gain or forward path gain

ex: Consider forward path.

$x_1 - x_2 - x_4 - x_5 - x_6 - x_7$

path gain $\rightarrow$ $1 \times a_{24} \times a_{45} \times a_{56} \times 1$

Feedback loop

A path which starts from a particular node and ends at same node, travelling through atleast one other node, and along which no node is traversed more than once. is known as feedback loop or closed loop.

ex: $x_3 - x_4 - x_5 - x_3$

11) **Self loop**: A path which starts from a particular node and ends at the same node

ex: $x_4 - x_4$

Hint: A self loop should not be considered while defining the forward path.

12) **Non-touching loop**: If there is no node common b/w 2 or more loops, such loops are said to be non-touching. loops

13) **loop gain**: The product of all the gains of the branches forming loop is known as loop gain.

Mason's Gain formula

Mason's gain formula is used for the determination of overall transfer function of a system.

- The number of steps involved in block diagram reduction technique are more & it is time consuming procedure and also task of solving for input--output relationship by algebraic manipulation could be quite tedius.

An advantage of Mason's gain formula is that the system transfer functions are readily obtained without manipulation of the graph

→ Mason's gain formula is given by

overall T.F $\quad T = \frac{1}{\Delta} \sum P_K \Delta_K$

where $\quad K$ = Number of forward path

P_K = path gain of K^{th} forward path

Δ = Determinant of the graph

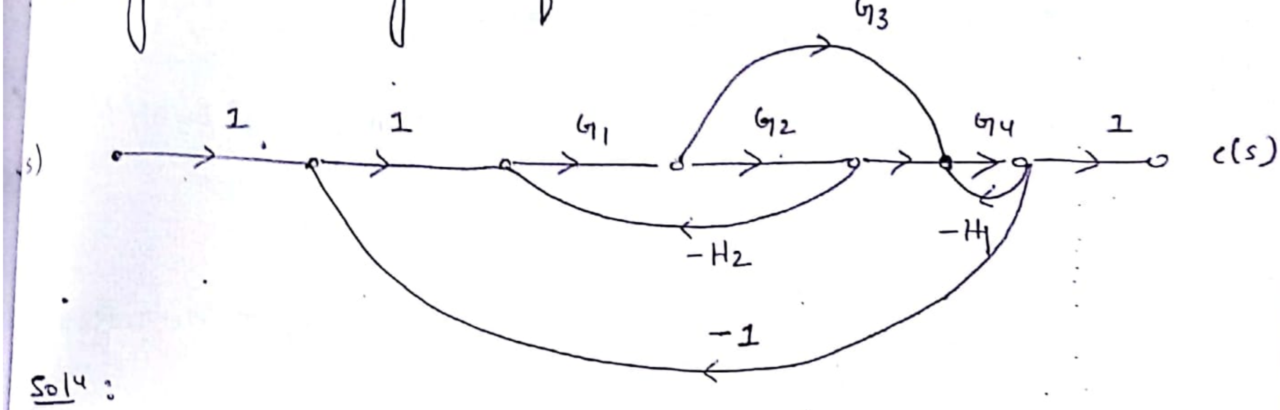
= 1 - (Sum of Individual loop gain) + (Sum of gain products of all Combinations of 2 Non touching loops) - (Sum of gain product of all combinations of 3 non-touching loops) + ...

$\Delta = 1 - \Sigma pm_1 + \Sigma pm_2 - \Sigma pm_3 + \dots$

Δ_K = Value of Δ by eliminating all loop gain and associated products which are touching to the K^{th} forward path.

Problems on Signal flow graph

For the system shown in fig. determine $\frac{C(s)}{R(s)}$ using Mason's gain formula.



Sol^n:

Step 1: Identify the number of forward path & there gain

∴ forward path gains are

$P_1 = 1 \times 1 \times G_1 \times G_2 \times G_4 \times 1 = G_1 G_2 G_4$

$P_2 = 1 \times 1 \times G_1 \times G_3 \times G_4 \times 1 = G_1 G_3 G_4$

} 2 forward paths $(K = 2)$

Step 2: Identify the Individual loops and there loop gain

∴ loop gains are

$L_1 = G_1 G_2 (-H_2) = -G_1 G_2 H_2$

$L_2 = G_4 (-H_1) = -G_4 H_1$

$L_3 = 1 \times G_1 G_2 G_4 (-1) = -G_1 G_2 G_4$

$L_4 = 1 \times G_1 G_3 G_4 (-1) = -G_1 G_3 G_4$

} 4 Individual loops

Step 3: Find the Combination of Non touching loops

1. Combination of 2 non-touching loops

$L_1 L_2 = (-G_1 G_2 H_2)(-G_4 H_1) = G_1 G_2 G_4 H_1 H_2$

No other Combination of 2 or more non touching loops step 4 : Find the Value of determinant (Δ)

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2]$$

$$= 1 - [-G_1G_2H_2 - G_4H_1 - G_1G_2G_4 - G_1G_3G_4] + G_1G_2G_4H_1H_2$$

$$\Delta = 1 + G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4 + G_1G_2G_4H_1H_2$$

step 5 : Δ_K = Value of Δ eliminating all loop gains and associated products which are touching to K^{th} forward path.

