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MODULE 1

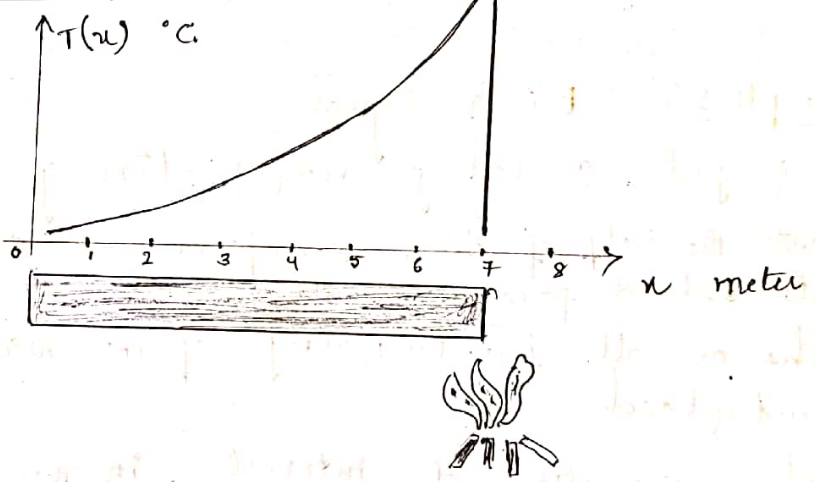
SIGNAL:

A Signal is a function of one or more variables, which conveys some sort of information on the nature of a physical phenomenon.

A signal is a dependent variable indicating the changes in a physical quantity due to changes in one or more independent variables.

Ex(1) Room temperature; it could be a function of position of interest in the room & time.
Symbolically $T(x, t)$

T : temperature
x : position
t : time.



Example(2):

Consider a metallic conductor which is heated at one of the extremes. The heat energy is transferred from heated end to other end.

Now the temperature is measured at various points on the surface of metallic rod, starting from the end which is not heated we observe that the temperature value increases as we go towards the heated end.

Here the temperature (T) is a dependent variable (the signal of interest) which depends on the distance (x) along the axis which is independent variable.

The relation b/w the two variables is denoted symbolically as : $T(x)$

T is said to be function of x.

Just for demonstration, this relation arbitrarily governed by the following Eqn.

$$T(x) = x^2$$

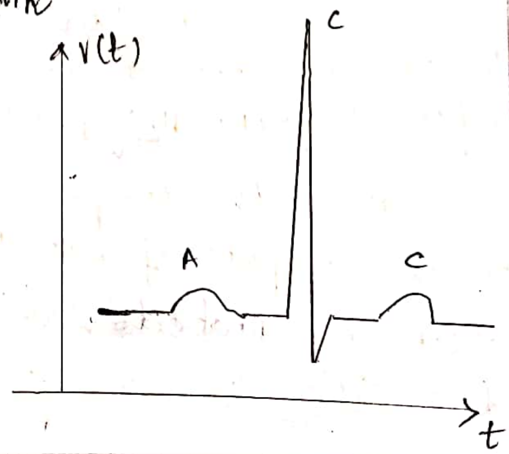
Stated as temperature increases as square of distance.

Thus variable 'T' is a signal which conveys temperature information.

Example 2: ECG Signal.

* It is just a voltage versus time graph.
* With the help of this waveform we can estimate heart rate, blood pressure, etc
* The overall functionality of the heart can be easily understood.
* The parameters of interest in the wave form for the estimation purpose could be
  - i) Time duration of pulse A or B or C ; width
  - ii) Amplitude of pulse A, B or C (hight)
  - iii) Time gap b/wn any two pulses.

A typical ECG Signal is shown in figure.



Example 3: Electrical signal through a coaxial cable for TV (Carries audio, video & colour information)

Example 4: An electromagnetic wave in a cellular communication system.

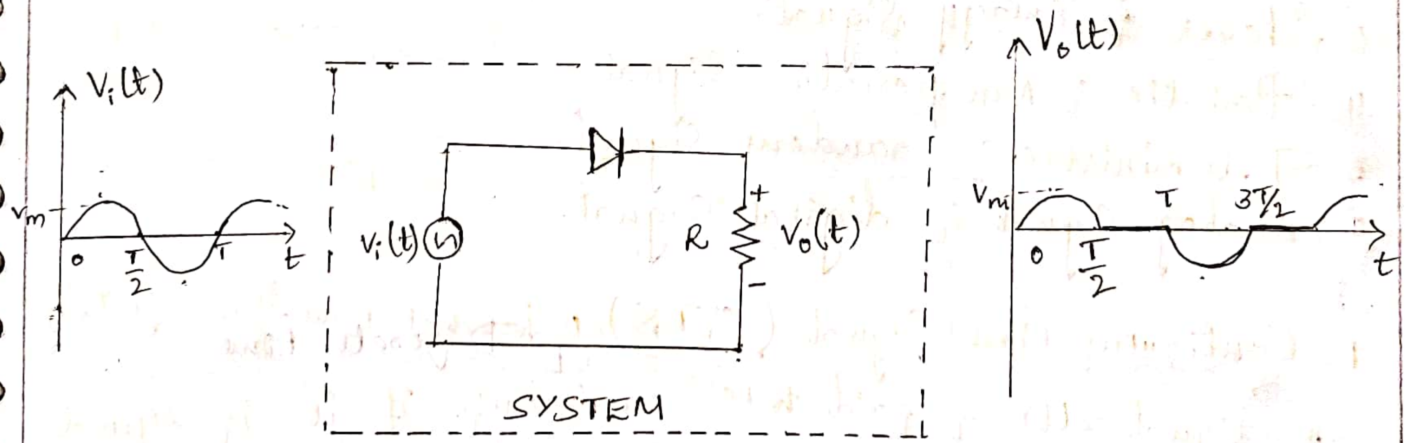
SYSTEM:

A system is formally defined as an entity that manipulates one or more signals (i/p) to accomplish a task, thereby yielding a new signal (o/p)

Respiratory system is an example for biological system which accepts oxygen (i/p) & produces carbon dioxide. (o/p)

Any electronic system does PROCESSING (set of operations done in a predefined sequence or order) of the input signal to produce output signal.

