UNIT - 1

Modeling of Systems:

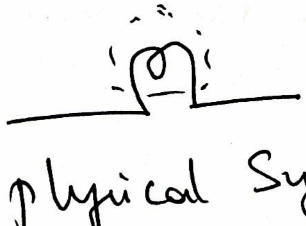
In recent years, concept of automatic control has achieved an important position in advancement of modern Science, which played an important role in the improvement of control system engineering and skills. The concept of control system also played an important role in space vehicles, satellites, guided missiles etc..

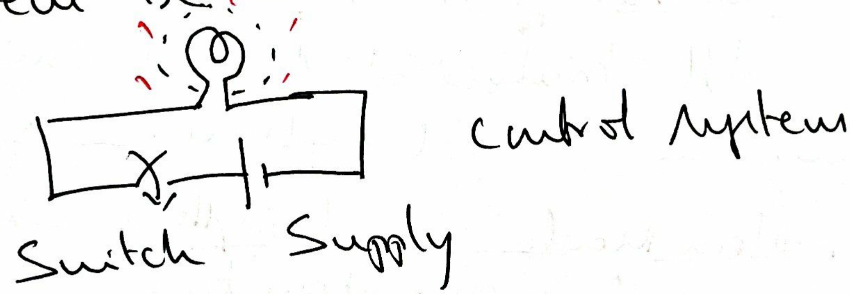
Definations!

System: is an arrangement of physical components connected or related in such a manner as to form or act as an entire unit.

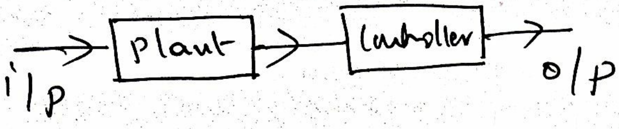
Control System: is an arrangement of physical components connected or related in such a manner so as to command or regulate itself or another system.

Ex: If a lamp is switched ON or OFF using switch, the system is called as control system.





Basic control System:



(Controlling Signal) (controlled o/p)

Plant:- The portion of the system which is to be controlled is called plant or the process.

Controller:- The element of the system itself or external to the system which controls the plant or the processes called controller.

Input:- It is an applied signal or an excitation applied to the control system from an external energy source to provide specified output.

Output: It is a particular signal of interest or the actual response obtained from a control system when input is applied to it.

Classification of the Control system:

1. **Natural control system**

All biological system existing in this world is natural control system. Ex: Human Being.

2. **Man Made control system:** The various systems which are used in our day to day life are designed and manufactured by human. Ex: Driving a car.

3. Combinational control system: It is the combination of natural and man made system. Ex: Traffic control by a police man (changing the signals)

4. Time varying and Time invarient Systems:

A system is time varying if its characteristics changes with time. Ex: Mass of a rocket.

In more general all physical systems are time varying because of ageing or due to the other factors. But over a definite period a good number of physical system can be modeled by a time-invarient system i.e parameter of a system are constant or not function of time.

5. Linear and non-linear Systems: A system is said to be linear if its response satisfy the superposition property i.e $f(x+y) = f(x) + f(y)$

$$f(x, \alpha) = \alpha \cdot f(x)$$

A system is said to be non linear if its response does not satisfy the superposition theorem.

6. Continuous time and Discrete time Control system:

If the inputs, outputs and inner variables of the system are defined for all time b/w the intervals of the systems are called continuous system (Ex: Analog signals.

In discrete time systems are one or more signals defined only at discrete instants of time b/w the internals. Ex: Digital systems.

7. Deterministic or stochastic control System:

If the signal can be defined precisely without any uncertainty or chance it is called deterministic, or in any control system if its response is unpredictable it is called stochastic system i.e Noise.

8. Lumped and distributed parameter control System

A lumped component is one which can be identified by a single single parameter b/w terminals.

Component like transmission line, heat insulating slaps are distributed parameters which needs partial differential equations for decription of the system.

9. SISO and MIMO Control system:

Single input Single out put systems. Ex:

proportional control system.

Multiple input multiple out put control system Ex:

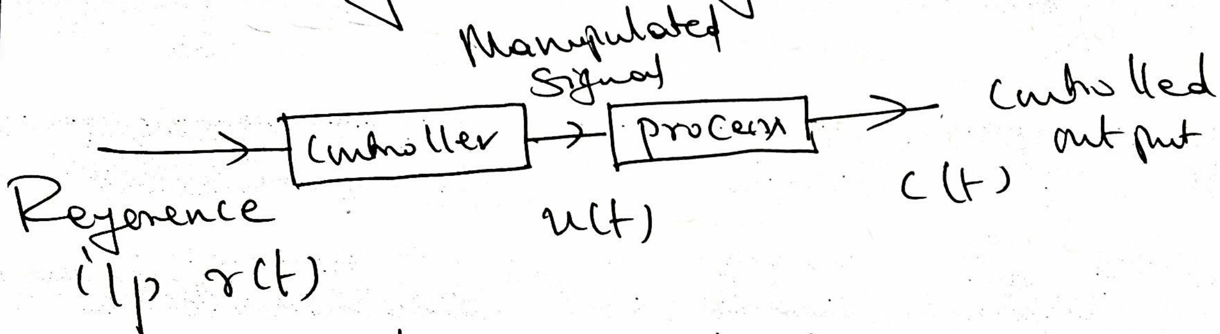
10. open loop and closed loop control Systems:

Open loop control System: An open loop control system is one in which the control action is independent of the o/p (out put).