Since from SFG, it is seen that all the loops are touching all the forward paths.

we have $\Delta_1 = \Delta_2 = 1$.

Thus Mason's gain formula.

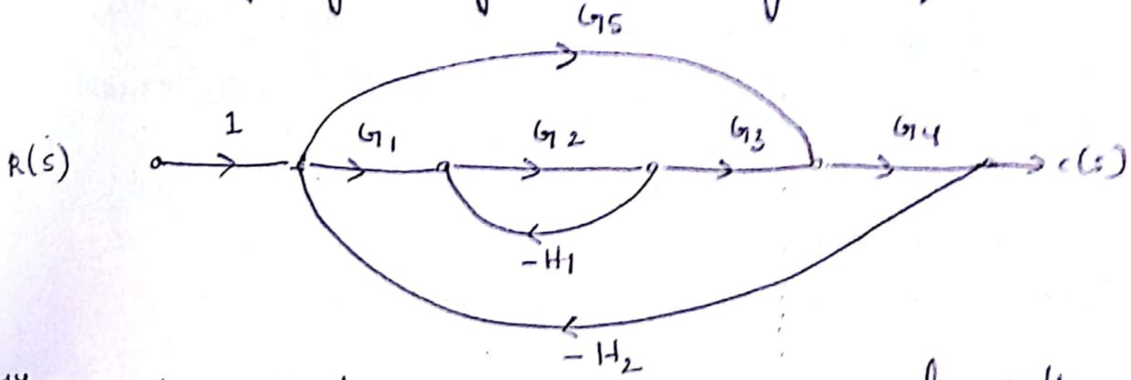
since $K = 2$, $\quad T = \frac{1}{\Delta} \sum_{K=1}^{2} P_K \cdot \Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$\therefore$ overall T.F.

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_4 + G_1G_3G_4}{1 + G_1G_2H_2 + G_4H_1 + G_1G_2G_4 + G_1G_3G_4 + G_1G_2G_4H_1H_2}$$

NOTE: If a forward path contains all the nodes of a graph or if the forward path touches all single loop present in graph $\boxed{\Delta_K = 1}$

Find the Controll ratio for the signal flow graph shown in fig by using Mason's gain formula.



Sol^n : step 1 : Identify the number of forward path and there gain

∴ forward path gains,

$P_1 = 1\,G_1\,G_2\,G_3\,G_4$
$P_2 = 1.\,G_5\,G_4$

} 2 forward paths ∴ $K = 2$

step 2 : Identify the Individual loops and there loop gain.

$L_1 = -G_2 H_1$
$L_2 = -G_1 G_2 G_3 G_4 - H_2$
$L_3 = -1\,G_5\,G_4\,H_2$

} There are 3 Individual paths.

step 3 : Find the combinations of non-touching loops and there gains

(1) Combination of 2 non-touching loops

$L_1 L_3 = G_2 \cdot G_5\,G_4\,H_1\,H_2$

step 4 : Find the Value of determinant Δ

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_3]$$

$$\Delta = 1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_5 G_4 H_2 + G_2 G_5 G_4 H_1 H_2$$

step 5: Find the value of Δ_K

Since from SFG, it is seen that all the loops are touching all the forward paths, we have

$$\Delta_1 = 1$$

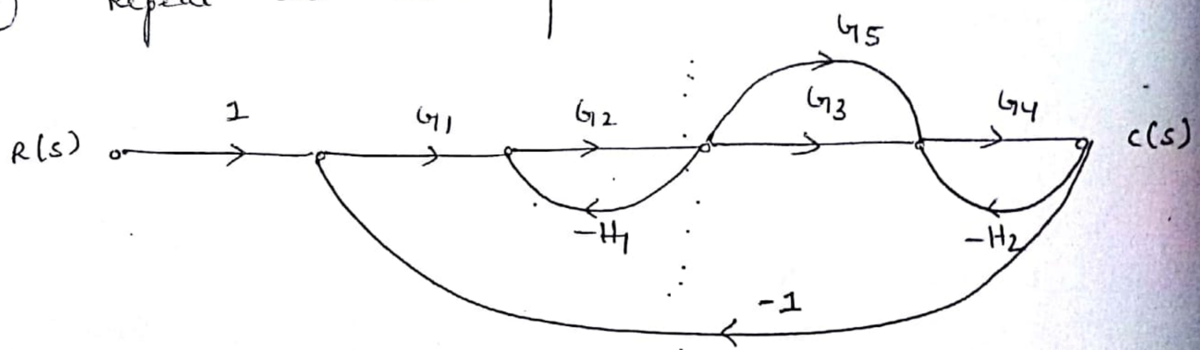
$$\Delta_2 = 1 - L_1 = 1 + G_2 H_1$$

Thus, from Mason's gain formula,

$$T.F = \frac{1}{\Delta} \sum_{K=1}^{2} P_K \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$T.F = \frac{C}{R} = \frac{G_1 G_2 G_3 G_4 + G_5 G_4 (1 + G_2 H_1)}{1 + G_2 H_1 + G_1 G_2 G_3 G_4 H_2 + G_4 G_5 H_2 + G_2 G_4 G_5 H_1 H_2}$$

(3) Repeat the above problem.



step: 1 forward paths.