Example: An Half wave Rectifier



$v_i(t) = V_m \sin\omega t \quad 0 < t < T$

$V_0(t) = V_m \sin\omega t \quad 0 < t \leq T/2$

$= 0 \quad \frac{T}{2} < t < T$

The above rectifier can be thought of as a system &

the processing operation done by it is the rectification
- i/p $v_i(t)$ is a sinusoidal signal.
- o/p $v_o(t)$ is a pulsating dc.

Typically a system is represented as follows.

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = H\{x(t)\}$$

$x(t)$: the i/p signal.
$y(t)$: the o/p signal.
H : the system operator / system operator.

$H\{x(t)\}$ is read as operated version of i/p signal $x(t)$, which is equal to o/p signal $y(t)$.

CLASSIFICATION OF SIGNALS

Signals can be classified in different ways as follows.

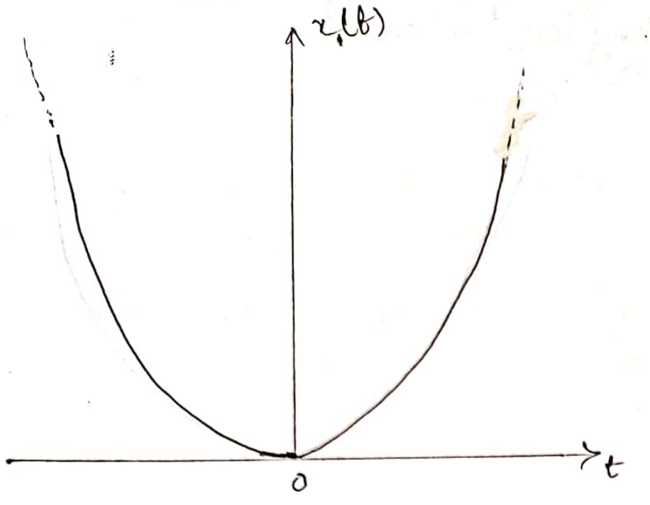
1. Continuous time signal & discrete time Signal.
2. Even Signal & odd Signal.
3. Power & Energy Signal.
4. Periodic & Non periodic Signal.
5. Deterministic & random Signal.
6. Analog Signal & digital Signal.

1. Continuous time Signal (C.T.S) & Discrete Time Signal (DTS)

A signal $x(t)$ is said to be continuous if it is defined for all values of time 't' (independent variable)

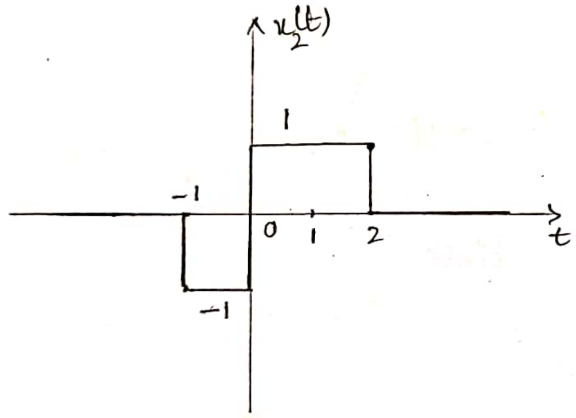
The variable 't' takes real values.
'x' can be real or complex.

Example ①



$x(t) = t^2 \quad \forall t$

Example ②

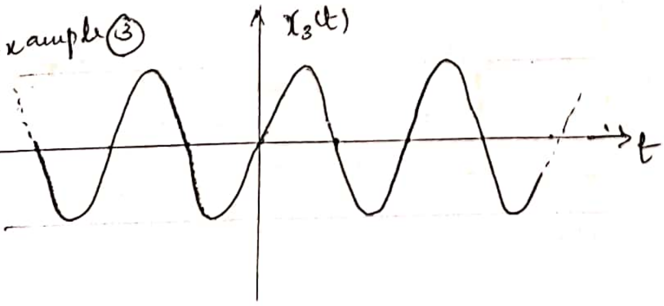


$x_2(t) = -1 \quad -1 \leq t < 0$

$\quad = 1 \quad 0 < t \leq 2$

$\quad = 0 \quad$ otherwise

Example ③



$x_3(t) = \sin t \quad \forall t$

A signal $x[n]$ is said to be discrete time signal if it is defined for discrete instants of time. i.e., the signal amplitude is specified at instants $t = nT$

$n = \ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

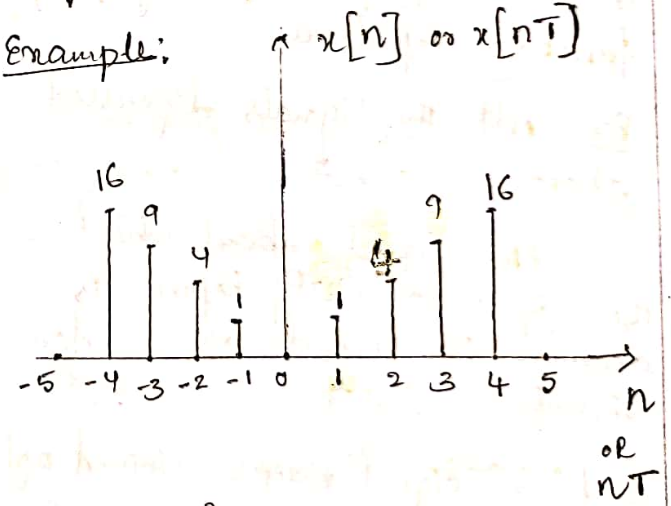
where 'T' is a fixed (discrete) time interval.

The signal amplitude is defined at $t = \ldots -2T, -T, 0, T, 2T, 3T \ldots$ so on.

and it is undefined elsewhere.

n - is also designated as time index. which is an integer.

Example:

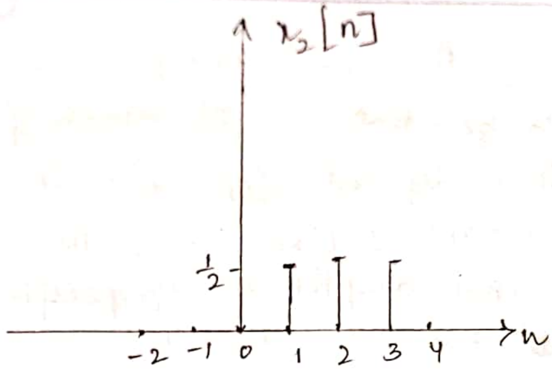


$x[n] = n^2 \quad -4 \leq n \leq 4$

$\quad = 0 \quad$ otherwise.