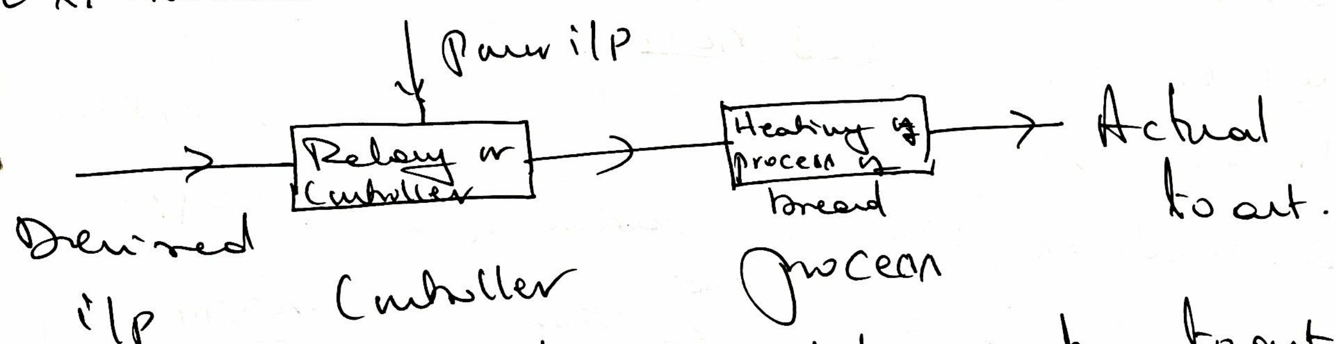
The outstanding features of the open loop control systems are,

(1) The ability to perform accurately is determined by their calibration.

(2) They are not generally instable.

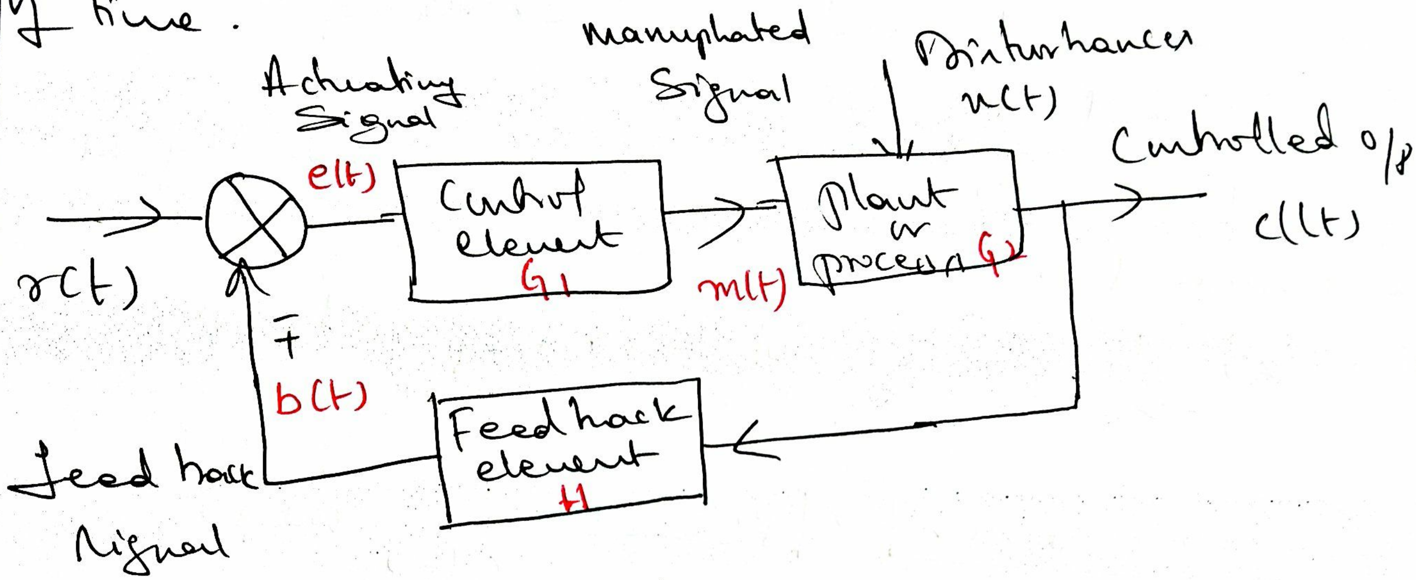


Ex: Automatic toaster system



In this system the quality of the toast depends on time for which it is toasted. Depending on the time the bread is toasted not on quality of the bread is made up of, the end user judge the effect of toast.

Closed loop control system:- In this type of system the controlling action somewhat dependent on the o/p. The output is continuous measure of time.



(+tive feedback is positive cumulative, unbounded w.r.t time and unstable)

Characteristic of feedback:

1. Increases accuracy
2. Reduces sensitivity ratio of o/p to i/p
3. Reduces effect of non-linearities
4. Increases band width.
5. Tendency towards oscillation or instability.

G_1: Gain of controller needed to generate manipulated signal m(t) to apply it to plant.

G_2: Gain of plant needed to control the system.

H : Gain of feed back component need to produce b(t) a primary feed back signal

r(t) : A Reference input signal.

C(t) : Controlled output signal.

b(t) : A primary feed back signal which is a function of algebraically summed with reference i/p to obtain actuating signal e(t).

