Expressing Signal in terms of basic signals

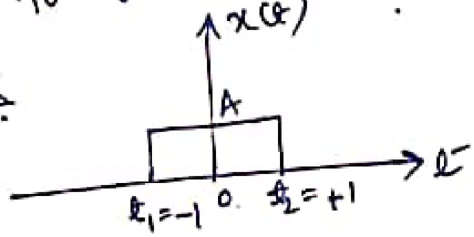
If a new signal $y(t)$ is obtained by shifting and scaling operations ~~it can be~~ on a original signal. it can be represented mathematically as follows.

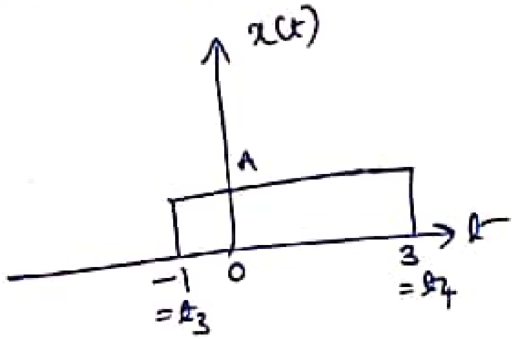
Let $x(t)$ be signal with duration extending from t_1 to t_2 & $y(t)$ be obtained by time shifting & scaling $x(t)$ (keeping magnitude unchanged) having time ~~vary~~ variations from t_3 to t_4.

then $a t_3 - b = t_1$ ① & $a t_4 - b = t_2$ ②

Solve ① & ② using known values of t_1, t_2, t_3 & t_4 to find a & b.

Eg.





$a(-1) - b = -1 \Rightarrow \quad -a - b = -1$

$a(3) - b = 1 \Rightarrow \quad \underline{\overset{-}{}3a \overset{+}{-} b = \overset{-}{}1}$

$-4a = -2$

$a = 1/2$ & $b = 1/2$.

$y(t) = x\left(\frac{t}{2} - \frac{1}{2}\right)$

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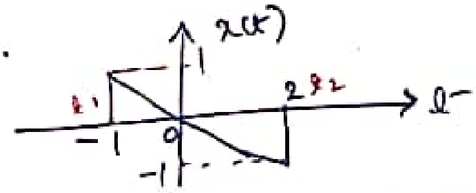
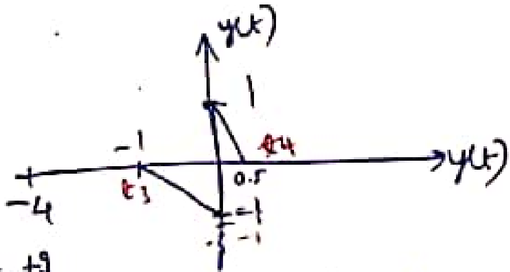




$a(-1) - b = +2 \Rightarrow \quad -a - b = +2$

$a(0.5) - b = -1 \quad \underline{(-)0.5a \overset{(+)}{-} b = \overset{(+)}{-}1}$

