Random Variables :-

• In a random experiment, if a real variable is associated with every outcome then it is called a random variable (or) Stochastic variable.

* Random Variable assigns a real number to every sample point in the sample space of a random experiments. denoted by X, Y, Z....

The set of all real no's of a random variable X is called range of X.

Eg:- Suppose a coin is tossed twice, we shall associate two different random variables X, Y as

$S = \{HH, HT, TH, TT\}$

$X = \{$No of heads in the outcome$\} = \{0, 1, 2\}$

$Y = \{$No of tails in the outcome$\} = \{0, 1, 2\}$

Discrete and Continuous random Variables:-

• If a random variable takes finite (or) countably infinite number of values then it is called a discrete random variables.

countable infinite means a sequence of real numbers.

ex:- 1) Tossing a coin & observing the outcome.

2) Tossing coins & observing the number of heads turing up.

3) Throwing a die and observing the number of on the face.

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If a random variable takes non-countable infinite number of values then it is called a non discrete (or) continuous random variable.

Eg:- 1) Weights of articles

2) Length of nails produced by a machine

3) Conducting a Survey on the life of electric bulbs.

* Generally counting problems correspond to discrete random variables and measuring problems lead to continuous random variables.

According to the category of the random variables we have two types of probability distributions.

① Discrete probability distributions

② Repeated trials

Discrete probability distribution :-

If for each x_i of a discrete random variable X we assign a real no $P(x_i)$ such that

(1) $p(x_i) \geq 0$

(2) $\sum_i p(x_i) = 1$

then the function $p(x)$ is called a probability function

If the probability that X takes the values x_i is p_i then $p(X = x_i) = p_i$ (or) $p(x_i)$

The set of values $[x_i, p(x_i)]$ is called a discrete (finite) probability distribution of the discrete random variable X.

The function $p(X)$ is called the probability density function (p.d.f) (or) the probability mass function (p.m.f)

• The distribution function $f(x)$ defined by,
$$f(x) = p(X \leq x) = \sum_{i=1}^{x} p(x_i),$$ x being an integer is called the cumulative distribution function (c.d.f)

* Mean, $$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$

Variance, $$\sigma^2 = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

* In this case $p(x_i)$ corresponds to f_i
we have, $\sum f_i = \sum p(x_i) = 1$

The Mean and Variance of the discrete probability distribution is defined as follows.

$$\text{Mean}, (\mu) = \sum_i x_i \cdot p(x_i)$$

$$\text{Variance}, (V) = \sum_i (x_i - \mu)^2 \cdot p(x_i)$$

$$\text{Standard deviation } (\sigma) = \sqrt{V}$$

Note :- Variance ~~and~~ can also be put in the form
$$V = \sum_i x_i^2 \, p(x_i) - \left[\sum_i x_i \, p(x_i)\right]^2$$

problems:-

1) A coin is tossed twice. A random variable X represent the number of heads turning up. find the discrete probability distribution for X. Also find its mean & variance.

→) $S = \{HH, HT, TH, TT\}$

The association of the elements of S to the random variable X are respectively 2, 1, 1, 0

$p(HH) = 1/4$, $p(HT) = 1/4$, $p(TH) = 1/4$, $p(TT) = 1/4$

$p(x=0$; i.e no head $) = p(TT) = 1/4$

$p(x=1$; 1 head $) = p(HT \cup TH) = p(HT) + p(TH)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$p(x=2$; 2 heads $) = p(HH) = 1/4$

The discrete probability distribution for X is as follows

$X = x_i$	0	1	2
$p(x_i)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$p(x_i) \geq 0$ & $\Sigma p(x_i) = 1$

Mean $= \mu = \sum_i x_i \cdot p(x_i) = 0 + \frac{1}{2} + \frac{2}{4} = 1$

Variance $= V = \Sigma (x_i - \mu)^2 \cdot p(x_i)$

$= (0-1)^2 \cdot \frac{1}{4} + (1-1)^2 \cdot \frac{1}{2} + (2-1)^2 \frac{1}{4}$

$= \frac{1}{2}$

Mean = 1 variance = 1/2

2) The Random Variable X has the following probability mass function, find (i) K (ii) $p(X<3)$ (iii) $p(3<x\le5)$

X	0	1	2	3	4	5
p(x)	k	3k	5k	7k	9k	11k

=)

The probability distribution is valid if $p(x)\ge0$ &

$$\Sigma p(x) = 1$$

$K\ge0$ & $K+3K+5K+7K+9K+11K=1$

$36K = 1$ (or) $K = \frac{1}{36}$

$p(x<3) = p(0)+p(1)+p(2)$

$= k+3k+5k = 9k = \frac{9}{36} = \frac{1}{4}$

$p(3<x\le5) = p(4)+p(5) = 9k+11k = 20k = \frac{20}{36} = \frac{5}{9}$

3) A random variable $X=x$ has the following probability distribution.

X	0	1	2	3	4	5	6	7
p(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

find (i) K (ii) $p(X<6)$ (iii) $p(X>6)$ also find the probability distribution & distribution function of x.

→)

we have $p(x)\ge0$ & $\Sigma p(x)=1$

The first condition is satisfied for $k>0$

& we have to find k $\Rightarrow \Sigma p(x) = 1$

i.e,

$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$

$10k^2+9k-1=0$

(or) $(10k-1)(k+1)=0$ (or) $k = 1/10$ & $k=-1$

If $k=-1$ The first condition fails $k \neq -1$ $\therefore k = 1/10$

x	0	1	2	3	4	5	6	7
$p(x)$	0	$1/10$	$2/10$	$2/10$	$3/10$	$1/100$	$1/50$	$17/100$

$$p(x<6) = p(0)+p(1)+p(2)+p(3)+p(4)+p(5)$$
$$= 0 + \frac{1}{10} + \frac{1}{5} + \frac{1}{5} + \frac{3}{10} + \frac{1}{100}$$
$$= \frac{81}{100} = 0.81$$

$$p(x>6) = p(7) = \frac{17}{100} = 0.17$$

The probability distribution is as follows:

x	0	1	2	3	4	5	6	7
$p(x)$	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

The distribution function of x is $f(x) = p(X \leq x)$

$$= \sum_{i=1}^{x} p(x_i)$$ is also called cumulative distribution function

x	0	1	2	3	4	5	6	7
$f(x)$	0	$0+0.1 = 0.1$	$0.1+0.2 = 0.3$	$0.3+0.2 = 0.5$	$0.5 + 0.3 = 0.80$	$0.80 + 0.01 = 0.81$	$0.81 + 0.02 = 0.83$	$0.83 + 0.17 = 1$

Discrete probability distribution:-

1) Binomial Distribution
2) Geometric Distribution
3) poisson Distribution.