e(t) : Control action or error signal

m(t) : manipulated signal

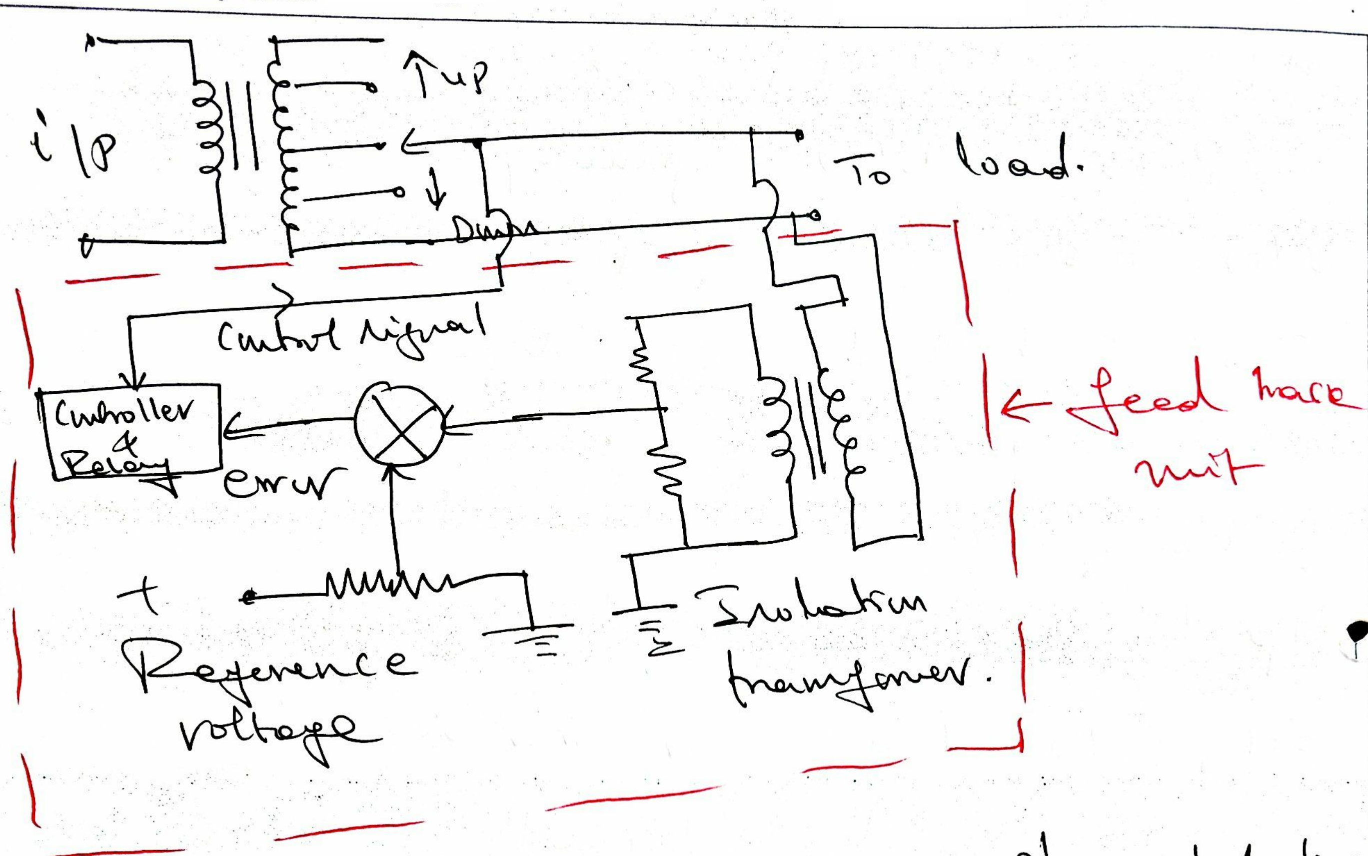
u(t) : undesired i/p signal called disturbance.

Forward path: It is the transmission path from actuating signal e(t) to controlled o/p c(t)

Feed back path: It is the transmission path from the controlled o/p c(t) to feed back signal b(t).

Voltage stabilizer example for CLFCS :

The basic working principle is to control the number of secondary turns as per requirement to increase or decrease as the variation in the i/p voltage to maintain constant voltage (o/p).



The mathematical modeling of physical system is characterised by linear, time-invariant, lumped and deterministic type, the two most common methods employed to describe the dynamic behaviour of such system are

1. Transfer Function representation
2. State Space representation

In Modeling of control system it is necessary to have some type of equivalent representation i.e (1) Mathematical Modelling to study dynamic behaviour (2) Block diagram representation using transfer function (3) Signal flow graph using Mason's gain formula.

Analysis of Mechanical System

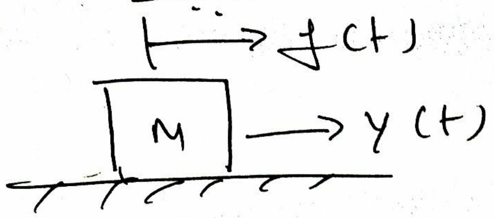
Most of the feedback control systems consists of Mechanical as well as electrical components, Mathematical point of view both of Mechanical and electrical elements are analogous to each other and it is easier to analyse the systems in terms of electrical terms or system because of its standard, simple symbol and simple laws and theorems.

In Mechanical system the motion can be of different types i.e translational (linear), Rotational or combination of both.

Translation System:- Here the motion takes place in straight line. These system are characterised by displacement, linear velocity, linear acc.

According to Newton's law of motion, the sum of force applied is equal to the sum of force consumed to produce displacement. The elements involved is Mass, Spring and Friction.

Mass: According to Newton's law of motion the algebraic sum of force acting on a rigid body in a given direction is equal to the product of the mass of the body and its acc in the same direction.