Note: In above example if $T = 2$ sec, the signal amplitude defined at the instants: .... -4 sec, -2 sec, 0 sec, 2 sec, 4 sec .... so on.
where as signal value at $t = 1.75$ sec, 2.3 sec etc is undefined.

Scanned by CamScanner



$x_2[n] = 1/2 \quad 1 \le n \le 3$
$\quad\quad = 0 \quad$ otherwise.

Any DTS can be written as SEQUENCE of values with n being time index. example
$x_2[n] = \{ \cdots 0 \;\; 0 \;\; \underset{\uparrow}{0} \;\; 1/2 \;\; 1/2 \;\; 1/2 \;\; 0 \cdots \}$

Deterministic & Random signals.

A signal about which there is no uncertainity with respect to its value at any time, is termed as deterministic signal.
Thus deterministic signal can be modeled as completely specified function of time.

Ex: All the signals discussed above.

A signal about which there is uncertainity before it occurs, is termed as random signal.

Such a signal may be viewed as belonging to an ensemble, or a group, of signals, with each signal in the ensemble having a different waveform.
Moreover each signal in the ensemble has a certain probability of occurance.
The ensemble of the signals is referred to as a random process.

Ex: The electrical noise generated in the amplifier of a radio or television receiver.

Analog Signal and Digital Signal

If a CTS $x(t)$ can take on any value in continuous interval (a, b) where a may be $-\infty$ & b may $+\infty$, the signal is called analog signal.

If a CTS can take on only a finite set of possible values then it is called as digital signal.

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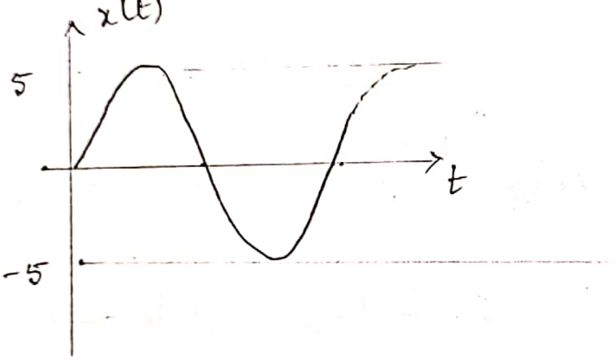
Examples:

Analog signals

① sinusoidal AC voltage.

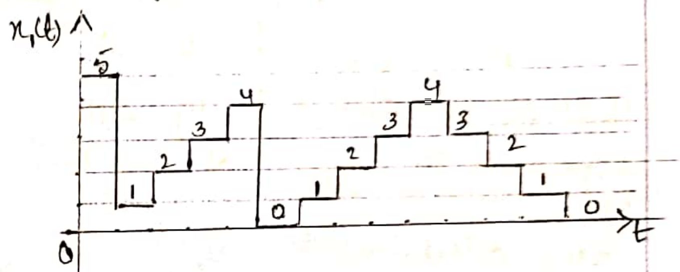


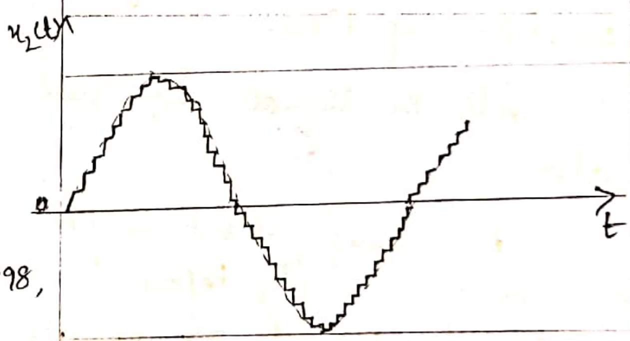
The signal $x(t)$ takes infinite no of possible values within the range $(-5\ \ +5)$, such as $-4.9, -4.98, -4.887$ ----- $0, 0.01, 0.02, 0.021$ --- $4.9, 4.99$, 4.999 --- etc.

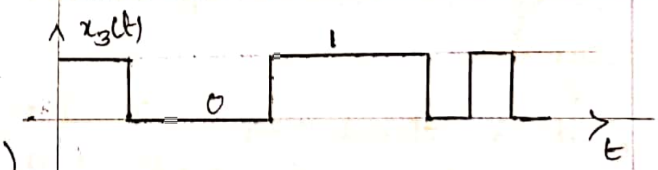
2) Signal coming out of a microphone (full of analog frequencies & harmonics)

device example: Analog multimeter.

Digital signals







The above signals x_1, x_2 & x_3 have taken one of the values of a finite set. at any instant of time.

Signal $x_3(t)$ is the o/p of a digital logic ckt which takes one the two values (0 & 1)

Ex device: Digital multimeter.

EVEN AND ODD SIGNALS

A continuous time signal $x(t)$ is said to be even, if it satisfies the condition

$$x(-t) = x(t) \quad \text{OR} \quad x(t) = x(-t)$$

Similarly for DTS to be Even

$$x(-n) = x(n) \quad \text{OR} \quad x(n) = x(-n)$$

Example: $x(t) = t^2$; a parabola ($y = x^2$)

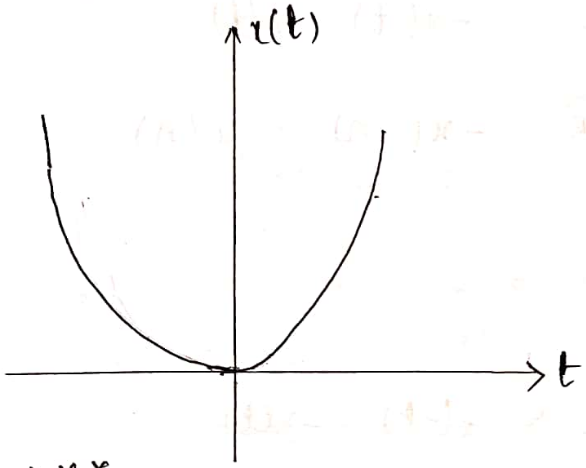
$t = 1$; $x(1) = 1$
$t = -1$; $x(-1) = (-1)^2 = 1$