Binomial Distribution :-

[If p is the probability of success and q is the probability of failure the probability of x successes out of n trials is given by

$$p(x) = {}^nC_x\, p^x q^{n-x}$$] → Bernoulli's theorem.

The probability distribution of $[x, p(x)]$ where

$$x = 0, 1, 2, \ldots n$$

x	0	1	2	......	n
$p(x)$	q^n	${}^nC_1 q^{n-1} p$	${}^nC_2 q^{n-2} p^2$	......	p^n

The value of $p(x)$ for different values $x = 0, 1, 2, \ldots n$ are the successive terms in the binomial expansion of $(q+p)^n$ and accordingly the distribution is called Binomial distribution (or) Bernoulli distribution.

$$\Sigma p(x) = q^n + {}^nC_1 q^{n-1} p + \ldots\ldots + p^n = (q+p)^n = 1^n = 1$$

$\therefore p(x)$ is a probability function.

~~Geometribution~~

Geometric Distribution :-

Consider a sequence of trials, where each trial has only two possible outcomes (designated failure & success). The probability of success is assumed to be the same for each trial. In such a sequence of trials, the geometric distribution is useful to model the number of failures before the first success.

The distribution gives the probability that there are zero failures before the first success, one failure before the first success, two failure before the first success. & so on.

i.e, To know the no of trials needed before a certain outcome occurs.

Geometric setting :- Each event falls into just one of two categories which are generally referred to as a success (or) failures.

Geometric formula :-

If X has a geometric distribution with probability 'p' of success & (1-p) of failure on each observation. The possible values of X are 1, 2, 3, ---. If n is any one of these values then the probability that the first success will occur on the n^{th} trial is

$$p(x=n) = (1-p)^{n-1}.p = (1-p)^{x-1}.p.$$

Geometric Distribution Mean :-

If X is a geometric random variable with probability p of success on each trial, the expected number of trials necessary to reach the first success is $\mu = \frac{1}{p}$

The Geometric distribution with parameter p ($0<p<1$) is discrete probability distribution with the following probability mass function $p(x:p) = p(1-p)^{x-1}$

Mean :- $\mu = \sum_{x=1}^{\infty} x\, p(x)$

$$= \sum_{x=1}^{\infty} x\,(1-p)^{x-1} \cdot p$$

$$= p \sum_{x=1}^{\infty} x\,(1-p)^{x-1}$$

$$= p\left[1 + 2(1-p) + 3(1-p^2) + \cdots\right]$$

1

x^{14} $(1-p)$

$$\mu(1-p) = p\left[(1-p) + 2(1-p)^2 + 3(1-p)^3 + \cdots\right] \longrightarrow (2)$$

(1) − (2)

$$\mu - \mu(1-p) = p\left[1 + (1-p) + (1-p)^2 + \cdots\right]$$

$$\mu - \mu + \mu p = p\left[\frac{1}{1-(1-p)}\right]$$

$$\mu \cancel{p} = \cancel{p}\left[\frac{1}{1-1+p}\right]$$

$$\mu = \frac{1}{p} \longrightarrow (3)$$

Variance :-

$$V = \sum_{x=1}^{\infty} x^2 p(x) - \left[\sum_{x=1}^{\infty} x\,p(x)\right]^2 \longrightarrow (4)$$

consider, $\sum_{x=1}^{\infty} x^2 p(x) = \sum_{x=1}^{\infty} x^2 (1-p)^{x-1} \cdot p$

$$= p\sum_{x=1}^{\infty} x^2 (1-p)^{x-1}$$

$$= p\left[\frac{1+1-p}{[1-(1-p)]^3}\right]$$

$$= p\left[\frac{2-p}{p^3}\right]$$

$$= \frac{2-p}{p^2} \longrightarrow (5)$$

Substitute (5) & (3) in (4)

$$V = \frac{2-p}{p^2} - \left(\frac{1}{p}\right)^2 = \frac{2-p-1}{p^2} = \frac{1-p}{p^2}$$

$$S.D = \frac{\sqrt{1-p}}{p}$$

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Example :-

If the dice is thrown repeatedly until the first time a three appears.
The probability distribution of the number of times its is thrown not getting a three. is a
(not-a-threes number of failures to get a three)
is a Geometric distribution with the success fraction.

$\frac{1}{6} = 0.1666$.

So, The probability of getting a three at the first thrown. (zero failures) is
The probability of getting a three at the second thrown (one failure) & so on.

problems:-

1) What is the probability that the marketing representative must select
(i) More than 6 people (ii) Exactly 6 people before he finds one who attended the last home p.d.f. of a Geometric random variables with
$1-p = 0.8$.

=)

$$1-p = 0.8 = q$$
$$p = 0.2$$
$$p(x) = p(1-p)^{x-1}$$
$$= (0.2)[0.8]^{x-1}$$

(i) $p(x>6) = 1 - p(x \le 6)$
$= 1 - [p(1) + p(2) + p(3) + p(4) + p(5) + p(6)]$
$= 1 - (0.2)[1 + 0.8 + (0.8)^2 + (0.8)^3 + (0.8)^4 + (0.8)^5]$

$p(x>6) = 1 - (0.2)(3.6892) = 0.2621$

(ii) $p(x=6) = (0.2)(0.8)^{6-1}$
$= (0.2)(0.8)^5$
$= 0.06554$

2) In a certain manufacturing process it is known that on the average of 1 in every 100 times are defective what is the probability that the fifth item inspected is the first defective item found.

=)

$$p = \frac{1}{100} = 0.01$$

$$q = (0.99) = 1-p, \quad x = 5$$

$$p(x) = p(1-p)^{x-1}$$
$$= (0.01)(0.99)^{5-1}$$
$$= (0.01)(0.99)^4$$
$$= 0.0096$$

3) The probability that a student pilot passes the written test for a private pilots license is 0.7 find the probability that the student will pass the test (a) on the third try (b) before the fourth try.