$$P_1 = 1\, G_1 G_2 G_3 G_4$$
$$P_2 = G_1 G_2 G_5 G_4$$

$\Big\}$ K = 2

(2) ~~path~~ loop gains

$$L_1 = -G_2 H_1$$
$$L_2 = -G_4 H_2$$
$$L_3 = G_1 G_2 G_3 G_4 (-1)$$
$$L_4 = G_1 G_2 G_5 G_4 (-1)$$

$\Big\}$ 4 Individual loops

step 3 : (i) Combination of 2 Non touching loops

$$L_1 L_2 = (-G_2 H_1)(-G_4 H_2) = G_2 G_4 H_1 H_2$$

No other combination of 2 or more non touching loops

step 4 :

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2]$$

$$\Delta = 1 - [-G_2 H_1 - G_4 H_2 - G_1 G_2 G_3 G_4 - G_1 G_2 G_5 G_4] + [G_2 G_4 H_1 H_2]$$

$$\Delta = 1 + G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 + G_1 G_2 G_5 G_4 + G_2 G_4 H_1 H_2$$

steps : $\Delta_1 = \Delta_2 = 1$

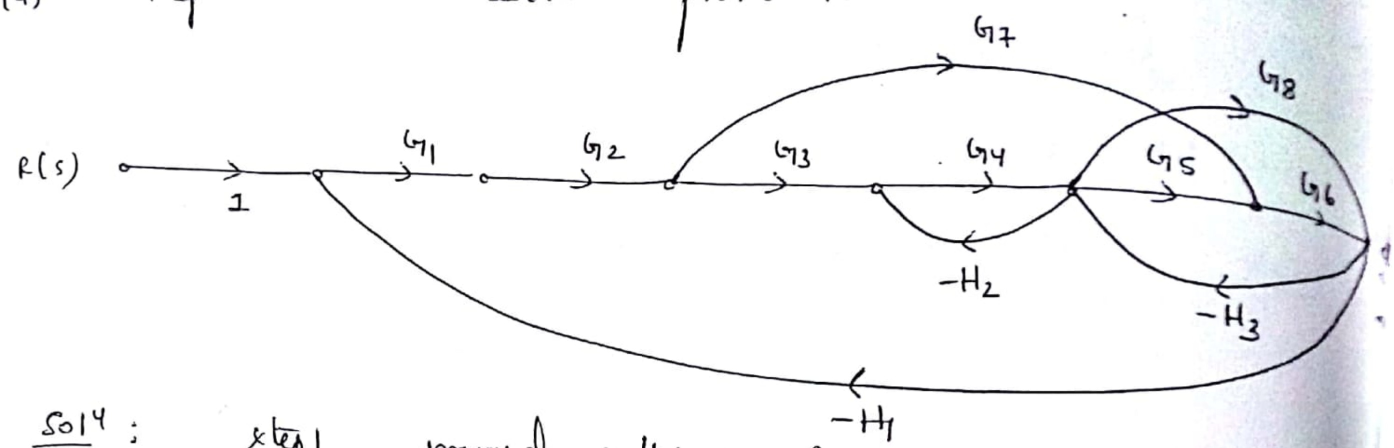
since from SFG, it is seen that all the loops are touchings all the forward paths.

Mason's gain formula,

$$T = \frac{1}{\Delta} \sum_{K=1}^{2} P_K \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_5 G_4}{1 + G_2 H_1 + G_4 H_2 + G_1 G_2 G_3 G_4 + G_1 G_2 G_5 G_4 + G_2 G_4 H_1 H_2}$$

(4) Repeat the above problem.



Sol^n : step 1 forward paths & gains

$P_1 = 1\, G_1 G_2 G_3 G_4 G_5 G_6 = G_1 G_2 G_3 G_4 G_5 G_6$

$P_2 = 1\, G_1 G_2 G_7 G_6 = G_1 G_2 G_7 G_6$

$P_3 = 1\, G_1 G_2 G_3 G_4 G_8 = G_1 G_2 G_3 G_4 G_8$

$\Bigg\}\; K = 3$

step 2 : Individual loops and loop gains.

$L_1 = -G_4 H_2$

$L_2 = -G_5 G_6 H_3$

$L_3 = -G_1 G_2 G_3 G_4 G_5 G_6 H_1$

$L_4 = -G_8 H_3$

$L_5 = -G_1 G_2 G_3 G_4 G_8 H_1$

$L_6 = -G_1 G_2 G_7 G_6 H_1$

step 3 : (1) Combination of 2 non-touching loops

$L_1 L_6 = -G_4 H_2 \times -G_1 G_2 G_7 G_6 H_1$

$L_1 L_6 = G_1 G_2 G_4 G_6 G_7 H_1 H_2$

There is no combination of 3 or more non-touching loops.

step 4 : Find the value of Δ.