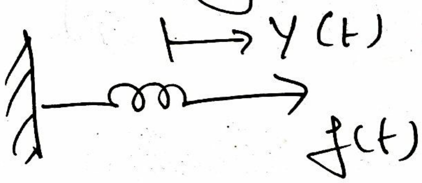


$$f(t) \propto \frac{d^2 y(t)}{dt}$$

or

$$f(t) = M\frac{d^2 y(t)}{dt^2} \quad \text{— (1)}$$

Linear Spring: In general Spring is considered as an element that stores potential energy which is analogous to capacitor in (F-V)



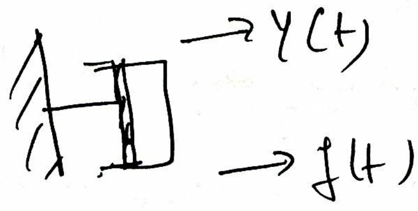
$$f(t) \propto y(t)$$

or

$$f(t) = k\,y(t) \quad \text{— (2)}$$

If the spring preloaded with a load tension of 'T', then $f(t) = T - Ky(t)$ — (3)

Friction: whenever there is motion or tendency of motion b/w two element friction exists.



$$f(t) \propto \frac{dy(t)}{dt}$$

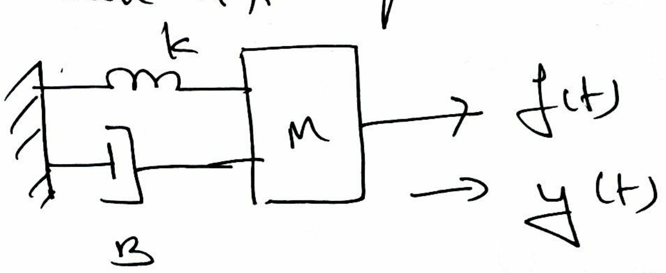
or

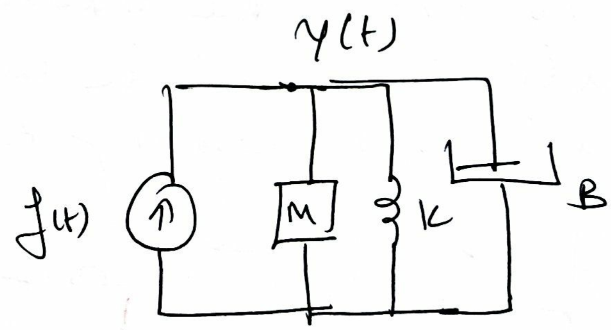
$$f(t) = B\frac{dy(t)}{dt} \quad \text{— (4)}$$

Electrical Analogous of Mechanical System

Mechanical quantity		F − V (KVL)	F − I (KCL)
Force	F	V	I
displacement	x	q	ϕ
Velocity	$\dot{x}$	i	v
Mass	M	L	C
Friction	B	R	1/R (G)
Spring stiffness	k	1/K	1/L

1 For the mechanical system shown below, obtain mechanical equivalent ckt, hence write the equilibrium equation based on F−V and F−I analogy and obtain it's equivalent ckt also.





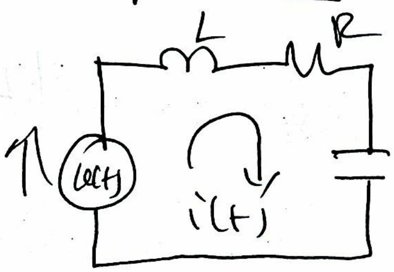
$$f(t) = M\frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + K\,y(t) \quad \text{— (1)}$$

Analogous equation based on F-V & F-I analogy are:

$$v(t) = L\frac{d^2 i(t)}{dt^2} + R\,i(t) + \frac{1}{C}\int i(t)\,dt \quad \text{— (2)}$$

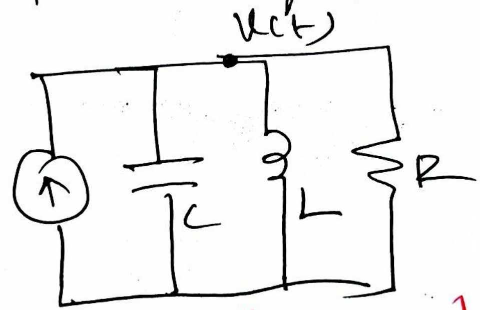
$$i(t) = C\frac{dv(t)}{dt} + \frac{v(t)}{R} + \frac{1}{L}\int v(t)\cdot dt \quad \text{— (3)}$$

From Equation (2)



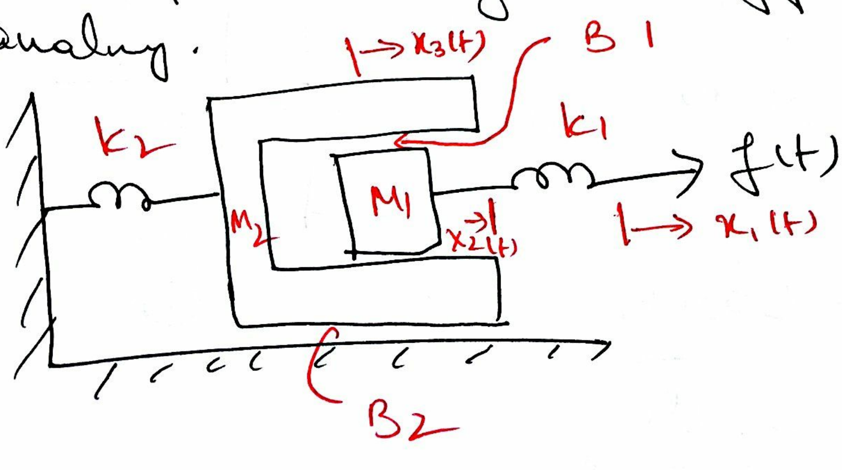
(KVL)

From equation (3)

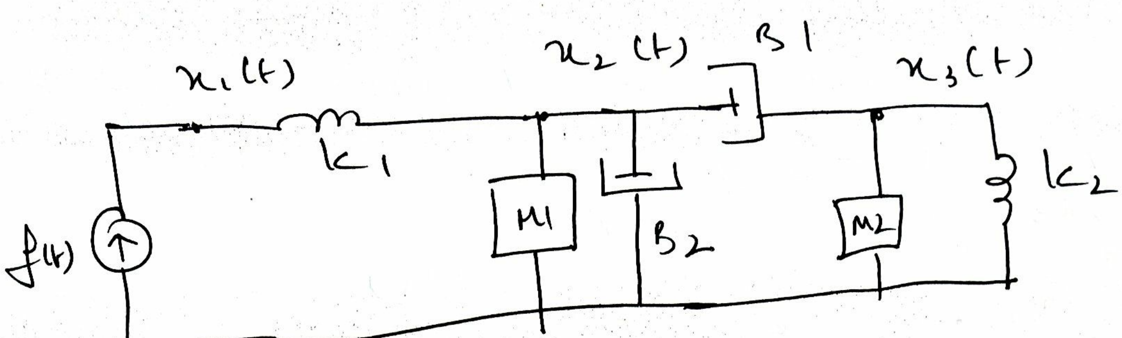


(KCL)

(2) Write the differential equations governing the behaviour of the mechanical system shown below. Also obtain the analogous electrical circuit based on (i) Force-voltage analogy (ii) Force-current analogy.