=)

$$p = 0.7, \quad q = 1-p = 0.3$$

$$p(x) = p(1-p)^{x-1}$$
$$p(x) = (0.7)(0.3)^{x-1}$$

(i) $p(x=3) = (0.7)(0.3)^{3-1} = 0.063$

(ii) $p(x<4) = p(1) + p(2) + p(3)$
$= 0.7 + (0.7)(0.3) + (0.7)(0.3)^2$
$= (0.7)[1 + 0.3 + (0.3)^2]$
$= 0.973$

4) 3% product produced by a machines are found to be defective. find probability that first defective occurs in the (i) fifth item inspected
(ii) first five inspected, (iii) Mean, (iv) variance

→

$p = 0.03 = \frac{3}{100}$

$q = 1-p = 1-0.03 = 0.97$

$p(x) = p(1-p)^{x-1}$

$= (0.03)(0.97)^{x-1}$

(i) $p(x=5) = (0.03)(0.97)^{5-1}$

$= \underline{\underline{0.0264}}$

(ii) $p(x \leq 5) = p(1) + p(2) + p(3) + p(4) + p(5)$

$= 0.03 + (0.03)(0.97) + (0.03)(0.97)^2 + (0.03)(0.97)^3 + (0.03)(0.97)^4$

$= \underline{\underline{0.1412}}$

(iii) Mean $= \mu = \frac{1}{p} = \frac{1}{0.03}$

(iv) Variance $= V = \frac{1-p}{p^2} = \frac{1-0.03}{(0.03)^2}$

$= \underline{\underline{1077.7}}$

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Poisson Distribution :-

poisson distribution is regarded as the limiting form of the binomial distribution when n is very large ($n \to \infty$) & p the probability of success is very small ($p \to 0$) so that np tends to a fixed finite constant say m.

We have in the case of binomial distribution, the probability of x success out of n trials,

$$p(x) = {}^nC_x \, p^x q^{n-x}$$

$$= \frac{n(n-1)(n-2)----(n-(x-1))}{x!} p^x q^{n-x}$$

$$= \frac{n.n\left(1-\frac{1}{n}\right)n\left(1-\frac{2}{n}\right)---n\left(1-\frac{(x-1)}{n}\right)}{x!} p^x q^{n-x}$$

$$= \frac{n^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)----n\left(1-\frac{(x-1)}{n}\right)}{x!} p^x q^{n-x}$$

$$= \frac{(np)^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)----\left(1-\frac{(x-1)}{n}\right)}{x!} q^{n-x}$$

$$= \frac{(np)^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)----\left(1-\frac{(x-1)}{n}\right)q^n}{x!\,q^x}$$

But $np = m$; $q^n = (1-p)^n = \left(1-\frac{m}{n}\right)^n$

$$= \left[\left(1-\frac{m}{n}\right)^{-n/m}\right]^{-m}$$

Denoting $-\frac{m}{n} = k$ we have,

$$q^n = \left\{(1+k)^{1/k}\right\}^{-m} \rightarrow e^{-m} \quad \text{as } n \rightarrow \infty \text{ (or) } k \rightarrow 0$$

$$\left\{ \text{Note: } \lim_{k \rightarrow 0} (1+k)^{1/k} = e \right\}$$

Further $q^x = (1-p)^x \rightarrow 1$ for a fixed x as $p \rightarrow 0$

Also the factors $\left(1-\frac{1}{n}\right)\cdot\left(1-\frac{2}{n}\right) \cdots \left(1-\frac{x-1}{n}\right) \rightarrow 1$ as $n \rightarrow \infty$

Thus we get,

$$p(x) = \frac{m^x e^{-m}}{x!}$$

This is known as poisson distribution of random Variable

$p(x)$ is called poisson probability function

x is called a poisson Variate.

The distribution of probabilities for $x = 0, 1, 2, 3, \ldots$ is as follows:

x	0	1	2	3	...
$p(x)$	e^{-m}	$\frac{me^{-m}}{1!}$	$\frac{m^2 e^{-m}}{2!}$	$\frac{m^3 e^{-m}}{3!}$	

we have $p(x) \geq 0$

$$\sum_{x=0}^{\infty} p(x) = e^{-m} + \frac{me^{-m}}{1!} + \frac{m^2 e^{-m}}{2!} + \cdots$$

$$= e^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots\right]$$

$$= e^{-m} \cdot e^{m} = e^{0} = 1$$

$$\therefore \sum p(x) = 1$$

Hence $p(x)$ is a probability function.

Mean and Variance :-

$$\text{Mean } (\mu) = \sum_{x=0}^{\infty} x \cdot p(x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{m^x e^{-m}}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{m^x \cdot e^{-m}}{(x-1)!}$$

$$= m e^{-m} \sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= m e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots \right]$$

$$= m e^{-m} \cdot e^{m} = m$$

$$\text{Mean} = \underline{\underline{\mu = m}}$$

$$\text{Variance } (V) = \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

Now,

$$\sum_{x=0}^{\infty} x^2 p(x) = \sum_{x=0}^{\infty} [x(x-1) + x] \cdot \frac{m^x e^{-m}}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{m^x e^{-m}}{(x-2)!} + \sum_{x=0}^{\infty} \frac{m^x e^{-m}}{(x-1)!}$$

$$= m^2 e^{-m} \sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!} + m$$

$$= m^2 e^{-m} \left[1 + \frac{m}{1!} + \frac{m^2}{2!} + \cdots \right] + m$$

$$= m^2 e^{-m} \cdot e^{m} + m$$

$$= m^2 + m$$

$$\therefore \quad V = m^2 + m - m^2$$

$$V = m$$

Standard deviation $(\sigma) = \sqrt{V} = \sqrt{m}$

Mean & variance are equal for the poisson distribution.

problems:-

1) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains (i) no defective fuses (ii) 3 (or) more defective fuses.