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5 + L_6] + [L_1 L_6]$$

$$\Delta = 1 - [-G_4 H_2 - G_5 G_6 H_3 - G_1 G_2 G_3 G_4 G_5 G_6 H_1 - G_8 H_3 - G_1 G_2 G_3 G_4 G_8 H_1 - G_1 G_2 G_7 G_6 H_1] + [(-G_4 H_2) * (-G_1 G_2 G_7 G_6 H_1)]$$

$$\Delta = 1 + \cancel{G_4 H_2} + G_5 G_6 H_3 + G_1 G_2 G_3 G_4 G_5 G_6 H_1 + G_8 H_3 + G_1 G_2 G_3 G_4 G_8 H_1 + G_1 G_2 G_7 G_6 H_1 \; \cancel{G_8 H_2} \; \cancel{-G_1 G_2 G_7 G_6 H_1} + G_1 G_2 G_4 G_6 G_7 H_1 H_2$$

step 5 : ① for P_1 forward path, all the loops are touching.

$$\Delta_1 = 1$$

② for P_2 forward path, only L_1 is Non-touching

$$\Delta_2 = 1 - L_1 = 1 + G_4 H_2.$$

③ for P_3 forward path, all the loops are touching

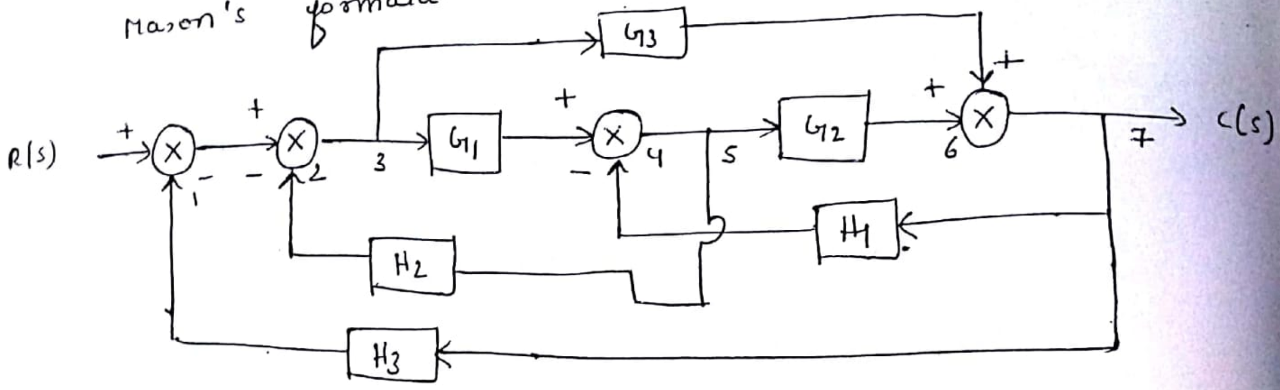
$$\Delta_3 = 1$$

from Mason's gain formula,

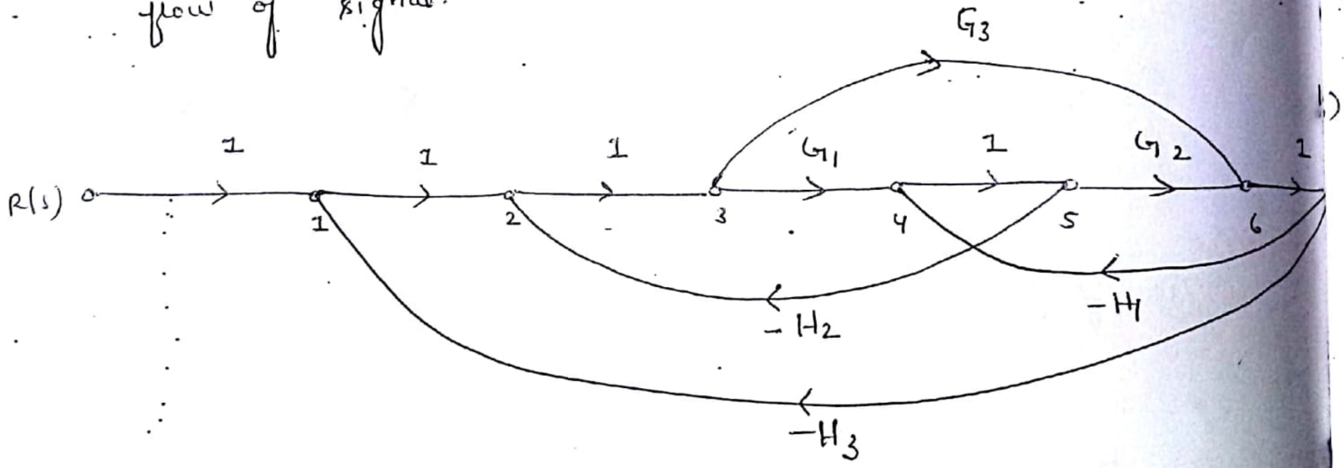
$$T = \frac{1}{\Delta} \sum_{K=1}^{3} \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 G_5 G_6 + G_1 G_2 G_7 G_6 (1 + G_4 H_2) + G_1 G_2 G_3 G_4 G_8}{1 + G_5 G_6 H_3 + G_1 G_2 G_3 G_4 G_5 G_6 H_1 + G_8 H_3 + G_1 G_2 G_3 G_4 G_8 H_1 + G_1 G_2 G_7 G_6 H_1 + G_1 G_2 G_4 G_6 G_7 H_1 H_2}$$

(5) Draw the signal flow graph for the block diagram shown in fig and find its control ratio using Mason's formula.