The mechanical equivalent ckt is :

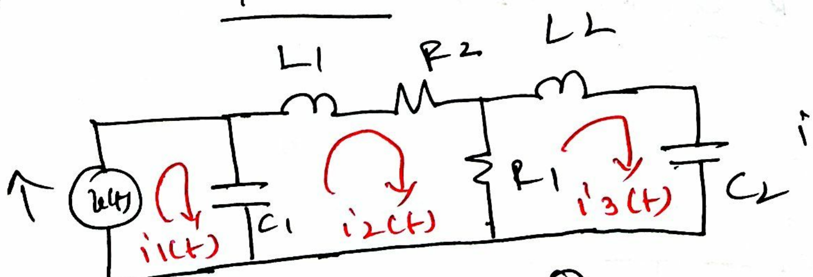


$$f(t) = k_1(x_1(t) - x_2(t)) \quad (1)$$

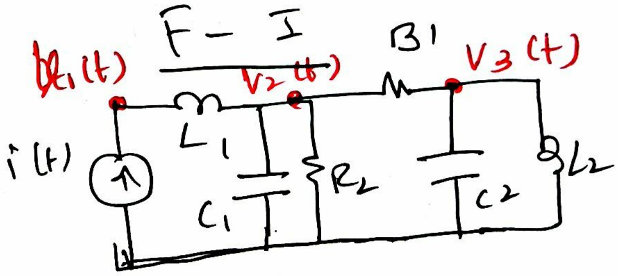
$$0 = M_1 \frac{d^2 x_2(t)}{dt^2} + k_1(x_2(t) - x_3(t)) + B_2 \frac{d x_2(t)}{dt} + B_1 \frac{d}{dt}(x_2(t) - x_3(t)) \quad (2)$$

$$0 = M_2 \frac{d^2 x_3(t)}{dt^2} + B_1 \frac{d}{dt}(x_3(t) - x_2(t)) + k_2 x_3(t) \quad (3)$$

F – V



F – I

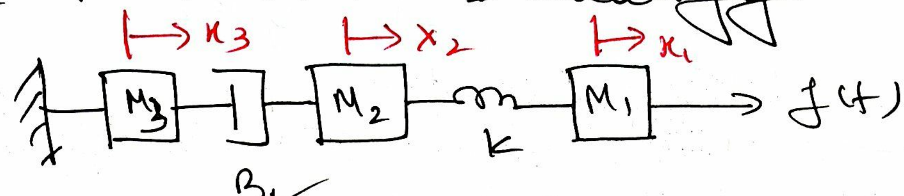


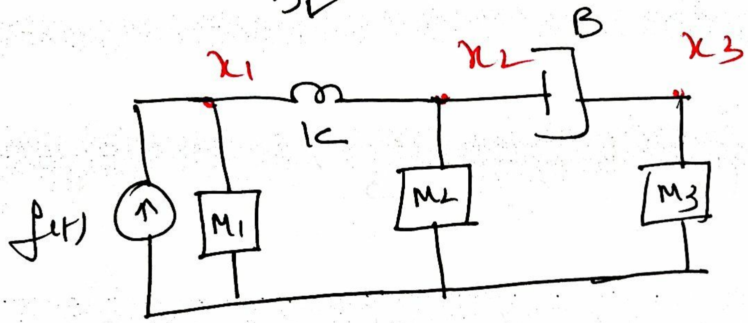
$$v(t) = \frac{1}{C_1} \int i_1(t) \cdot dt \quad (1)$$

$$0 = L_1 \frac{d i_2(t)}{dt^2} + R_2 i_2(t) + \frac{1}{C_1} \int i_2(t) - i_1(t) \quad (2)$$

$$0 = R_1 (i_3(t) - i_2(t)) + L_2 \frac{d i_3(t)}{dt} + \frac{1}{C_2} \int i_3(t) \cdot dt$$

③ Write the mechanical equivalent diagram for the mechanical system shown below, hence obtain electrical analogous system based on (1) F – V (2) F – I analogy.





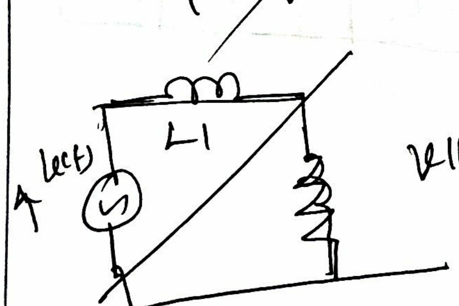
$$f(t) = M_1 \frac{d^2x_1}{dt^2} + K(x_1 - x_2) \quad \text{—— ①}$$

$$0 = M_2 \frac{d^2x_2}{dt^2} + B\frac{d(x_2 - x_1)}{dt} + K(x_2 - x_1) \quad \text{— ②}$$