Here

$$x(-1) = x(1) \quad \text{i.e.,} \quad x(-t) = x(t)$$

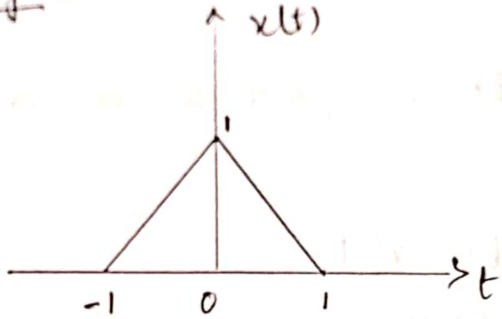
$\therefore$ the signal above $x(t)$ is a Even signal.

Graphical representation:

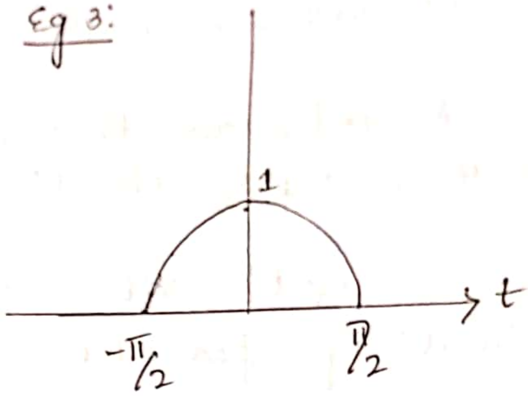


*** Note: A signal is said to be EVEN, if it is SYMMETRICAL w.r.t vertical axis or dependent axis

Eg 2:

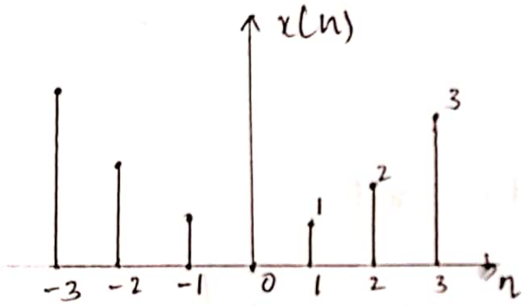


Eg 3:



Above waveform is cosine function.

Ex 4:



Ex 5:

A CTS is said to be ODD if it satisfies the condition

$$x(-t) = -x(t) \quad \text{OR} \quad -x(-t) = x(t)$$

For DTS

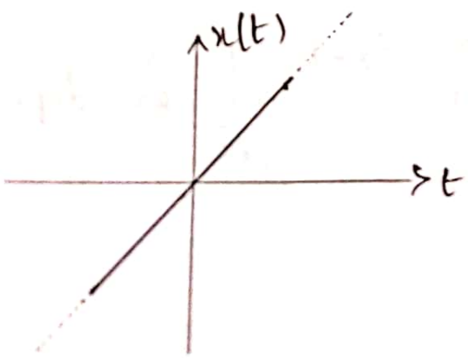
$$x(-n) = -x(n) \quad \text{OR} \quad -x(-n) = x(n)$$

Example: $x(t) = t$

$t=1 \quad x(1) = 1$

$t=-1 \quad x(-1) = -1$

$$x(-1) = -x(1) \Longrightarrow x(-t) = -x(t)$$



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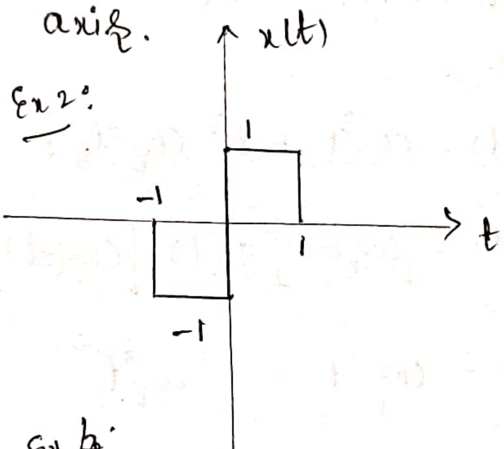
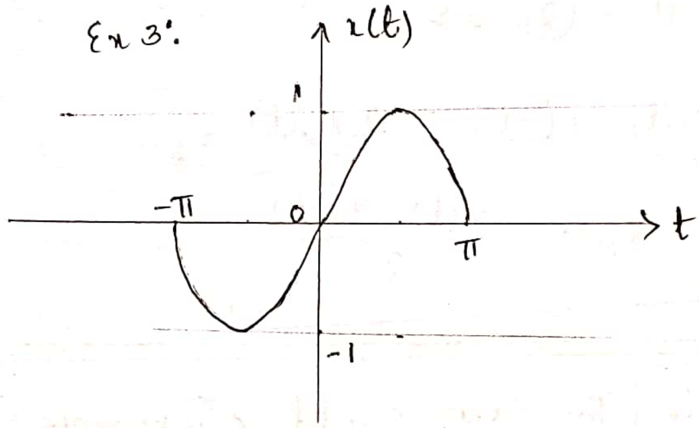
<u>Note</u>: A signal x(t) is said to be ODD if it is ANTI-SYMMETRIC w.r.t vertical axis or dependent axis.

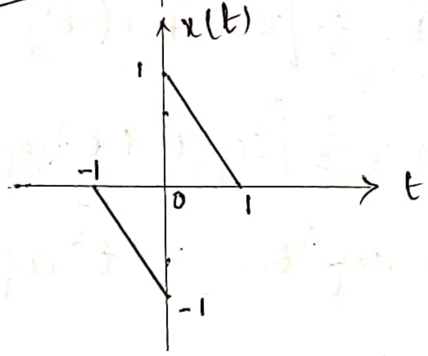
<u>Ex 2</u>:



Ex 3:



<u>Ex 4</u>:



<u>Ex 5</u>:

If x(t) is a CTS which is neither Even nor odd, then it can be seperated into Even & odd part.

Let $x_e(t)$ be the Even part of $x(t)$ & $x_o(t)$ be the odd part of $x(t)$.