$\Rightarrow$ p = probability of a defective fuse $= \frac{2}{100} = 0.02$

$\therefore$ mean no of defective $\mu = m = np = 200 \times 0.02 = 4$

The poisson distribution is given by $p(x)$

$$= \frac{m^x e^{-m}}{x!}$$

i.e, $p(x) = \frac{4^x e^{-4}}{x!}$ But $e^{-4} = 0.0183$

$\therefore p(x) = 0.0183 \cdot \frac{4^x}{x!}$

(i) probability of no defective fuse $= p(0) = 0.0183$

(ii) probability of 3 (or) more defective fuses.

$$= 1 - [p(0) + p(1) + p(2)]$$

$$= 1 - 0.0183\left[1 + \frac{4}{1!} + \frac{4^2}{2!}\right]$$

2) If the probability of a bad reaction from a certain injection is 0.001, determine the chance that out of 2000 individuals, more than two will get a bad reaction

=) As the probability of occurrence is very small, this follows poisson distribution & we have,

$$p(x) = \frac{m^x e^{-m}}{x!}$$

$$\text{Mean} = m = np = 2000 \times 0.001 = 2$$

To find $p(x>2) = 1 - p(x \le 2)$

$$= 1 - [p(0) + p(1) + p(2)]$$

$$= 1 - e^{-m}\left[1 + \frac{m}{1!} + \frac{m^2}{2!}\right]$$

$$= 1 - e^{-2}[1+2+2]$$

$$= 1 - 5e^{-2} = \underline{\underline{0.3233}}$$

3) Fit a poisson distribution for the following data & calculate the theoretical frequencies:-

x	0	1	2	3	4
f	122	60	15	2	1

=) First compute the mean (μ) of the given distribution

$$\mu = \frac{\sum f(x)}{\sum f} = \frac{(0)(122) + (1)(6) + (2)(15) + (3)(2) + (4)(1)}{122+60+15+2+1}$$

$$\mu = 0.5 = m.$$

$$p(x) = \frac{m^x e^{-m}}{x!}$$

Let $f(x) = 200 \cdot p(x)$

$$f(x) = 200 \cdot \frac{(0.5)^x e^{-0.5}}{x!}$$

But $e^{-0.5} = 0.6065$

$$f(x) = 121.3 \frac{(0.5)^x}{x!}$$

put $x = 0, 1, 2, 3, 4$ in $f(x)$.
we obtain,

$x = 0$; $f(x) = 121.3 \frac{(0.5)^0}{0!} = 121.3$

$x = 1$; $f(x) = 121.3 \frac{(0.5)^1}{1!} \approx 61$

$x = 2$; $f(x) = 121.3 \frac{(0.5)^2}{2!} \approx 15$

$x = 3$; $f(x) = 121.3 \frac{(0.5)^3}{3!} \approx 3$

$x = 4$; $f(x) = 121.3 \frac{(0.5)^4}{4!} \approx 0$

The required frequencies are 121, 61, 15, 3, 0

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Continuous Probability Distribution :

A Random variable which takes non countable infinite number of values is called a continuous random variable.

Defn :- If for every x belonging to the range of a continuous random variable X.
we assign a real number $f(x)$ satisfying the conditions
(i) $f(x) \geq 0$ & (ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

then $f(x)$ is called a continuous probability function (or) probability density function.

If (a, b) is a Subinterval of the range space of X then the probability that x lies in (a, b) is defined to be the integral of $f(x)$ between a & b. that is,

$$p(a \le x \le b) = \int_a^b f(x)\,dx \longrightarrow (1)$$

Cumulative distribution function:-

If X is a continuous random variable with probability density function $f(x)$ then The function $F(x)$ defined by,

$$F(x) = p(X \le x) = \int_{-\infty}^{x} f(x)\,dx. \longrightarrow (2)$$

is called the cumulative distribution function of X

$$(2) \Rightarrow F(x) = p(X \le x) = p(-\infty < X \le x)$$

$$\text{and } \frac{d}{dx}\left[F(x)\right] = f(x)$$

Note :- * The probability of a continuous random Variable taking a particular Value is Zero.

* The probability take it take values in an interval is a positive Quantity.

If r is any real number then,

(i) $p(x \geq r) = \int_{r}^{\infty} f(x)\, dx$

(ii) $p(x < r) = 1 - p(x \geq r)$

$$p(x<r) = 1 - \int_{r}^{\infty} f(x)\, dx$$

* $\int_{-\infty}^{\infty} f(x)\, dx = 1$. Geometrically means that the area bounded by the curve $f(x)$ and the x-axis is equal to unity.

Also $p(a \leq x \leq b)$ = the area of the region bounded by the curve $f(x)$, the x-axis & the ordinates $x=a$ & $x=b$

Mean and Variance:-

If X is a continuous random variable with probability density function $f(x)$ where $-\infty < x < \infty$, the mean μ (or) expectation $E(X)$ and the variance (σ^2) of X is defined as follows.

$$\text{Mean}, \mu = \int_{-\infty}^{\infty} x.\, f(x)\, dx$$

$$\text{Variance}, (\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 . f(x)\, dx$$

Two continuous probability distributions:-

1) Exponential Distribution

2) Normal Distribution.

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Exponential Distribution :-

The continuous probability distribution having the probability density function $f(x)$ given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}, \text{ where } \alpha > 0$$

is known as the exponential distribution. Evidently $f(x) \geq 0$ & we have,

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{\infty} \alpha e^{-\alpha x}\,dx = \left[\alpha \frac{e^{-\alpha x}}{-\alpha}\right]_0^{\infty}$$

$$= -(0-1) = 1$$

Thus $\int_{-\infty}^{\infty} f(x)dx = 1$

$f(x)$ satisfy the both conditions required for a continuous probabity function / p.d.f.

