

Department of Mathematics
Question Bank for IT(EC, TC, EE, ML, EI, IS)
Unit 2: Complex Variable

Q.No	Question
1.	a) Find the modulus and amplitude of $\frac{(1+i)^2}{(3+i)}$. b) Define the neighborhood of a complex variable with Geometrical interpolation.
2.	a) Determine the region in the z-plane represented by i) $1 < z + 2i \leq 3$ ii) $\operatorname{Re}(z) > 3$ iii) $\frac{\pi}{6} \leq \operatorname{amp}(z) \leq \frac{\pi}{3}$ b) Define the following : i) Limit of a complex function ii) Continuity of a complex function iii) Differentiability of a complex function
3.	a) If $f(z) = u + iv$ is analytic, then prove that Cauchy- Riemann equations $u_x = v_y, v_x = -u_y$ are true. b) If $f(z) = u(r, \theta) + i v(r, \theta)$ is analytic, then prove that Cauchy- Riemann equations in polar form $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$ are true.
4.	a) If $u = x^2 - y^2, v = x^3 - 3xy^2$ Show that u and v are harmonic functions but $f(z) = u + iv$ is not analytic. b) If $u = \frac{x^2}{y}, y \neq 0$ and $v = x^2 + 2y^2$, then show that the curve $u = \text{constant}$ and $v = \text{constant}$ are orthogonal but $f(z) = u + iv$ is not an analytic function.
5.	a) Show that $f(z) = z^n$ is analytic. Hence find its derivative. b) Show that the function $f(z) = \log z$ is analytic and hence find its derivative.
6.	a) Show that the function $f(z) = \cosh z$ is analytic and hence find its derivative. b) Show that $w = z + e^z$ is analytic and hence find $\frac{dw}{dz}$.
7.	a) Show that the function $f(z) = \sin 2z$ is analytic and hence find its derivative. b) Find the analytic function $f(z) = u + iv$, where $u = x^2 - y^2 + \frac{x}{x^2 + y^2}$
8.	a) Find the analytic function whose real part is $\log \sqrt{x^2 + y^2}$. b) Find the analytic function $f(z) = u + iv$, where $v = \frac{y}{x^2 + y^2}$.
9.	a) Find the analytic function $f(z) = u + iv$, where $v = e^x(x \sin y + y \cos y)$ using Milne Thompson method. b) Find the analytic function $f(z) = u + iv$, where $u = e^{-x}\{(x^2 - y^2) \cos y + 2xysin y\}$ using Milne Thompson method.
10.	a) Find the analytic function $f(z) = u(r, \theta) + iv(r, \theta)$, where $v(r, \theta) = r^2 \cos 2\theta - r \cos \theta + 2$ using Milne Thompson method. b) Find the analytic function $f(z) = u(r, \theta) + iv(r, \theta)$, where $u(r, \theta) = \frac{\cos 2\theta}{r^2}$ using Milne Thompson method.
11.	a) Find the analytic function $f(z) = u + iv$, where $v = -\frac{\sin \theta}{r}$ using Milne Thompson method. b) Find the analytic function $f(z) = u + iv$, where $u = r^2 \cos 2\theta$
12.	a) Show that $u = e^{2x}(x \cos 2y - y \sin 2y)$ is harmonic and find its harmonic conjugate.

	b) Show that $u = e^x \cos y + xy$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function.
13.	a) Show that $v = \cos x \sin y$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function. b) Show that $u = x^3 - 3xy^2 - 3x^2 + 3y^2 + 1$ is harmonic and find its harmonic conjugate.
14.	a) Show that $u = \left(r + \frac{1}{r}\right) \cos \theta$ is harmonic and find its harmonic conjugate. b) Show that $v = \left(r - \frac{1}{r}\right) \sin \theta$ is harmonic and find its harmonic conjugate.
15.	a) Show that $v = r \sin \theta + \frac{\cos \theta}{r}$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function. b) Show that $u = \frac{\cos \theta}{r}$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function.
16.	a) Show that $u = x^2 + 4x - y^2 + 2y$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function. b) Show that $v = 2xy - 2x + 4y$ is harmonic and find its harmonic conjugate and also find the corresponding analytic function.
17.	a) Find the analytic function $f(z) = u + iv$ if $u + v = \frac{1}{r^2} (\cos 2\theta - \sin 2\theta)$ ($r \neq 0$). b) Find the analytic function $f(z) = u + iv$ if $u + v = r(\cos \theta + \sin \theta) + \frac{1}{r}(\cos \theta - \sin \theta)$
18.	a) Find the analytic function $f(z) = u + iv$ if $u + v = (x + y) + e^x(\cos y + \sin y)$. b) Find the analytic function $f(z)$ as a function of z gives the sum of its real and imaginary part is $x^3 - y^3 + 3xy(x - y)$.
19.	a) Find the analytic function $f(z) = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$ b) Find the analytic function $f(z) = u + iv$ if $u - v = e^x(\cos y - \sin y)$
20.	a) An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2y - y^3$, find the stream function. b) Two concentric circular cylinders of radii r_1, r_2 ($r_1 < r_2$) are kept at potentials ϕ_1 and ϕ_2 respectively. Using complex function $\omega = a \log z + c$, prove that the capacitance per unit length of the capacitor formed by them is $\frac{2\pi\lambda}{\log(r_2/r_1)}$ where λ is the dielectric constant of the medium.