The signal x(t) is the sum of Even & odd part.

$$x(t) = x_e(t) + x_o(t) \longrightarrow ①$$

put $t = -t$ in ①

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \longrightarrow ②$$

$\because$ $x_e(t)$ is Even & $x_o(t)$ is odd.

Now

① + ② $\Rightarrow$

$$x(t) + x(-t) = 2x_e(t)$$

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}}$$

① − ② ⇒

$$x(t) - x(-t) = 2x_o(t)$$

$$\boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}$$

Find the even & odd components of the following.

1) $x(t) = \cos t + \sin t + \sin t \cos t$

Soln.

$$x(-t) = \cos(-t) + \sin(-t) + \sin(-t)\cos(-t)$$

$$x(-t) = \cos t - \sin t - \sin t \cos t$$

Even component is given by

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$= \frac{1}{2}[\cos t + \sin t + \sin t \cos t + \cos t - \sin t - \sin t \cos t]$$

$$= \frac{1}{2} 2\cos t$$

$$x_e(t) = \cos t$$

Odd component is given by.

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}[\cos t + \sin t + \sin t \cos t - \cos t + \sin t + \sin t \cos t]$$

$$x_o(t) = \frac{1}{2}[2\sin t + 2\sin t \cos t]$$

$$x_o(t) = \sin t + \sin t \cos t.$$

2) $x(t) = (1+t^3)\cos^3 t$

Soln

$$x(t) = \cos^3 t + t^3 \cos^3 t$$

$$x(-t) = [\cos(-t)]^3 + (-t)^3 [\cos(-t)]^3$$

$$x(-t) = \cos^3 t - t^3 \cos^3 t$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_e(t) = \frac{1}{2}[\cos^3 t + t^3 \cos^3 t + \cos^3 t - t^3 \cos^3 t]$$

$$x_e(t) = \cos^3 t$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

$$= \frac{1}{2}[\cos^3 t + t^3 \cos^3 t - \cos^3 t + t^3 \cos^3 t]$$

$$= \frac{1}{2} 2t^3 \cos^3 t$$

$$x_o(t) = t^3 \cos^3 t.$$

3)

$x(t) = 1+t^2+5t^3+9t^4$

soln

$x(-t) = 1+(-t)^2+5(-t)^3+9(-t)^4$

$x(-t) = 1+t^2-5t^3+9t^4$

$x_e(t) = \frac{1}{2}\left[x(t)+x(-t)\right]$

$= \frac{1}{2}\left[1+t^2+5t^3+9t^4 \right.$
$\left. 1+t^2-5t^3+9t^4\right]$

$x_e(t) = 1+t^2+9t^4$

$x_o(t) = \frac{1}{2}\left[x(t)-x(-t)\right]$

$= \frac{1}{2}\left[1+t^2+5t^3+9t^4 \right.$
$\left. -1-t^2+5t^3-9t^4\right]$

$x_o(t) = 5t^3$

$x_o(t) = \frac{1}{2}\left[x(t)-x(-t)\right]$

$= \frac{1}{2}\left[e^{-jt}-e^{jt}\right] \times \frac{j}{j}$

$x_o(t) = -j\sin t \quad \because \sin t = \frac{e^{jt}-e^{-jt}}{2j}$

6) $x(t) = (\cos\pi t+\sin\pi t)^2$

$= \cos^2\pi t+\sin^2\pi t+2\cos\pi t\sin\pi t.$

$x(t) = 1+2\cos\pi t\sin t \rightarrow (1)$

$x(-t) = 1+2\cos\pi(-t)\sin\pi(-t)$

$x(-t) = 1-2\cos\pi t\sin\pi t. \rightarrow (2)$

$x_o(t) = \frac{1}{2}\left[x(t)-x(-t)\right]$

$x_o(t) = 2\cos\pi t\sin\pi t$

4) $x(t) = 1+t\cos t+t^2\sin t+t^3\sin t\cos t$

Ans: $x_e(t) = 1+t^3\sin t\cos t$

$x_o(t) = t\cos t+t^2\sin t$

5) $x(t) = e^{-jt}$

$x(-t) = e^{+jt}$

$x_e(t) = \frac{x(t)+x(-t)}{2}$

$= \frac{1}{2}\left[e^{-jt}+e^{jt}\right]$

$x_e(t) = \cos t$

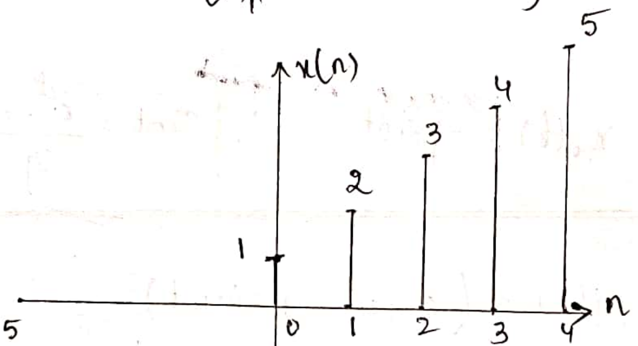
$x_e(t) = \frac{1}{2}\left[x(t)+x(-t)\right]$

$= \frac{1}{2}\left[1+2\cos\pi t\sin t. \right.$
$\left. 1-2\cos\pi t\sin t\right]$

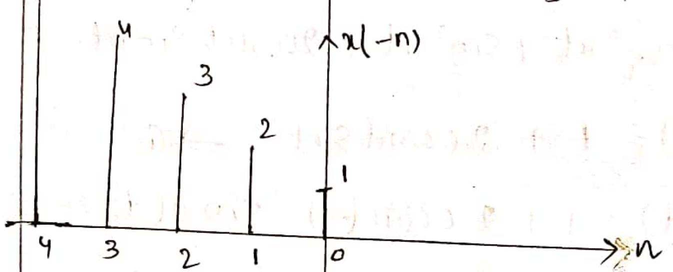
$x_e(t) = 1$

① Find the Even & odd component for the DT signal given.