Mean and Variance and Standard deviation of the Exponential Distribution

$$\text{Mean } (\mu) = \int_{-\infty}^{\infty} x f(x)\,dx = \int_{-\infty}^{\infty} x \cdot \alpha e^{-\alpha x}\,dx$$

$$\mu = \alpha \int_{0}^{\infty} x e^{-\alpha x}\,dx.$$

Apply Bernoulli's rule of integration by parts we have,

$$\mu = \alpha \left[x \left(\frac{e^{-\alpha x}}{-\alpha}\right) - 1 \left(\frac{e^{-\alpha x}}{\alpha^2}\right)\right]_0^{\infty}$$

(Here $x/e^{\alpha x} \to 0$ as $x \to \infty$ by L'Hospital Rule)

$$\mu = \alpha\left[0 - \frac{1}{\alpha^2}(0-1)\right]$$

$$\mu = \alpha\left[\frac{1}{\alpha^2}\right] = \frac{1}{\alpha}$$

$$\therefore \mu = \frac{1}{\alpha}$$

Variance, $(\sigma^2) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

$$\sigma^2 = \alpha\int_0^{\infty} (x-\mu)^2 e^{-\alpha x}\,dx$$

Apply Bernoulli's rule we have,

$$\sigma^2 = \alpha\left[(x-\mu)^2\left(\frac{e^{-\alpha x}}{-\alpha}\right) - 2(x-\mu)\left(\frac{e^{-\alpha x}}{\alpha^2}\right) + 2\left(\frac{+e^{-\alpha x}}{-\alpha^3}\right)\right]_0^{\infty}$$

$$\sigma^2 = \alpha\left[\frac{-1}{\alpha}(0-\mu^2) - \frac{2}{\alpha^2}(0-(-\mu)) - \frac{2}{\alpha^3}(0-1)\right]$$

$$\sigma^2 = \alpha\left[\frac{\mu^2}{\alpha} - \frac{2\mu}{\alpha^2} + \frac{2}{\alpha^3}\right]$$

But $\mu = \frac{1}{\alpha}$

$$\sigma^2 = \alpha\left[\frac{1}{\alpha^3} - \frac{\cancel{2}}{\alpha^3} + \frac{\cancel{2}}{\alpha^3}\right]$$

$$\sigma^2 = \alpha\left(\frac{1}{\alpha^{\cancel{3}2}}\right) = \frac{1}{\alpha^2}$$

Variance, $\sigma^2 = \frac{1}{\alpha^2}$

Standard deviation, $\sigma = \sqrt{\frac{1}{\alpha^2}} = \frac{1}{\alpha}$

$\therefore$ Mean and S.D are equal for the exponential distribution.

Problems:-

1) A random variable X has the density function
$f(x) = \begin{cases} kx^2 & \text{for } -3 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$ find k
& find (i) $p(1 \le x \le 2)$ (ii) $p(x \le 2)$, (iii) $p(x>1)$

$\Rightarrow$ $f(x) \ge 0$ if $k \ge 0$ & also we must have
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

i.e., $$\int_{-3}^{3} kx^2\,dx = 1$$

$$\left[\frac{kx^3}{3}\right]_{-3}^{3} = 1$$

$\therefore$

$$\frac{k}{3}\left[3^3 - (-3)^3\right] = 1$$

$$\frac{k}{3}\left[27+27\right] = 1$$

$$\Rightarrow \frac{k(\cancel{54})^{18}}{\cancel{3}} = 1$$

$$k = \frac{1}{18}$$

(i) $p(1 \le x \le 2) = \int_1^2 \frac{x^2}{18} dx = \left(\frac{x^3}{54}\right)_1^2 = \frac{1}{54}(8-1) = \frac{7}{54}$.

(ii) $p(x \le 2) = \int_{-3}^{2} \frac{1}{18} x^2 dx = \frac{1}{18}\left(\frac{x^3}{3}\right)_{-3}^{2} = \frac{1}{54}(8+27)$

$= \frac{35}{54}$

(iii) $p(x \ge 1) = \int_1^3 \frac{1}{18} x^2 dx = \frac{1}{18}\left(\frac{x^3}{3}\right)_1^3 = \frac{1}{54}(27-1)$

$= \frac{26}{54} = \frac{13}{27}$

2) The probability density $f(x)$ of continuous random variable is given by $f(x) = ke^{-|x|}$, $-\infty < x < \infty$ P.T $k = \frac{1}{2}$ & also find mean & variance.

=)

(i) $f(x) \ge 0$ if $k > 0$, (ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\int_{-\infty}^{\infty} ke^{-|x|} dx = 1$

$f(x) = ke^{-|x|} \Rightarrow f(-x) = ke^{-|x|}$

$f(-x) = f(x)$

$2\int_0^{\infty} ke^{-x} dx = 1$

$2k\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$

$2k\left[-e^{-\infty} + e^0\right] = 1$

$2k = 1$

$k = \frac{1}{2}$

(ii) Mean : $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$

$= \int_{-\infty}^{\infty} x \cdot k e^{-|x|}\, dx.$

$f(x)$ ~~f(x)~~ $= x k e^{-|x|}$

$f(-x) \Rightarrow -x k e^{-|x|}$

$f(-x) = -[x k e^{-|x|}] = -f(x)$

$\therefore$ the function is odd.

$= 2\int_{0}^{\infty} x k e^{-|x|}\, dx = 0$

$\therefore \mu = 0$

(iii) Variance, $\sigma^2 = V = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$

$= \int_{-\infty}^{\infty} x^2 \cdot k e^{-|x|}\, dx.$

$f(x) = x^2 \cdot k e^{-|x|}$

$f(-x) = (-x)^2 k e^{-|x|} = x^2 \cdot k e^{-|x|}$

$f(-x) = f(x)$

The function is even

$V = 2\int_{0}^{\infty} x^2 k e^{-x}\, dx$

$= 2k \left[x^2 \frac{e^{-x}}{-1} - 2x \frac{e^{-x}}{1} + 2 \frac{e^{-x}}{-1} \right]_0^{\infty}$

$V = 2k \left[+2 \right]$

$V = \cancel{2} \frac{1}{\cancel{2}} [1\cdot 2]$

$V = 2$

3) The sales per day in a shop is exponentially distributed with the average sale amounting to Rs. 100 & net profit is 8%. find the probability that the net profit exceeds Rs. 30 on two consecutive days.