$-1.5a = +3$

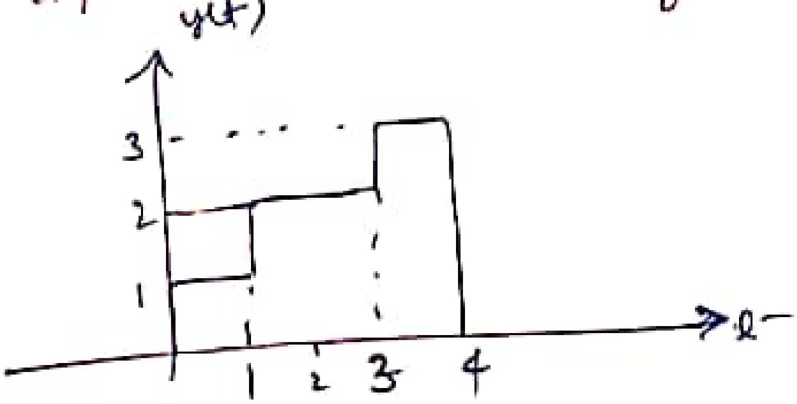
$a = -2 \quad b = 0$

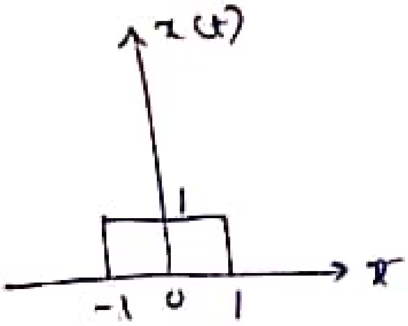
$t_1 \quad t_2$

$t_3 \quad t_4 \Rightarrow$ replicated

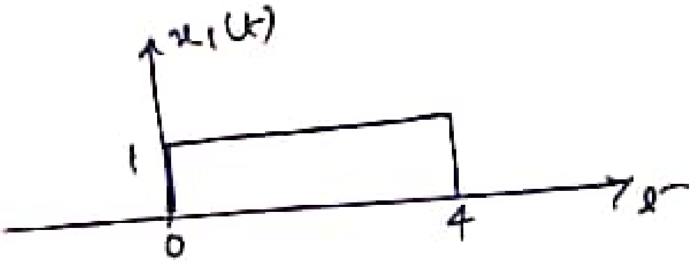
$y(t) = x(-2t)$

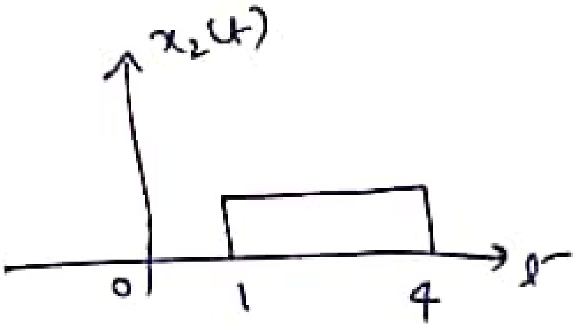
Express $y(t)$ in terms of $x(t)$

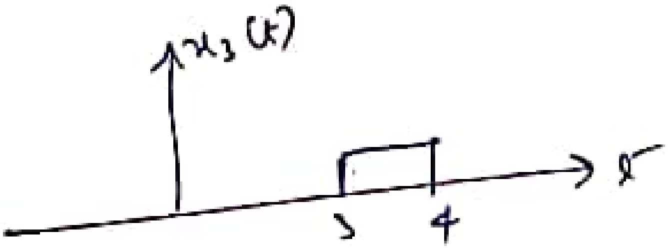




$$y(t) = x_1(t) + x_2(t) + x_3(t)$$







for $x_1(t)$

$a(-0) - b = -1$

$a(4) - b = 1$

$b = 1, \quad a = \frac{1}{2}$

$\therefore x_1(t) = x\left(\frac{t}{2} - 1\right)$

for $x_2(t)$

$a(1) - b = -1$

$a(4) - b = 1$

$a = \frac{2}{3} \qquad b = \frac{5}{3}$

$x_2(t) = x\left(\frac{2}{3}t - \frac{5}{3}\right)$

|||ly **$x_3(t)$**

$3a - b = -1$

$4a - b = 1$

$\Rightarrow a = 2, \quad b = 7$

$$\therefore y(t) = x\left(\frac{t}{2} - 1\right) + x\left(\frac{2}{3}t - \frac{5}{3}\right) + x(2t - 7)$$

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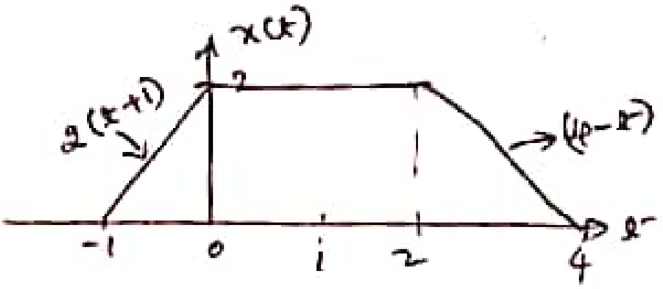
Expressing signal in mathematical form.