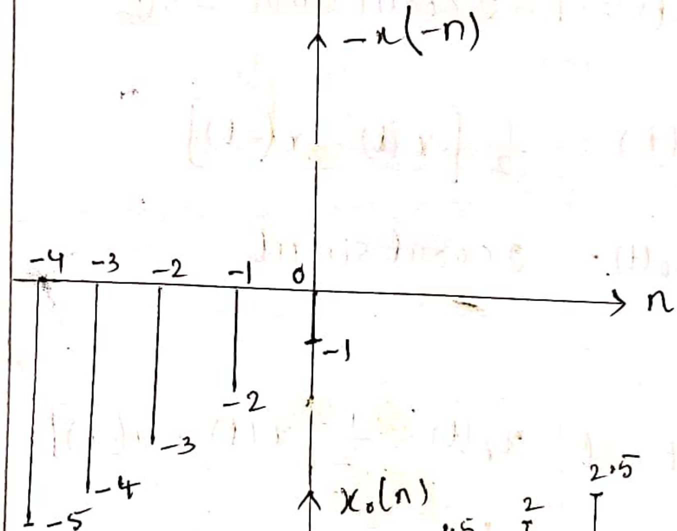
$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 5\}$$



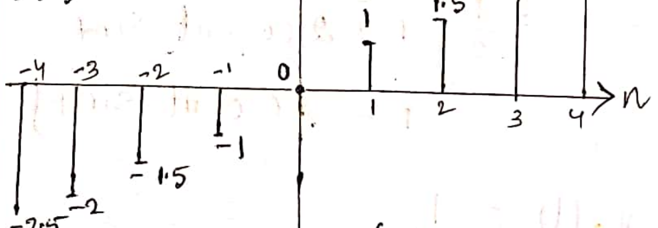
← given signal $x(n)$



← Reflection of $x(n)$

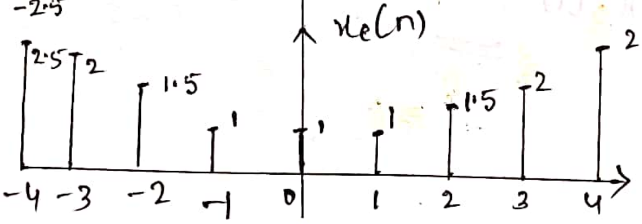


← inversion of $x(-n)$



← odd component; formula used

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$



← Even component; formula used

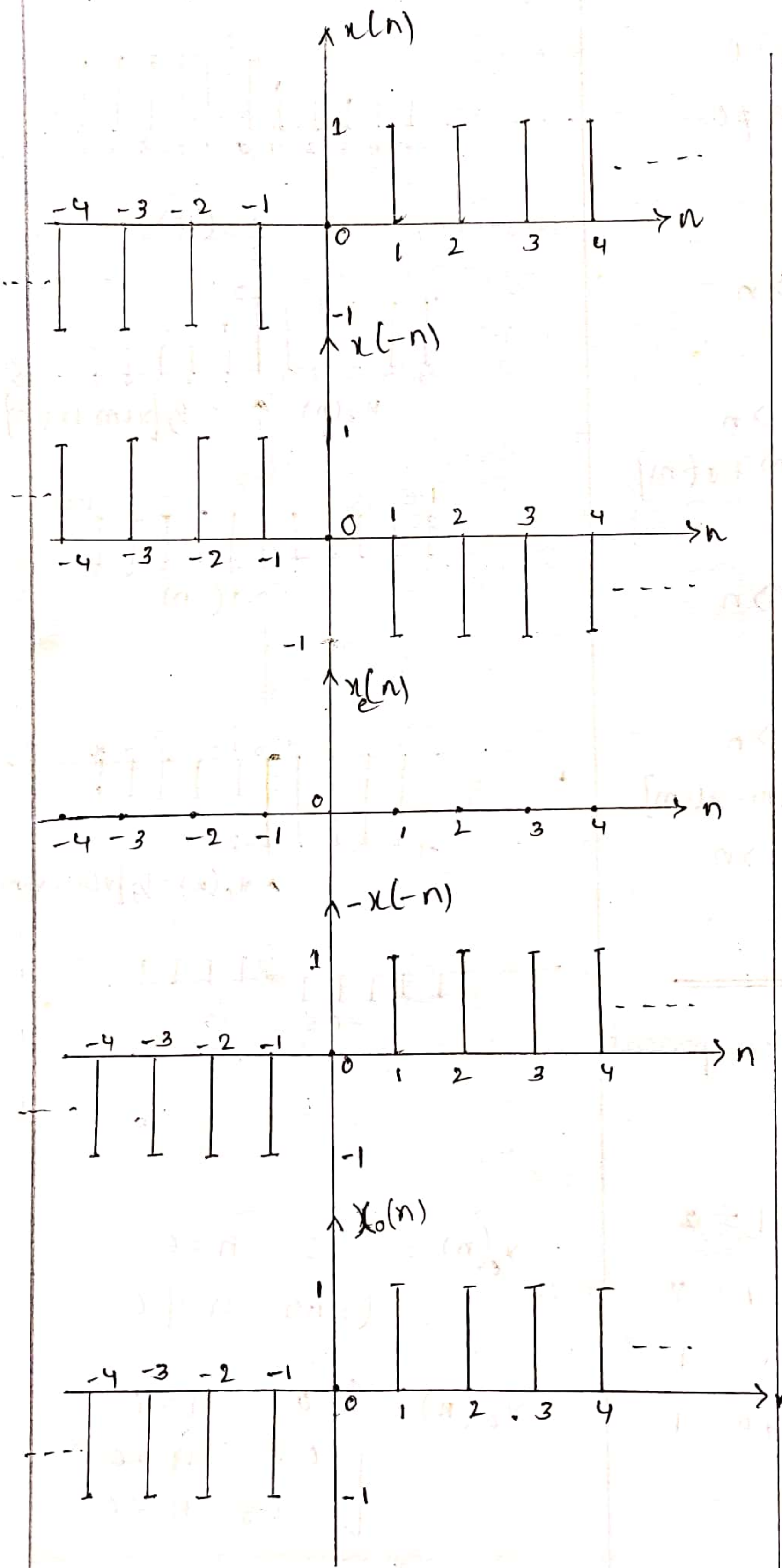
$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

Note! Symmetry can be observed.