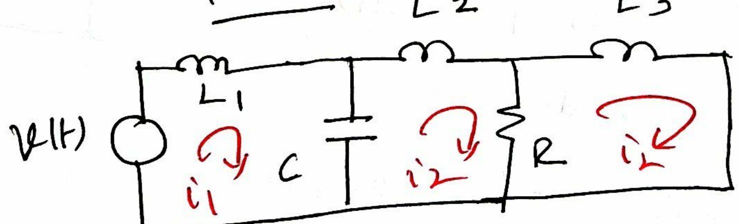
$$0 = M_3 \frac{d^2x_3}{dt^2} + B\frac{d(x_3 - x_2)}{dt} \quad \text{—— ③}$$

F – I

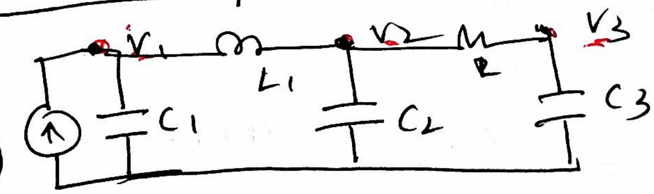
~~F – V~~



F – V

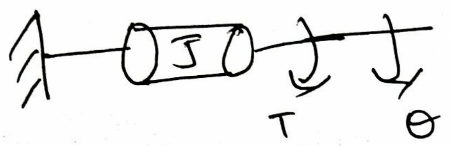


F – I



Rotational Motion | Rotational motion of a body is defined as motion about a fixed axis. The variable generally used are torque, angular acc(α), angular velocity (ω) and angular displacement (θ).

The inertia J is the property of the element which stores kinetic energy of rotational motion.

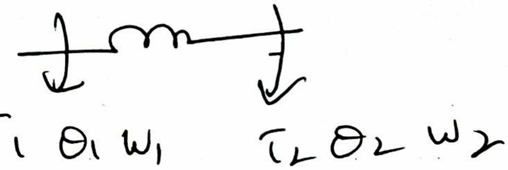


$$T(t) \propto \frac{d^2\theta}{dt^2}$$

or

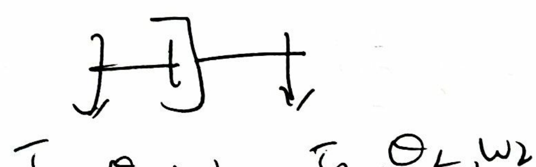
$$T(t) \propto \frac{d\omega}{dt} \quad \text{or} \quad T(t) = J\frac{d\omega}{dt}$$

Torsional Spring :



$$T = k(\theta_1 - \theta_2)$$

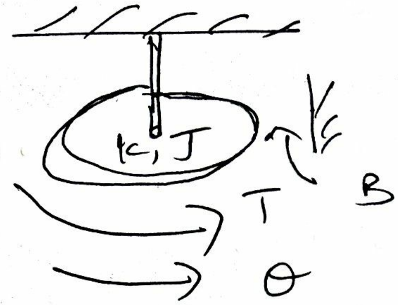
The damper element



$$T = B\frac{d(\theta_1 - \theta_2)}{dt}$$

Rotational Element	F-V	F-I
T	v	i
θ	q	φ
ω	i	v
J	L	C
k	c	L
B	R	1/R (G)

1. obtain the transfer function of the system shown below, hence obtain F-V, F-I equivalent ckt dia.



$$T(t) = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

$$T(s) = s^2 J\theta(s) + sB\theta(s) + K\theta(s)$$

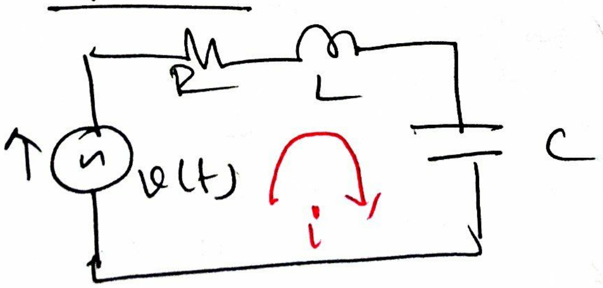
$$T(s) = \theta(s)\left[s^2 J + sB + K\right]$$

$$\boxed{\frac{\theta(s)}{T(s)} = T.F = \frac{1}{Js^2 + Bs + K}}$$

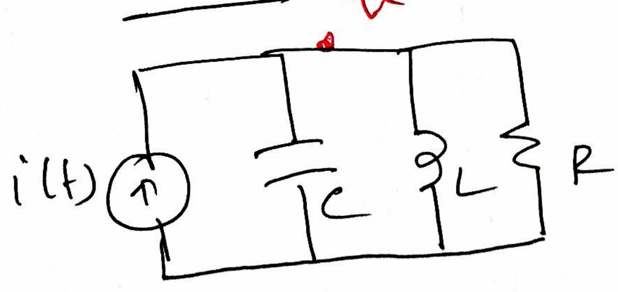
$$v(t) = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$

