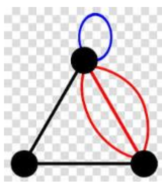
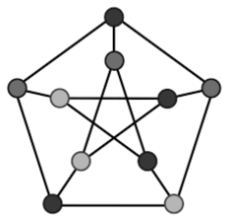
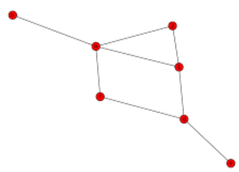
Linear Algebra and Graph Theory

Q.No	QUESTION BANK
1.	a) Define a vector space and a subspace of a vector space. Give an example each. b) For what values of h will y be in Span$\{v_1, v_2, v_3\}$ if $v_1 = \begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}, v_3 = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$ $and\ y = \begin{bmatrix} -4 \\ 3 \\ h \end{bmatrix}$
2.	a) Write the vector $\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ as a linear combination of the vectors, $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 4 \end{bmatrix}$. b) Express the vector $b = \begin{bmatrix} 2 \\ 13 \\ 6 \end{bmatrix}$ as a linear combination of the vectors, $v_1 = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$.
3.	a) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, v_3 = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$, and $w = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ (i) Is w in $\{v_1, v_2, v_3\}$? How many vectors are in $\{v_1, v_2, v_3\}$? (ii) How many vectors are in Span$\{v_1, v_2, v_3\}$? (iii) Is w in the subspace spanned by $\{v_1, v_2, v_3\}$? why? b) Show that w is in the subspace of $\mathcal{R}^4$ spanned by v_1, v_2, v_3, where $w = \begin{bmatrix} -9 \\ 7 \\ 4 \\ 8 \end{bmatrix}, v_1 = \begin{bmatrix} 7 \\ -4 \\ -2 \\ 9 \end{bmatrix}, v_2 = \begin{bmatrix} -4 \\ 5 \\ -1 \\ -7 \end{bmatrix}, v_3 = \begin{bmatrix} -9 \\ 4 \\ 4 \\ -7 \end{bmatrix}$
4.	a) Define Null Space and Column space of a matrix b) Find a spanning set for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$
5.	a) Let $= \begin{bmatrix} 2 & 4 & -2 & 1 \\ -2 & -5 & 7 & 3 \\ 3 & 7 & -8 & 6 \end{bmatrix}$. Let $u = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 0 \end{bmatrix}$ and $v = \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}$ i) Determine if u is in NulA. Could u be in ColA? ii) Determine if v is in ColA. Could v be in NulA?

	b) Let $A = \begin{bmatrix} 1 & 0 & 3 & -2 \\ 0 & 3 & 1 & 1 \\ 1 & 3 & 4 & -1 \end{bmatrix}$. For each of the following vectors, determine whether the vectors are in the null space $\mathcal{N}(A)$. (a) $\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} -4 \\ -1 \\ 2 \\ 1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Then describe the null space $\mathcal{N}(A)$ of the matrix A
6.	a) Define Linearly independent and linearly dependent vectors with an example b) Determine whether the following set of vectors is linearly independent or linearly dependent. If the set is linearly dependent, express one vector in the following set as a linear combination of the other: $\left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 7 \\ 11 \end{bmatrix} \right\}$
7.	**a)** Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ a \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 4 \\ b \end{bmatrix}$ be vectors in $\mathbb{R}^3$. Determine a condition on the scalars a, b so that the set of vectors $\{v_1, v_2, v_3\}$ is linearly dependent. **b)** Find the value(s) of h for which the following set of vectors $\left\{ v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} h \\ 1 \\ -h \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 2h \\ 3h+1 \end{bmatrix} \right\}$ is linearly independent.
8	a) Define the basis for a vector. Find a basis for span(s), where $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \right\}$. b) Let $S = \{v_1, v_2, v_3, v_4, v_5\}$ where $v_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 5 \\ -1 \\ 5 \end{bmatrix}, v_4 = \begin{bmatrix} 1 \\ 1 \\ 4 \\ -1 \end{bmatrix}, v_5 = \begin{bmatrix} 2 \\ 7 \\ 0 \\ 2 \end{bmatrix}$. Find a basis for the span(s).
9	a) Prove that if a vector space V has a basis $\mathcal{B} = \{b_1, b_2, \ldots \ldots b_n\}$, then any set in V containing more than n vectors must be linearly dependent. b)Find the dimension of the subspace spanned by the given vectors $v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, v_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, v_3 = \begin{bmatrix} 9 \\ 4 \\ -2 \end{bmatrix}, v_4 = \begin{bmatrix} -7 \\ -3 \\ 1 \end{bmatrix}$

10	a) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation. Let $u = \begin{bmatrix}1\\2\end{bmatrix}, v = \begin{bmatrix}3\\5\end{bmatrix}$ be 2-dimensional vectors. Suppose that $T(u) = T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}-3\\5\end{bmatrix}$ and $T(v) = T\left(\begin{bmatrix}3\\5\end{bmatrix}\right) = \begin{bmatrix}7\\1\end{bmatrix}$. Let $w = \begin{bmatrix}x\\y\end{bmatrix} \in \mathbb{R}^2$. Find the formula for $T(w)$ in terms of x and y. b) Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T\left(\begin{bmatrix}3\\2\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}, T\left(\begin{bmatrix}4\\3\end{bmatrix}\right) = \begin{bmatrix}0\\-5\\1\end{bmatrix}$. Find the matrix representation of T (with respect to the standard basis for $\mathbb{R}^2$).
11	Define a graph. Draw the diagram of a graph where the degrees of vertices are 1,1,1,2,3,4,5,7.
	Define simple and multi graph with an example.
12	Show that in a graph the number of vertices of odd degree is even.
	Define degree of a vertex. State Handshaking property.
13	Show that every simple graph has at least two vertices of the same degree.
	Indicate the degree of each vertex and verify the handshaking property. 
14	Show that every cubic graph has an even number of vertices.
	Explain a Regular graph with an example.
15	Discuss the Konigsberg bridge problem.
	Define with an example: (a) Subgraph of a graph (b) spanning sub graph (c) Complement of a graph.
16	Define complete bipartite graph. Find the complement of the complete bipartite graph $K_{3,3}$.
	Define Bipartite graph. Show that the complement of a bipartite graph need not be a bipartite graph.
17	Define Pendant and isolated vertex with an example
	Draw the complete graphs K_2, K_3, K_4, K_5 and K_6.
18	Define Eulerian trail and Eulerian circuit with an example.
	Define Hamiltonian path and Hamiltonian cycle with an example.
19	Explain Adjacency matrix and incidence matrix with an example.
	Draw the graph represented by the given adjacency matrices: i) $\begin{bmatrix}1 & 0 & 1\\0 & 0 & 1\\1 & 1 & 1\end{bmatrix}$ ii) $\begin{bmatrix}0 & 1 & 1\\1 & 0 & 0\\1 & 0 & 0\end{bmatrix}$

20	Write the adjancency matrix for the following graphs i)  ii) 
	Write the incidence matrix for the above graphs.