=> Let x be the random variable of the sale in the shop. since x is an exponential variate the p.d.f

$f(x) = \alpha e^{-\alpha x}, \quad x > 0$

Mean $= \frac{1}{\alpha} = 100 \quad \therefore \alpha = \frac{1}{100} = 0.01$

Hence $f(x) = 0.01\, e^{-0.01x}, \quad x > 0$

Let A be the amount for which profit is 8%

$\frac{A \cdot 8}{100} = 30$

$\therefore A = 375$

probability of profit exceeding Rs. 30 is equal to

$1 - \text{prob}(\text{profit} \leq \text{Rs } 30)$

$= 1 - \text{prob}(\text{sales} \leq \text{Rs}.375)$

$= 1 - \int_0^{375} (0.01) e^{-0.01x}\, dx$

$= 1 + \left[e^{-0.01x} \right]_0^{375}$

$= \cancel{1} + e^{-3.75} - 1$

$= e^{-3.75}$

The probability that profit exceeds Rs 30 on a single day is $e^{-3.75}$

Thus the probability that it repeats on the following day is $e^{-3.75} \cdot e^{-3.75} = e^{-7.5}$

$= 0.00055$

4) In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for:

(i) 10 minutes (or) more

(ii) less than 10 minutes

(iii) between 10 & 12 minutes

⇒ The p.d.f of the exponential distribution is given by,

$f(x) = \alpha e^{-\alpha x}, \quad x > 0$ & the mean $= 1/\alpha$

$\frac{1}{\alpha} = 5 \Rightarrow \alpha = 1/5$

Hence $f(x) = \frac{1}{5} e^{-1/5 x}$

(i) $p(x \geq 10) = \int_{10}^{\infty} \frac{1}{5} e^{-x/5} dx = -\left[e^{-x/5} \right]_{10}^{\infty}$

$= -\left[0 - e^{-2} \right] = e^{-2}$

$= 0.1353$

(ii) $p(x<10) = \int_0^{10} \frac{1}{5} e^{-x/5} dx = -\left[e^{-x/5}\right]_0^{10}$

$= -\left[e^{-2} - 1\right] = 1 - e^{-2} = \underline{0.8647}$

(iii) $p(10<x<12) = \int_{10}^{12} \frac{1}{5} e^{-x/5} dx = -\left[e^{-x/5}\right]_{10}^{12}$

$= -\left[e^{(-12/15)} - e^{-2}\right]$

$= \underline{0.0446}$

5) If x is an exponential variate with mean 3 find (i) $p(x>1)$ (ii) $p(x<3)$

$\Rightarrow$ The p.d.f of the exponential distribution is given by,

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & 0<x<\infty \\ 0 & \text{otherwise} \end{cases}$$

The mean of this distribution is given by $1/\alpha$

$$\text{Mean} = \frac{1}{\alpha} = 3 \quad \therefore \alpha = \frac{1}{3}$$

Hence $$f(x) = \begin{cases} \frac{1}{3} e^{-x/3} & 0<x<\infty \\ 0 & \text{otherwise} \end{cases}$$

(i) $p(x>1) = 1 - p(x \le 1)$

$= 1 - \int_0^1 f(x) dx$

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$$p(x>1) = 1 - \int_0^1 \frac{1}{3} e^{-x/3}\, dx = \int_1^\infty \frac{1}{3} e^{-x/3}\, dx$$

$$= 1 + \left[e^{-x/3} \right]_0^1 = \left(\frac{\frac{1}{3} e^{-x/3}}{-1/3} \right)_1^\infty$$

$$= 1 + e^{-1/3} - 1 = e^{-1/3}$$

$$= e^{-1/3} = \underline{\underline{0.7165}}$$

(ii) $$p(x<3) = \int_0^3 f(x)\, dx$$

$$= \int_0^3 \frac{1}{3} e^{-x/3}\, dx$$

$$= \left[e^{-x/3} \right]_0^3$$

$$= \left[-e^{-1} + 1 \right]$$

$$= 1 - \frac{1}{e}$$

$$= \underline{\underline{0.6321}}$$

Normal Distribution:-

The continuous probability distribution having the probability density function $f(x)$ given by,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

where $-\infty < x < \infty$, $-\infty < \mu < \infty$ and $\sigma > 0$ is known as the normal distribution.

Evidently $f(x) \geq 0$

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-\mu)^2/2\sigma^2} dx$$

put $t = \frac{x-\mu}{\sqrt{2}\sigma}$ (or) $x = \mu + \sqrt{2}\sigma t$

$dx = \sqrt{2}\sigma\, dt$

t also varies from $-\infty$ to ∞

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2} \sqrt{2}\sigma\, dt$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt$$

$$= \frac{1}{\sqrt{\pi}} 2 \int_{0}^{\infty} e^{-t^2} dt$$

$$= \frac{2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = 1$$

$\therefore$ $f(x)$ represents a probability density function.

Mean, variance and standard deviation :-

Mean, $\mu = \int_{-\infty}^{\infty} x f(x)\, dx$

$$\mu = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{-(x-\mu)^2/2\sigma^2}\, dx.$$

put $t = \frac{x-\mu}{\sqrt{2}\sigma}$ (or) $x = \mu + \sqrt{2}\sigma t$

$dx = \sqrt{2}\sigma\, dt$

t varies from $-\infty$ to ∞

$$\mu = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sqrt{2}\sigma t)\, e^{-t^2} \sqrt{2}\sigma\, dt$$

$$\mu = \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2} dt + \frac{\sigma\sqrt{2}}{\sqrt{\pi}} \int_{-\infty}^{\infty} t e^{-t^2} dt$$

$$\mu = \frac{2\mu}{\sqrt{\pi}} \int_{0}^{\infty} e^{-t^2} dt + \frac{\sigma\sqrt{2}}{\sqrt{\pi}} \int_{-\infty}^{\infty} t e^{-t^2} dt$$

the second integral is zero by a standard property since te^{-t^2} is an odd function.

$$\mu = \frac{2\mu}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} + 0 = \mu$$

Hence we can say that the mean of the normal distribution is equal to the ~~same~~ mean of the given distribution.