Sol^n : step 1: Represent all the summing points and takeoff points each by a separate node and show flow of signal.



step 2: forward paths & gains.

$P_1 = G_1 G_2$

$P_2 = G_3$

} $K = 2$.

step 3: Individual loops & their gains.

$L_1 = -G_1 H_2$

$L_2 = -G_2 H_1$

$L_3 = 1 \times 1 \times G_1 \times G_2 \times 1 \times -H_3 = -G_1 G_2 H_3$

$L_4 = 1 \times 1 \times G_3 \times 1 \times -H_3 = -G_3 H_3$

$L_5 = 1 \times G_3 \times 1 \times -H_1 \times 1 \times -H_2 = G_3 H_1 H_2$

step 4: there is no combination of Non-touching loops

step 5: $\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5]$

$$\Delta = 1 + G_1H_2 + G_2H_1 + G_1G_2H_3 + G_3H_3 - G_3H_1H_2$$

step 6: $\Delta_K = 1$,

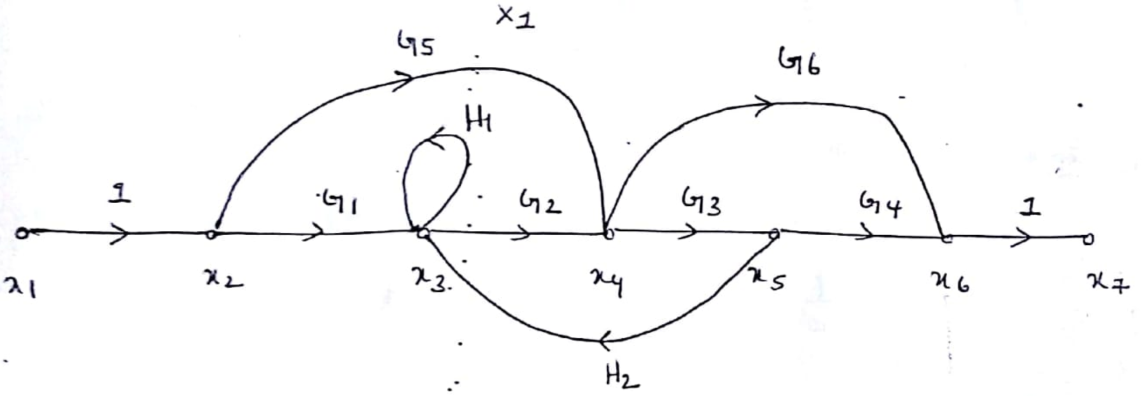
$\Delta_1 = \Delta_2 = 1$, all the loops are touching all forward paths.

from Mason's gain formula,

$$\text{Gain} = \frac{1}{\Delta}\sum_{K=1}^{2} P_K\Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2 + G_3}{1 + G_1H_2 + G_2H_1 + G_1G_2H_3 + G_3H_3 - G_3H_1H_2}$$

In SFG, obtain $\frac{X_7}{X_1}$



st 1: Identify the forward path and gains

$P_1 = G_1G_2G_3G_4$

$P_2 = G_5G_6$

$P_3 = G_5G_3G_4$

$P_4 = G_1G_2G_6$

$K = 4$

step 2: Individual loops and gains.

$$L_1 = H_1$$
$$L_2 = G_2 G_3 H_2$$

step 3: There is no combination of non-touching loops.

step 4:
$$\Delta = 1 - [L_1 + L_2]$$
$$\Delta = 1 - H_1 - G_2 G_3 H_2 .$$

step 5: $\Delta_K = ??$

$$\Delta_1 = 1 .$$
$$\Delta_2 = 1 - L_1 = 1 - H_1$$ (within the loop)

$$\Delta_3 = 1 - L_1 = 1 - H_1 .$$
$$\Delta_4 = 1 .$$

step 6: Mason's gain formula.

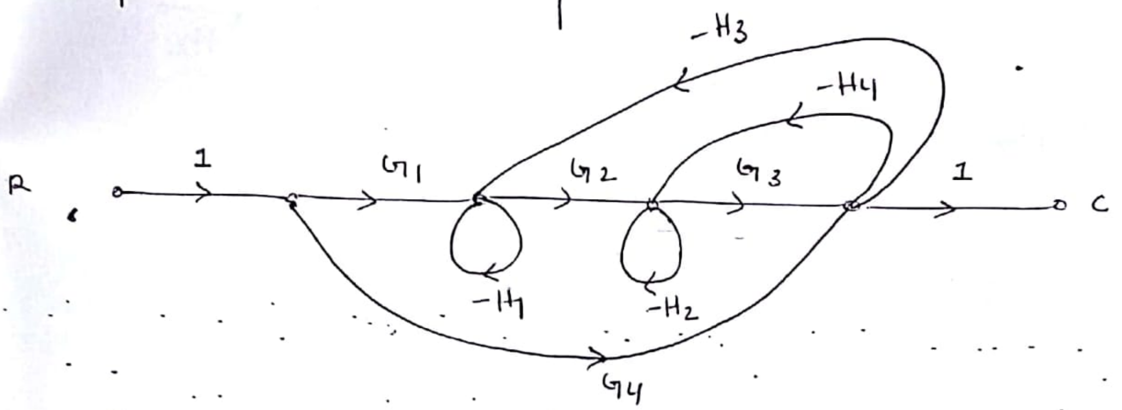
$$T = \frac{1}{\Delta} \sum_{K=1}^{4} P_K \Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$\frac{X_7}{X_1} = \frac{G_1 G_2 G_3 G_4 + G_5 G_6 (1 - H_1) + G_3 G_4 G_5 (1 - H_1) + G_1 G_2 G_6}{1 - (H_1 + G_2 G_3 H_2)} .$$