method 1

$-1 < t < 0$. It is a ramp with slope = 2 & shifted left by 1

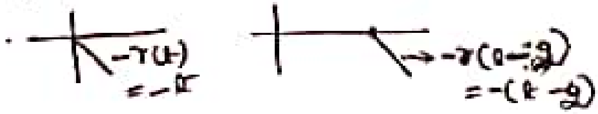
ie $r(t+1) \Rightarrow 2(t+1)$

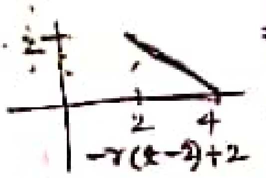
$0 < t < 2$ $x(t)$ is constant with mag = 2. $\Rightarrow 2$.

$2 < t < 4$ It is is -ve ramp with slope = 1

shifted to right by 2 units & shifted in mag by 2 units

$\therefore -r(t-2)+2. \Rightarrow -(t-2)+2 = 4-t.$





method 2

$-1 < t < 0 \Rightarrow$ slope × shift = $2 \times (t+1) = 2(t+1)$

$0 < t < 2 \Rightarrow$ constant with mag. = 2. $\Rightarrow 2$.

$2 < t < 4 \Rightarrow$ slope × shift = $-1(t-2) = 4-t$

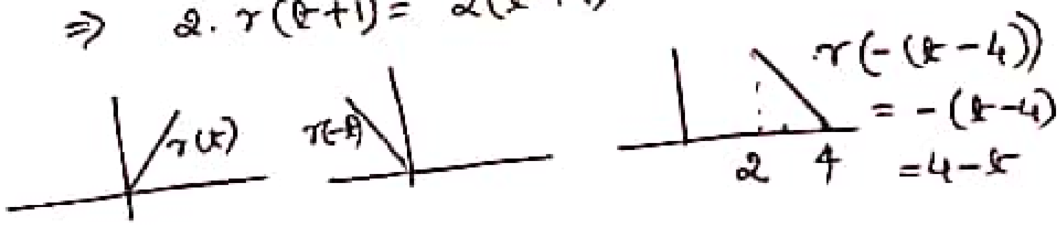
slope is -1 $\because$ It is -ve going ramp.

method 3

$-1 < t < 0 \Rightarrow$ +ve ramp shifted to left by 1 with slope = 2

$\Rightarrow 2.r(t+1) = 2(t+1)$

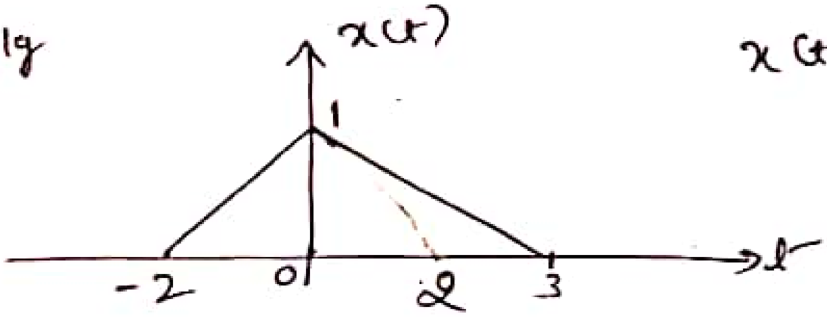
$2 < t < 4 \Rightarrow$



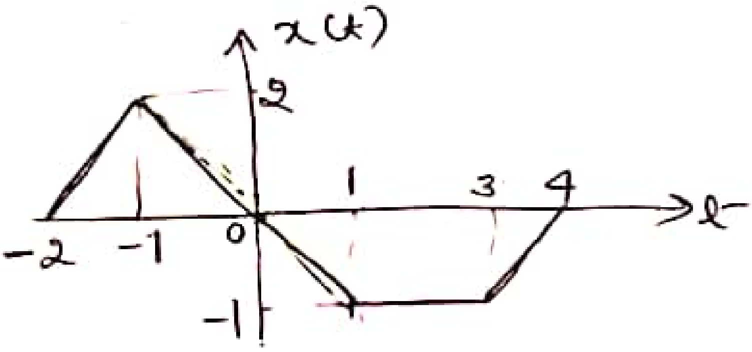
Use any method to express signal in mathematical form as

$$x(t) = \begin{cases} 2t+2 & -1 < t < 0 \\ 2 & 0 < t < 2 \\ 4-t & 2 < t < 4 \end{cases}$$

iii'g



$$x(t) = \begin{cases} \frac{1}{2}(t+2), & -2<t<0 \\ \frac{-1}{3}(t-3) & 0<t<3 \end{cases}$$