Variance, $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-(x-\mu)^2/2\sigma^2}\, dx$$

$t = \frac{x-\mu}{\sqrt{2}\sigma}$ 't' varies from $-\infty$ to ∞

$dx = \sqrt{2}\sigma\, dt$

$$\sigma^2 = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} 2\sigma^2 t^2 e^{-t^2} \cdot \sqrt{2}\sigma\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t^2 e^{-t^2}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot 2 \int_{0}^{\infty} t^2 e^{-t^2}\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{0}^{\infty} t\,(2te^{-t^2})\, dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \left\{ \left[t\,(-e^{-t^2}) \right]_0^{\infty} - \int_0^{\infty} -e^{-t^2}\, dt \right\}$$

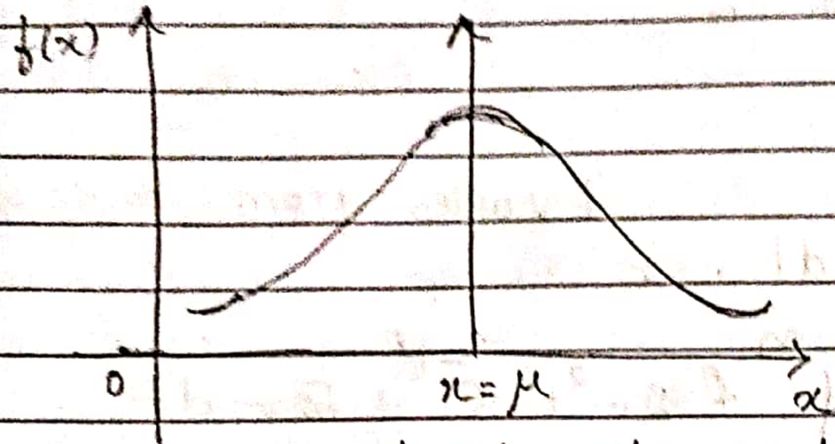
$$= \frac{2\sigma^2}{\sqrt{\pi}} \left[\int_0^{\infty} e^{-t^2}\, dt \right]$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{2} = \sigma^2$$

Hence we can say that the variance / S.D of the normal distribution is equal to the variance / S.D of the given distribution.

Note:-

The graph of the probability function $f(x)$ is a bell shaped curve symmetrical about the line $x=\mu$ & is called the Normal probability curve. The shape of the curve is as follows.



The line $x=\mu$ divides the total area under the curve which is equal to 1 into two equal parts.

The area to the right as well as to the left of the line $x=\mu$ is 0.5.

Standard Normal distribution:-

We have, $p(a \le x \le b) = \int_a^b f(x)\,dx$.

In case of normal distribution,

$$p(a \le x \le b) = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b e^{-(x-\mu)^2/2\sigma^2}\,dx. \longrightarrow (1)$$

put $z = \frac{x-\mu}{\sigma}$ (or) $x = \mu + \sigma z$

$dx = \sigma\,dz$.

$z_1 = \frac{a-\mu}{\sigma}$ & $z_2 = \frac{b-\mu}{\sigma}$

(1) =)

$$p(a \le x \le b) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-z^2/2}\, dz = p(z_1 \le z \le z_2)$$

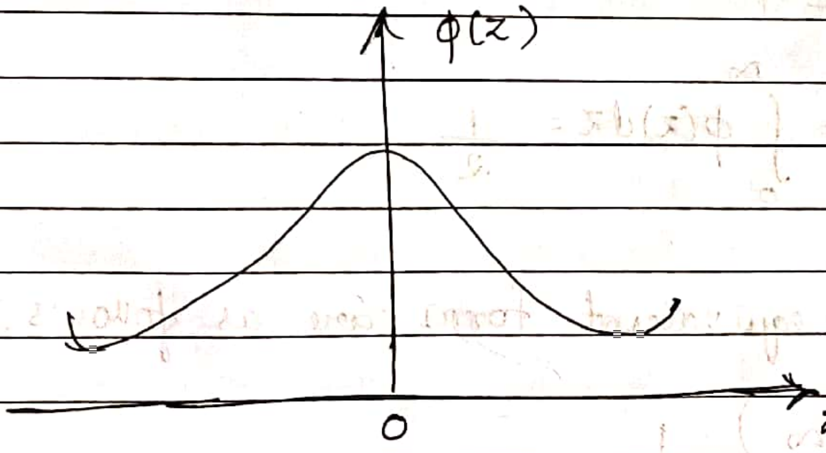
$\longrightarrow$ (2)

If $F(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$ (Standard normal probability density function)

$\therefore$ The Normal probability density function with $\mu = 0$ & $\sigma = 1$ is the standard normal probability density function. $N(\mu, \sigma)$

$Z = \frac{x-\mu}{\sigma}$ is called the Standard Normal Variate.

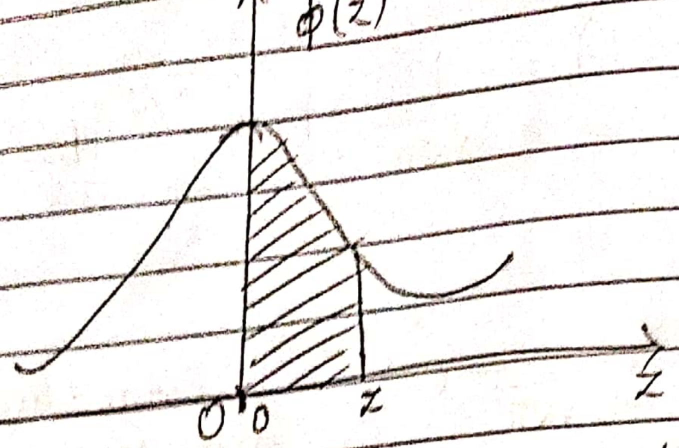
$F(z)$ is called the standard normal curve which is symmetrical about the line $z = 0$ & $x = \mu$. The curve is as follows.



The integral in the RHS of (2) geometrically represents the area bounded by the standard normal curve $F(z)$ b/n $z = z_1$ & $z = z_2$

If $z_1 = 0$ $\phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz$

This represents the area under the curve standard normal curve from $z = 0$ to z.



$\phi(z)$ is also denoted as $A(z)$ represents the area as shown in the fig. ∴ the total area is 1, the area on either side of $z=0$ is 0.5.

* Tabulated values which gives the area for different positive values of z are available & this helps us in practical problems (Normal probability table).

Note:-

1) $\int_{-\infty}^{\infty} \phi(z)\,dz = 1$

2) $\int_{-\infty}^{0} \phi(z)\,dz = \int_{0}^{\infty} \phi(z)\,dz = \frac{1}{2}$

This result is equivalent form are as follows.