$$\text{Gain} = \frac{1}{\Delta}\sum_{K=1}^{-} P_K \Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{C}{R} = T = \frac{G_1G_2G_3 + G_1G_4G_3(1+G_2H_3)}{1+G_2H_3 + G_1G_2H_1 + G_1G_3H_2 - G_1G_4G_3H_2G_2H_1}$$

) Repeat the above problem.



Sol^n: (1) Forward paths and gains.

$P_1 = G_1G_2G_3$ } $K = 2$
$P_2 = G_1G_4$

(2) loop gains

$L_1 = -H_1$ } self loops
$L_2 = -H_2$

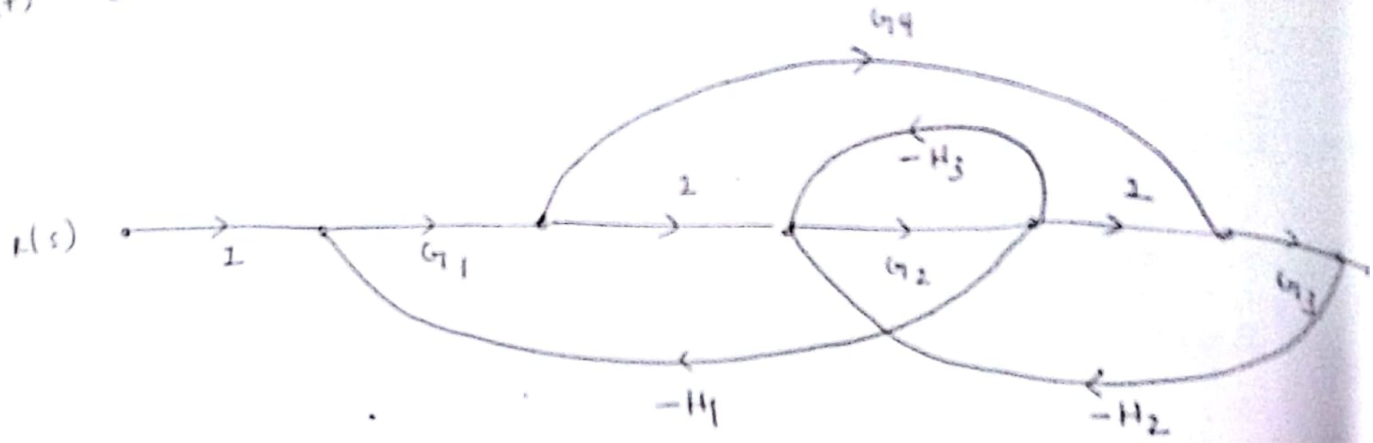
$L_3 = -G_3H_4$ } 4 Individual loops
$L_4 = -G_2G_3H_3$

(3) Combination of 2 non-touching loops

$L_1L_2 = H_1H_2$

$L_1L_3 = H_1G_3H_4$.

(7) obtain the overall T.F $\frac{C}{R}$ from SFG



Sol^n: step 1: Forward path and gains.

$P_1 = G_1 G_2 G_3$
$P_2 = G_1 G_4 G_3$ } $K = 2$

step 2: Individual loops and gains

$L_1 = -G_2 H_3$
$L_2 = -G_1 G_2 H_1$
$L_3 = -G_2 G_3 H_2$ } 4 loops
$L_4 = G_1 G_4 G_3 H_2 G_2 H_1$

step 3: No combination of 2 or more Non-touching loops

step 4: $\Delta = 1 - [L_1 + L_2 + L_3 + L_4]$

$\Delta = 1 + G_2 H_3 + G_1 G_2 H_1 + G_2 G_3 H_2$
$+ G_1 G_4 G_3 H_2 G_2 H_1$

step 5: $\Delta_1 = 1$

$\Delta_2 = 1 - L_1 = 1 + G_2 H_3$

step 4:

$$\Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1 L_2 + L_2 L_3]$$

$$\Delta = 1 - [-H_1 - H_2 - G_3 H_4 - G_2 G_3 H_3] + [H_1 H_2 + H_1 G_3 H_4]$$

$$\Delta = 1 + H_1 + H_2 + G_3 H_4 + G_2 G_3 H_3 + H_1 H_2 + H_1 G_3 H_4$$

step 5: Δ_K,

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - L_1 - L_2 + L_1 L_2$$

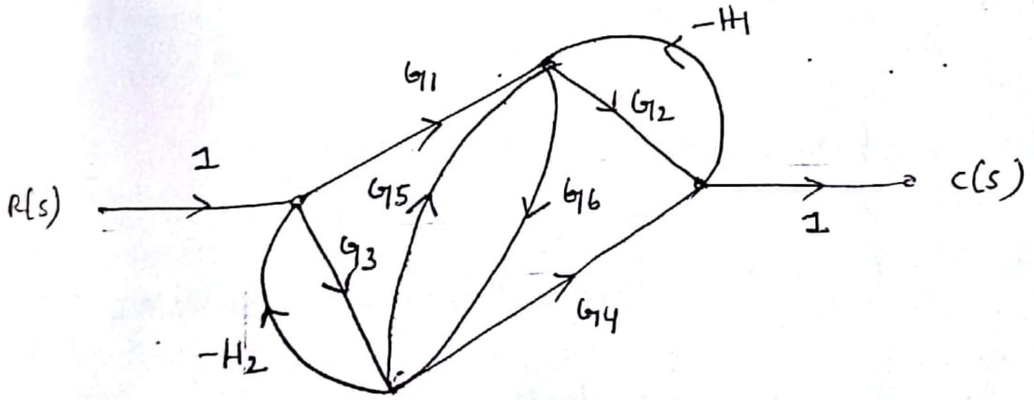
$$\Delta_2 = 1 + H_1 + H_2 + H_1 H_2$$

Mason's gain formula

$$T = \frac{1}{\Delta} \sum_{K=1}^{2} P_K \Delta_K = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\frac{C}{R} = \frac{G_1 G_2 G_3 + G_4 [1 + H_1 + H_2 + H_1 H_2]}{1 + [H_1 + H_2 + G_3 H_4 + G_2 G_3 H_3] + [H_1 H_2 + H_1 G_3 H_4]}$$

For the signal flow graph shown in fig determine the $T.F = \frac{C(s)}{R(s)}$ using Mason's gain formula.



Soln: ① forward paths and gains

$P_1 = 1 \times G_1 \times G_2 \times 1 = G_1 G_2$

$P_2 = 1 \times G_3 \times G_4 \times 1 = G_3 G_4$

$P_3 = 1 \times G_1 \times G_6 \times G_4 \times 1 = G_1 G_6 G_4$

$P_4 = 1 \times G_3 \times G_5 \times G_2 \times 1 = G_3 G_5 G_2$

$K = 4$

② loop gains

$L_1 = -G_2 H_1$

$L_2 = -G_3 H_2$

$L_3 = G_5 G_6$

$L_4 = -G_4 H_1 G_6$

$L_5 = -G_1 G_6 H_2$

③ Combination of 2 non-touching loops

$L_1 L_2 = (-G_2 H_1)(-G_3 H_2) = G_2 G_3 H_1 H_2$

There is no combination of 3 or more non-touching loops

④ $\Delta = 1 - [L_1 + L_2 + L_3 + L_4 + L_5] + [L_1 L_2]$.

$\Delta = 1 - [-G_2 H_1 - G_3 H_2 + G_5 G_6 - G_4 H_1 G_6 - G_1 G_6 H_2]$
$+ [G_2 G_3 H_1 H_2]$

$= 1 + G_2 H_1 + G_3 H_2 - G_5 G_6 + G_4 G_6 H_1 + G_1 G_6 H_2$
$+ G_2 G_3 H_1 H_2$

$\Delta K = 1$,

since from fig it is seen that for all the forward paths all the loops are touching.

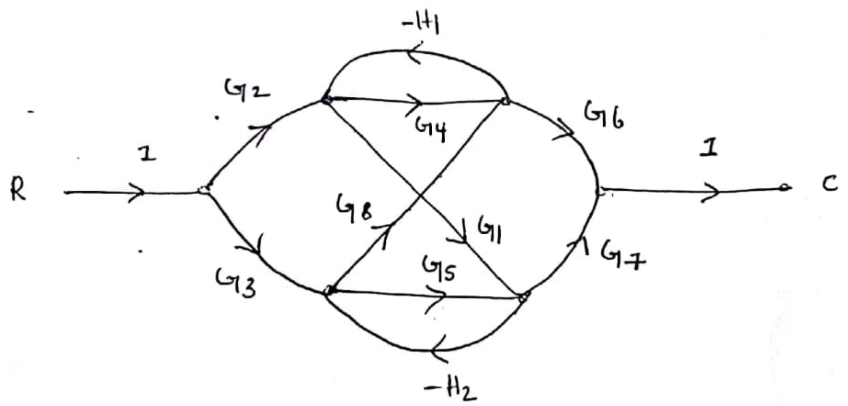
$\therefore \Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$

Mason's gain formula,

$$T = \frac{1}{\Delta}\sum_{K=1}^{4} P_K \Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2 + G_3G_4 + G_1G_6G_4 + G_2G_3G_5}{1 + G_2H_1 + G_3H_2 - G_5G_6 + G_4H_1G_6 + G_1G_6H_2 + G_2G_3H_1H_2}$$

obtain the overall T.F $\frac{C}{R}$ from SFG:



① forward path and gains

$P_1 = G_2 G_4 G_6$

$P_2 = G_3 G_5 G_7$

$P_3 = G_2 G_1 G_7$

$P_4 = G_3 G_8 G_6$

$P_5 = 1 \times G_2 \times G_1 \times -H_2 \times G_8 \times G_6 \times 1 = -G_2G_1H_2G_8G_6$

$P_6 = 1 \times G_3 \times G_8 \times -H_1 \times G_1 \times G_7 \times 1 = -G_3G_8H_1G_1G_7$.