$$i(t) = C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v.dt$$

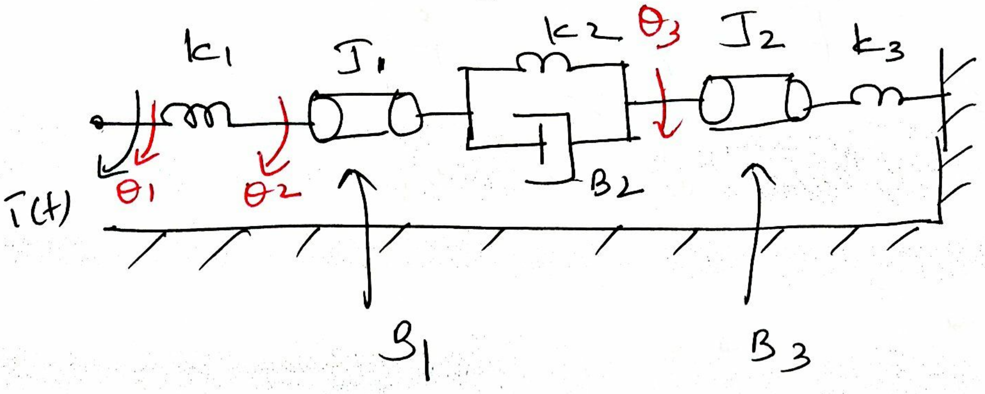
F - V

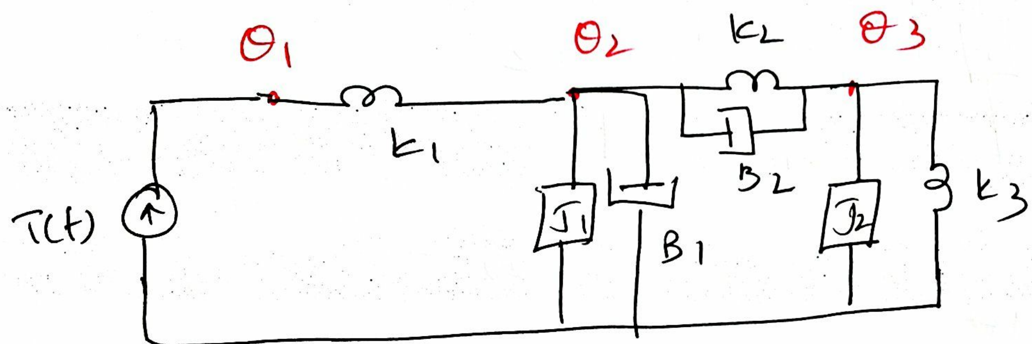


F - I



(2) For the given rotational system, obtain the electrical analogous system based on F-V and F-I analogous method.



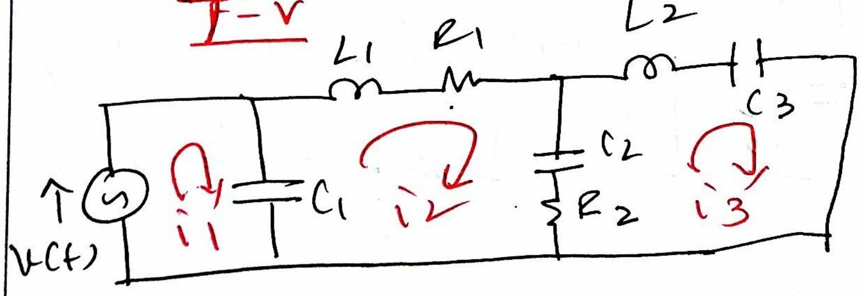


$$T(t) = k_1(\theta_1 - \theta_2) \quad \text{—} \ (1)$$

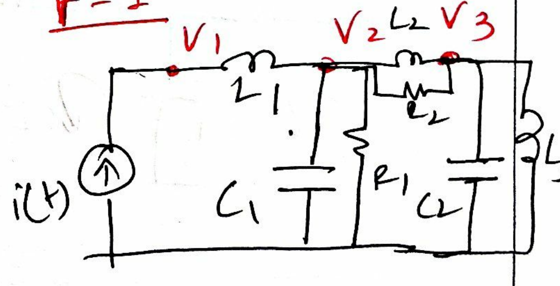
$$0 = J_1 \frac{d^2\theta_2}{dt} + B_1 \frac{d\theta_2}{dt} + k_1(\theta_2 - \theta_1) + k_2(\theta_2 - \theta_3) + B_2 \frac{d(\theta_2 - \theta_3)}{dt} \quad \text{—} \ (2)$$

$$0 = J_2 \frac{d^2\theta_3}{dt^2} + k_3\theta_3 + k_2(\theta_3 - \theta_2) + B_2 \frac{d(\theta_3 - \theta_2)}{dt} \quad \text{—} \ (3)$$

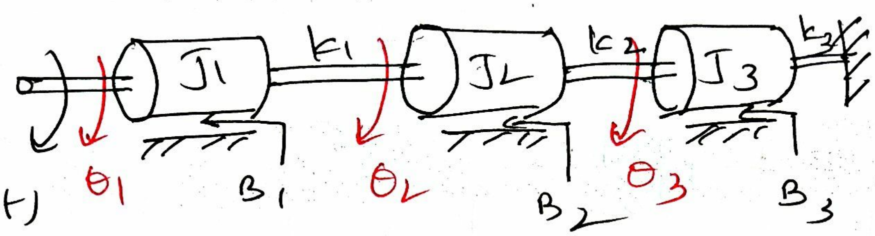
F-V

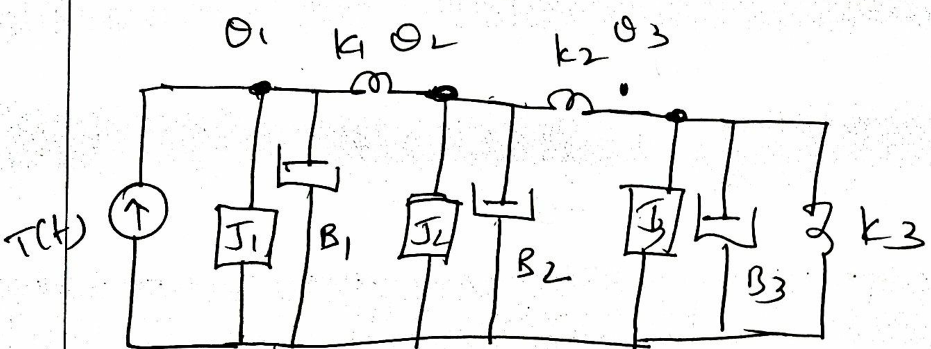


F-I



(3) Write the differential equation for the system shown. Also draw electrical analogous circuit based on T-v and T-I analogy.