1) $p(-\infty \le z \le \infty) = 1$
2) $p(-\infty \le z \le 0) = 1/2$
3) $p(0 \le z \le \infty)$ (or) $p(z > 0) = 1/2$

Also $p(-\infty < z < z_1) = p(-\infty < z \le 0) + p(0 \le z < z_1)$

$p(z < z_1) = 0.5 + \phi(z_1)$

$\phi(z > z_1) = 0.5 - \phi(z_1)$

$$p(z > z_2) = p(z \geq 0) - p(0 \leq z < z_2)$$

$$p(z > z_2) = 0.5 - \phi(z_2)$$

$$p(z < z_2) = 0.5 + \phi(z_2)$$

problems:-

1) In an examination 7% of students score less than 35% marks & 89% of students score less than 60% marks. Find the mean & S.D if the marks are normally distributed. It is given that if,

$$p(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2}\, dz \quad \text{then} \quad p(1.2263) = 0.39$$
$$p(1.4757) = 0.43$$

=) Let μ & σ be the mean & S.D of normal distribution

$$p(x < 35) = 0.07 \quad , \quad p(x < 60) = 0.89$$

We have S.N.V is, $z = \frac{x - \mu}{\sigma}$

When $x = 35$, $z = \frac{35 - \mu}{\sigma} = z_1$

$x = 60$, $z = \frac{60 - \mu}{\sigma} = z_2$

$$p(z < z_1) = 0.07 \quad \& \quad p(z < z_2) = 0.89$$

$$0.5 + \phi(z_1) = 0.07 \quad \& \quad 0.5 + \phi(z_2) = 0.89$$

$$\phi(z_1) = -0.43 \quad \& \quad \phi(z_2) = 0.39$$

Using the given data in R.H.S of these we have

$$\phi(z_1) = -\phi(1.4757) \quad \& \quad \phi(z_2) = \phi(1.2263)$$

$$z_1 = -1.4757 \quad \& \quad z_2 = 1.2263$$

i.e, $\frac{35-\mu}{\sigma} = -1.4757$, $\frac{60-\mu}{\sigma} = 1.2263$

$\mu - 1.4757\sigma = 35$ & $\mu + 1.2263\sigma = 60$

By solving we get

Mean, $\mu = 48.65$ and $\sigma = 9.25 = S.D.$

2) The mean weight of 1000 students during medical examination was found to be 70kg & S.D weight is 5 kg. Assume that the weight are normally distributed, find the number of students having weight (i) less than 65 kg (ii) More than 75 kg (iii) b/n 65 kg to 75 kg

$[\cancel{p(0.83) = 0.2967}]$, $p(1) = 0.3413$

$\Rightarrow$

Let x represents the marks of students.

By data $\mu = 70$, $\sigma = 5$

Hence S.N.V $\Rightarrow$ $z = \frac{x-\mu}{\sigma} = \frac{x-70}{6}$

(i) If $x = 65$ $z = -1$ & we have to find $p(z<-1)$

$$p(z<-1) = p(z>1)$$
$$= p(z \geq 0) - p(0<z<1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

$\therefore$ No of students scoring less than 65 marks $\Rightarrow$ 1000×0.1587

$= 158.7 \approx \underline{159}$

(ii) If $x = 75$, $z = 1$ $p(z > 1)$

$$p(z>1) = p(z \geq 0) - p(0 < z < 1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413$$
$$= 0.1587$$

No of students scoring more than 75 marks

$$1000 \times 0.1587 = 158.7 \approx 159$$

(iii) We have to find $p(-1 < z < 1)$

$$p(-1 < z < 1) = 2p(0 < z < 1)$$
$$= 2\phi(1) = 2(0.3413) = 0.6826.$$

No of students scoring marks b/n 65 & 75

$$1000 \times 0.6826 = 682.6 \approx 683.$$

3) A sample of 100 dry battery cells tested to find the length of life produced by a company & following results are recorded; mean life is 12 hrs. S.D is 3 hrs Assuming data to be normally distributed find the expected life of a dry cell

(i) have more than 15 hrs. $\phi(1) = 0.3413$

(ii) between 10 & 14 hrs $\phi(0.67) = 0.2486$

=)

$\mu = 12$, $\sigma = 3$

$$S.N.V = z = \frac{x - \mu}{\sigma} = \frac{x - 12}{3}$$

(i) If $x = 15$ $z = \frac{15-12}{3} = 1$

$$p(z>1) = p(z \geq 0) - p(0 < z < 1)$$
$$= 0.5 - \phi(1)$$
$$= 0.5 - 0.3413 = 0.1587$$

In 100 dry battery cells ~~having life b/n 10 to 14 hrs~~ have more than 15hrs $\Rightarrow 100 \times 0.1587$

$$\Rightarrow 15.87 \approx 16$$

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$x=10$ & $x=14$

(ii) $z_1 = \frac{10-12}{3} = \frac{-2}{3}$, $z_2 = \frac{14-12}{3} = \frac{2}{3}$

$z_1 = -0.67$ $z_2 = 0.67$

$$p(-0.67 < z < 0.67) = 2p(0 < z < 0.67)$$
$$= 2\phi(0.67)$$
$$= 2(0.2486)$$
$$= 0.4972$$

In 100 dry battery cells having life b/n 10 ~~&~~ 14 hrs is

$$100 \times 0.4972 = 49.72 \approx 50$$

Note :-

1) A S.N.V 'z' is denoted by $z \sim N(0,1)$.
2) $\phi(z)$ are the same for positive as well as negative value of z $[\phi(-z) = \phi(z)]$. Hence the S.N.P curve is symmetric about the line $z=0$.
3) Why do we need standard normal distribution

A normal distribution is characterized by two parameters (i) Mean → locate any where on the x-axis (ii) S.D(σ) → determines the spread of its bell shapes curve along the x-axis $p(a \le x \le b)$ under any normal probability curve $N(\mu, \sigma)$! we need to construct a infinite no of tables for different combinations of the value of μ & σ which is practically impossible.

In practice standardize the variable X to obtain the ~~z-score~~ Z-score, $z = \frac{x-\mu}{\sigma}$ & then construct the table. & the areas under standard probability curve.

4) The curve is symmetrical about the line $x = \mu$ $(z = 0)$

a) $p(z > z_1) = p(z \geq 0) - p(0 < z < z_1)$

$= 0.5 - \phi(z_1)$

b) $p(z < z_1) = 0.5 + p(0 < z < z_1)$

$= 0.5 + \phi(z_1)$

c) $p(z < -z_1) = p(z > z_1)$

$= 0.5 - p(0 \leq z < z_1)$

d) $p(z > z_2) = p(z \geq 0) - p(0 \leq z < z_2)$

$= 0.5 - \phi(z_2)$