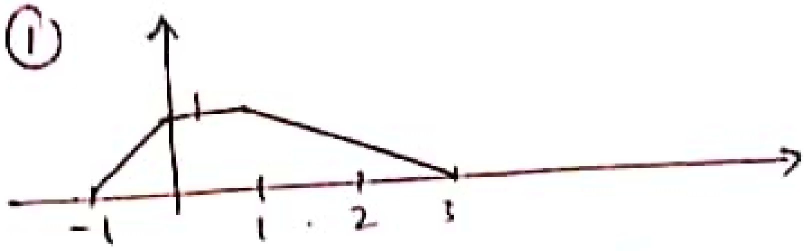


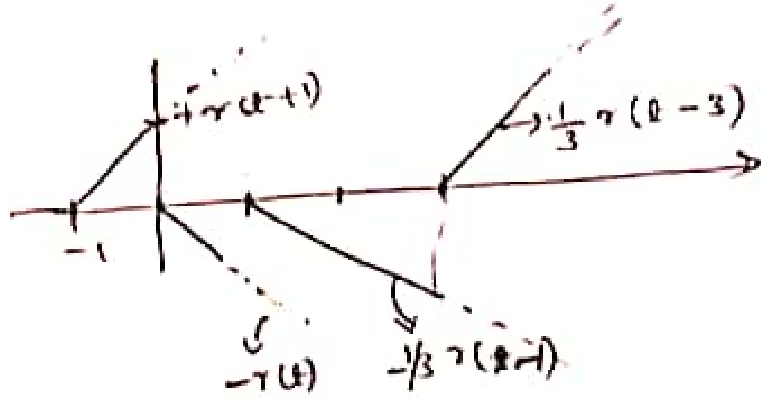
$$x(t) = \begin{cases} 2(t+2) & -2<t<-1 \\ -2(t+1) & -1<t<0 \\ -t & 0<t\ 1 \\ -1 & 1<t<3 \\ t-4 & 3<t<4 \end{cases}$$

Expressing signals in terms of basic signals

Express sig $x(t)$ in terms of basic signals.

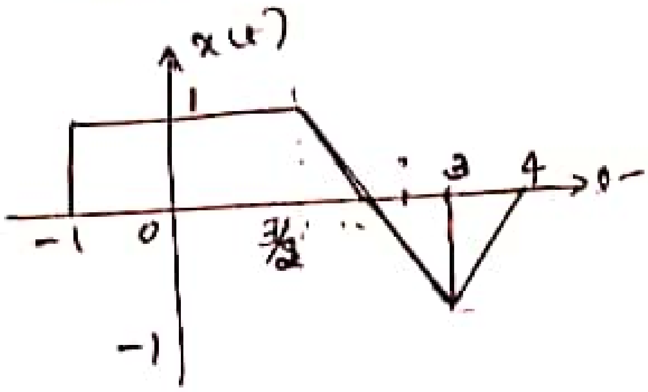
①





$$x(t) = r(t+1) - r(t) - \frac{1}{3} r(t+1) + \frac{1}{3} r(t-3)$$

②



$$x(t) = u(t+1) - u\left(t - \frac{3}{2}\right) - \frac{2}{1.5} r\left(t - \frac{3}{2}\right) + \frac{2}{1.5} r\left(t - \frac{3}{2}\right)$$
$$+ \frac{2}{1.5} r(t-3) - r(t-4)$$

$$x(t) = u(t+1) - u(t - 1.5) - \frac{4}{3} r\left(t - \frac{3}{2}\right)$$
$$+ \frac{4}{3} r(t-3) + r(t-3) - r(t-4)$$