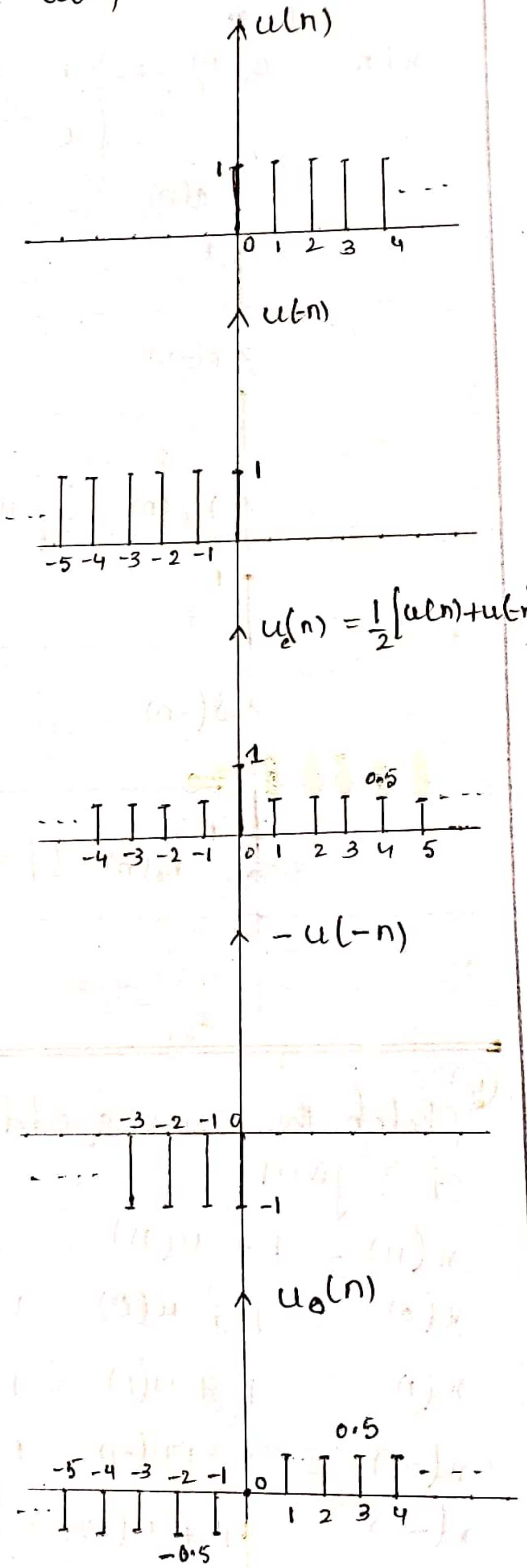
$$x_o(n) = \{-2.5, -2, -1.5, -1, \underset{\uparrow}{0}, 1, 1.5, 2, 2.5\}$$

$$x_e(n) = \{2.5, 2, 1.5, 1, \underset{\uparrow}{1}, 1, 1.5, 2, 2.5\}$$

(2) Sketch the even & odd component of x(n)



(3) Sketch even & odd part of u(n)



$$u_e(n) = \begin{cases} 1 & n=0 \\ 0.5 & n\neq 0 \end{cases}$$

OR

$$u_e(n) = \frac{1}{2} + \frac{1}{2}\delta(n)$$

$$u_o(n) = \begin{cases} 0 & n=0 \\ 0.5 & n>0 \\ -0.5 & n<0 \end{cases}$$

$$u_o(n) = \frac{1}{2}\text{signam}[\delta(n)]$$

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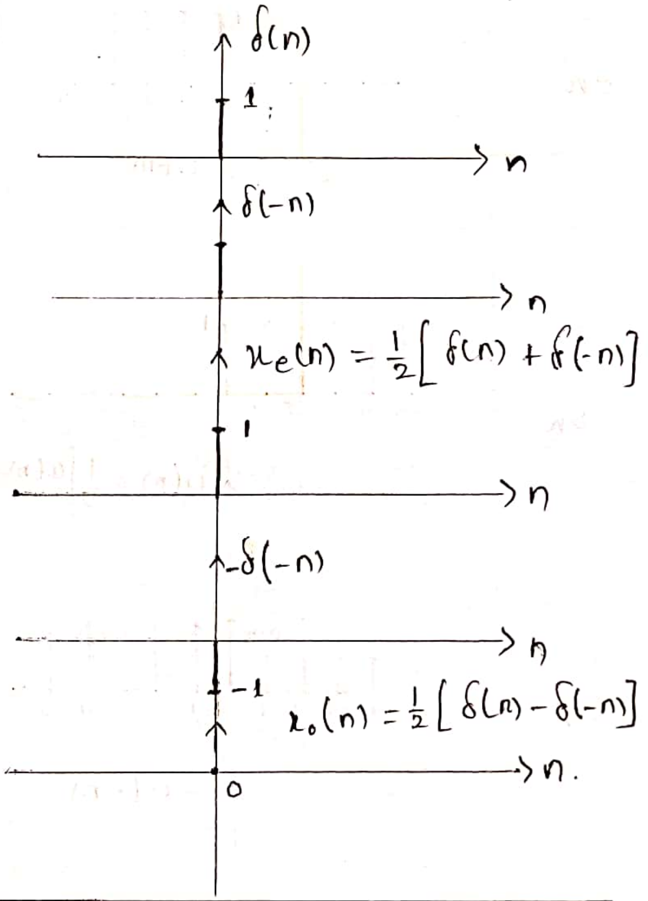
(4) Find the even & odd component of the signal $x(n)$

$$x(n) = \delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



(5) Sketch the even & odd component of given.