② loop gains

$L_1 = -G_4 H_1$

$L_2 = -G_5 H_2$

$L_3 = +G_1 H_2 G_8 H_1$

③ Find the combination of 2 Non-touching loops.

$L_1 L_2 = G_4 H_1 G_5 H_2$.

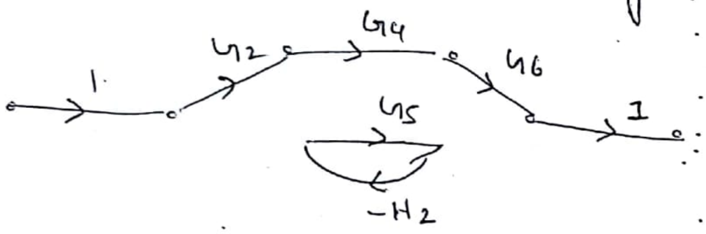
④ $\Delta = 1 - [L_1 + L_2 + L_3] + [L_1 L_2]$

$= 1 - [-G_4 H_1 - G_5 H_2 + G_1 H_2 G_8 H_1] + [G_4 H_1 G_5 H_2]$

$\Delta = 1 + G_4 H_1 + G_5 H_2 - G_1 H_2 G_8 H_1 + G_4 H_1 G_5 H_2$

⑤ $\Delta_K = ?$

① To find Δ_1, P_1 is Non-touching to loop 1.



$\therefore \Delta_1 = 1 - L_2 = 1 - [-G_5 H_2] = 1 + G_5 H_2$

② To find Δ_2, for P_2, L_1 is Non touching.

$\therefore \Delta_2 = 1 - L_1 = 1 - [-G_4 H_1] = 1 + G_4 H_1$

③ To find Δ_3, for P_3, all the loops are touching it

Δ_4,	P_4	—"—	$\Delta_4 = 1$
Δ_5,	P_5	—"—	$\Delta_5 = 1$
Δ_6,	P_6	—"—	$\Delta_6 = 1$

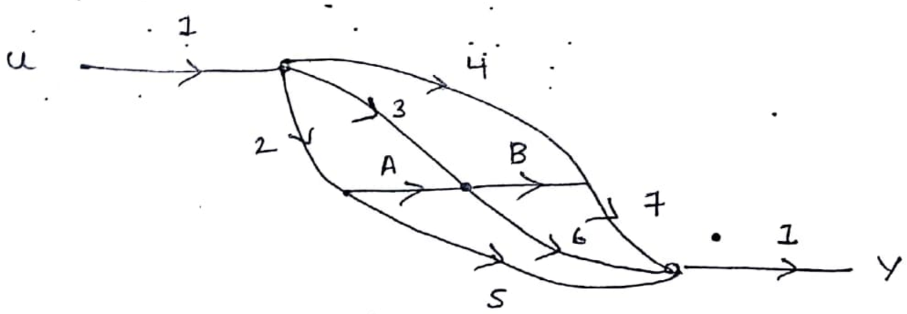
Mason's gain formula,

$$T = \frac{1}{\Delta}\sum_{K=1}^{6} P_K\Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_5}{\Delta}$$

$$\frac{C}{R} = \frac{G_2G_4G_6(1+G_5H_2) + G_3G_5G_7(1+G_4H_1) + G_2G_1G_7(1) + G_3G_8G_6(1) - G_2G_1H_2G_8G_6(1) - G_3G_8H_1G_1G_7}{1+G_4H_1 + G_5H_2 - G_1H_2G_8G_1 + G_4H_1G_5H_2}$$

Explain Mason's gain formula, use it to determine the transmittance of the flow graph shown in fig.

$$A = B = \frac{1}{s+1}$$



(1) forward path and gains.

$P_1 = 1\times 2\times 5\times 1 = 10$

$P_2 = 1\times 3\times 6\times 1 = 18$

$P_3 = 1\times 4\times 7\times 1 = 28$

K=6

$P_4 = 1\times 2\times A\times 6\times 1 = 12A = \frac{12}{s+1}$

$P_5 = 1\times 3\times B\times 7\times 1 = 21B = \frac{21}{s+1}$

$P_6 = 1\times 2\times A\times B\times 7\times 1 = 14AB = \frac{14}{(s+1)^2}$

(2) There is no loops.

(3) $\Delta = ?$

since there is no loops.

$$\Delta = 1.$$

$$\Delta_K = 1, \quad \text{for} \quad K = 1, 2, 3, 4, 5, 6.$$

(4) Mason's gain formula

$$T = \frac{1}{\Delta} \sum_{K=1}^{6} P_K \Delta_K = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4 + P_5\Delta_5 + P_6\Delta_6}{\Delta}$$

$$\frac{Y}{U} = \frac{10 + 18 + 28 + \frac{12}{s+1} + \frac{14}{(s+1)^2} + \frac{21}{(s+1)}}{1}$$

$$\frac{Y}{U} = \frac{56(s+1)^2 + 12(s+1) + 14 + 21(s+1)}{(s+1)^2}$$

Thus transmittance, $\frac{Y}{U} = \frac{56s^2 + 145s + 103}{(s+1)^2}$