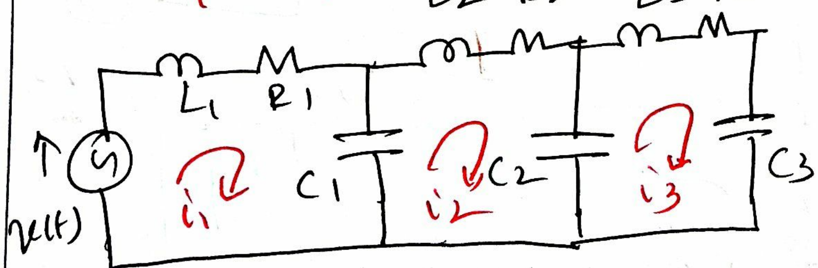


$$T(t) = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + k_1(\theta_1 - \theta_2) \quad -(1)$$

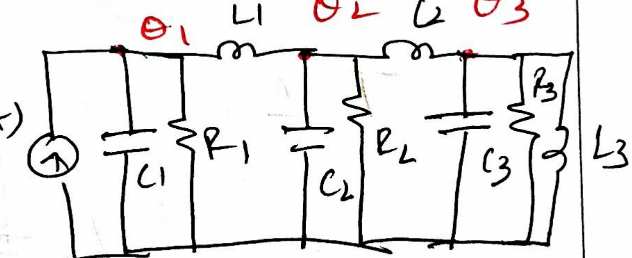
$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + k_1(\theta_2 - \theta_1) + k_2(\theta_2 - \theta_3) \quad -(2)$$

$$0 = J_3 \frac{d^2\theta_3}{dt^2} + B_3 \frac{d\theta_3}{dt} + k_3\theta_3 + k_2(\theta_3 - \theta_2) \quad -(3)$$

T - v



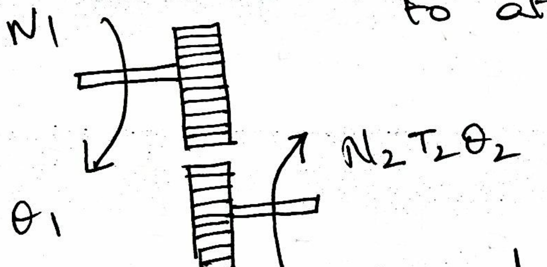
T - I



Gear trains :-

Gears are used in mechanical rotational system, These transmit energy from one part of the system to another part in such a way that force, torque, speed and displacement are altered.

Transformation of torque is accompanied by a corresponding change in speed decided by the transformation ratio. Gear trains are used to attain magnification or reduction in speed.

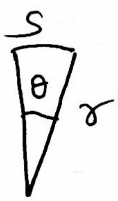


(i) The number of teeth on the surface of the gears is proportional to the radii r_1 & r_2 of the gears.

$N_1 \propto r_1$ or $\frac{N_1}{N_2} = \frac{r_1}{r_2}$ — (1)

$N_2 \propto r_2$

(ii) The linear displacement travelled on the surface of each gear remains the same.



$S = r\theta$, $r_1\theta_1 = r_2\theta_2$

or

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} \quad \text{— (2)}$$

(iii) Work done by one gear is equal to that of other, since there are no losses i.e $T_1\theta_1 = T_2\theta_2$

$\frac{T_1}{T_2} = \frac{\theta_2}{\theta_1}$ — (3)

$$\boxed{\frac{T_1}{T_2} = \frac{N_1}{N_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}}$$

where T = applied Torque.

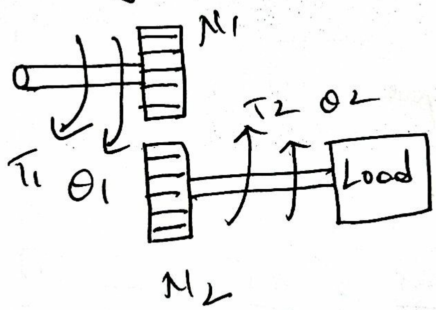
θ_1, θ_2 = angular displacement

T_1, T_2 = Torque transmitted to gears

N_1, N_2 - No of teeth

B_1, B_2 - viscous friction co-efficient.

(1) The system shows a load connected through a gear train having ratio N_2/N_1. Find load moment of inertia J_2 and friction B_2. Find transfer function $\theta_1(s)/T_1(s)$, Neglecting inertial and friction of the gears.



w.k.t

$$\boxed{\frac{T_1}{T_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}}$$

$$T_2(t) = J_2 \frac{d^2\theta_2}{dt} + B_2 \frac{d\theta_2}{dt}, \quad \text{— (1)} \qquad T_1 = T_2\left(\frac{N_1}{N_2}\right) \text{— (2)}$$

$$T_1(t) = \frac{N_1}{N_2} J_2 \frac{d^2\theta_2}{dt} + \frac{N_1}{N_2} B_2 \frac{d\theta_2}{dt} \quad \text{— (3)}$$

$$\boxed{\theta_2 = \frac{N_1}{N_2}\theta_1} \quad \text{— (4)}$$

$$T_1(t) = \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt}$$

Taking Laplace transformation.

$$T_1(s) = \left(\frac{N_1}{N_2}\right)^2 J_2\, s^2 \theta_1(s) + \left(\frac{N_1}{N_2}\right)^2 B_2\, s\, \theta_1(s)$$

$$T_1(s) = J_{eq}\, s^2 \theta_1(s) + B_{eq}\, s\, \theta_1(s)$$

where $J_{eq} = \left(\frac{N_1}{N_2}\right)^2 J_2$ and $B_{eq} = \left(\frac{N_1}{N_2}\right)^2 B_2$

$$\therefore T_1(s) = \theta_1(s)\left[J_{eq} s^2 + B_{eq} s\right]$$

or

$$\boxed{\frac{\theta_1(s)}{T_1(s)} = T.F = \frac{1}{s(sJ_{eq} + B_{eq})}}$$