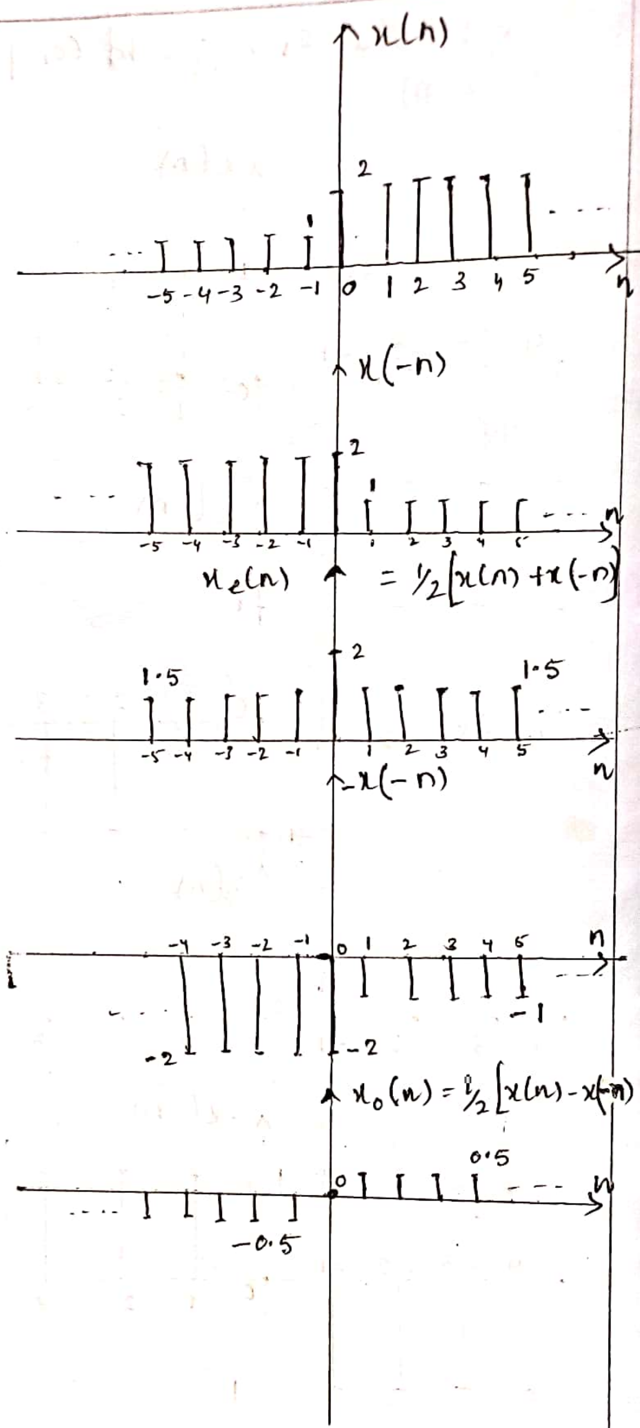
$x(n) = 1 + u(n)$

$x(0) = 1 + u(0) = 1 + 1 = 2$

$x(1) = 1 + u(1) = 1 + 1 = 2$

$x(-1) = 1 + u(-1) = 1 + 0 = 1$

$x(-2) = 1 + u(-2) = 1 + 0 = 1$



$$x_e(n) = \begin{cases} 2 & n = 0 \\ 1.5 & n \neq 0 \end{cases}$$

$$x_o(n) = \begin{cases} 0 & n = 0 \\ 0.5 & n > 0 \\ -0.5 & n < 0. \end{cases}$$

6 $x(n) = (0.5)^n u(n)$

$x(0) = 0.5^0 \, u(0) = 1 \times 1 = 1$

$x(1) = 0.5^1 \, u(1) = 0.5 \times 1 = 0.5 = \frac{1}{2}$

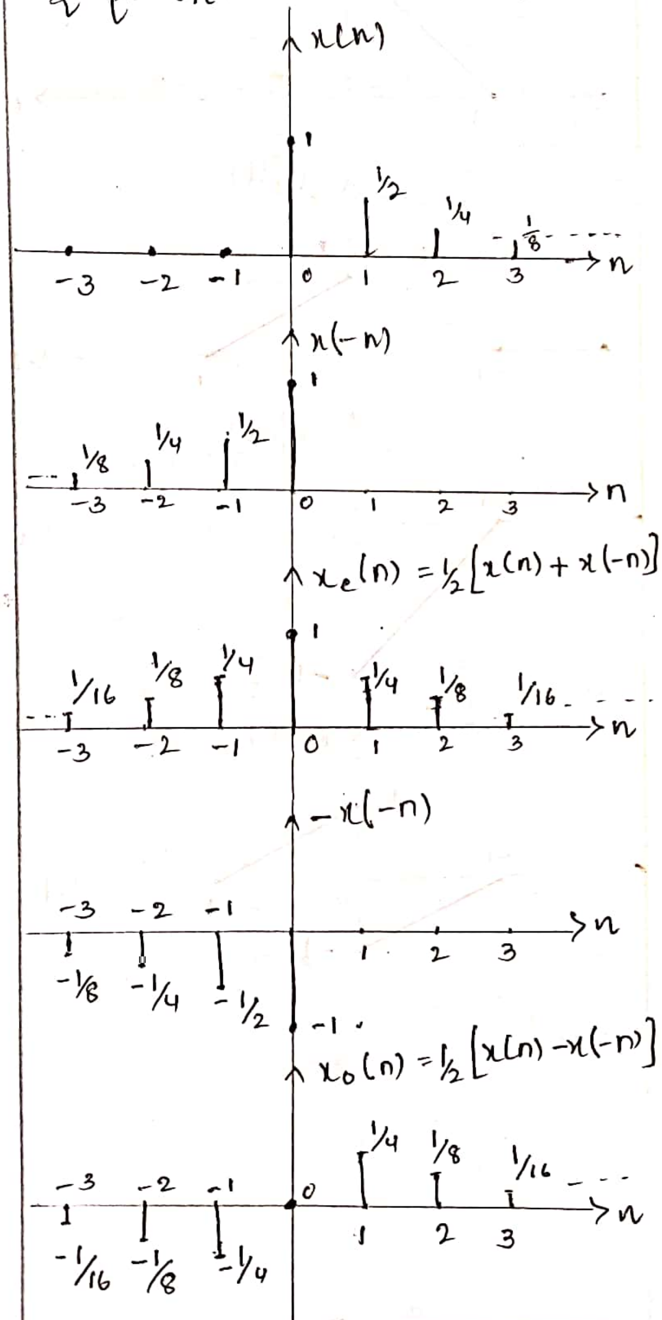
$x(2) = 0.5^2 \, u(2) = 0.25 \times 1 = 0.25 = \frac{1}{4}$

$x(3) = \frac{1}{8}$ and so on.

$x(-1) = (0.5)^{-1} u(-1) = (0.5)^{-1} \times (0) = 0$

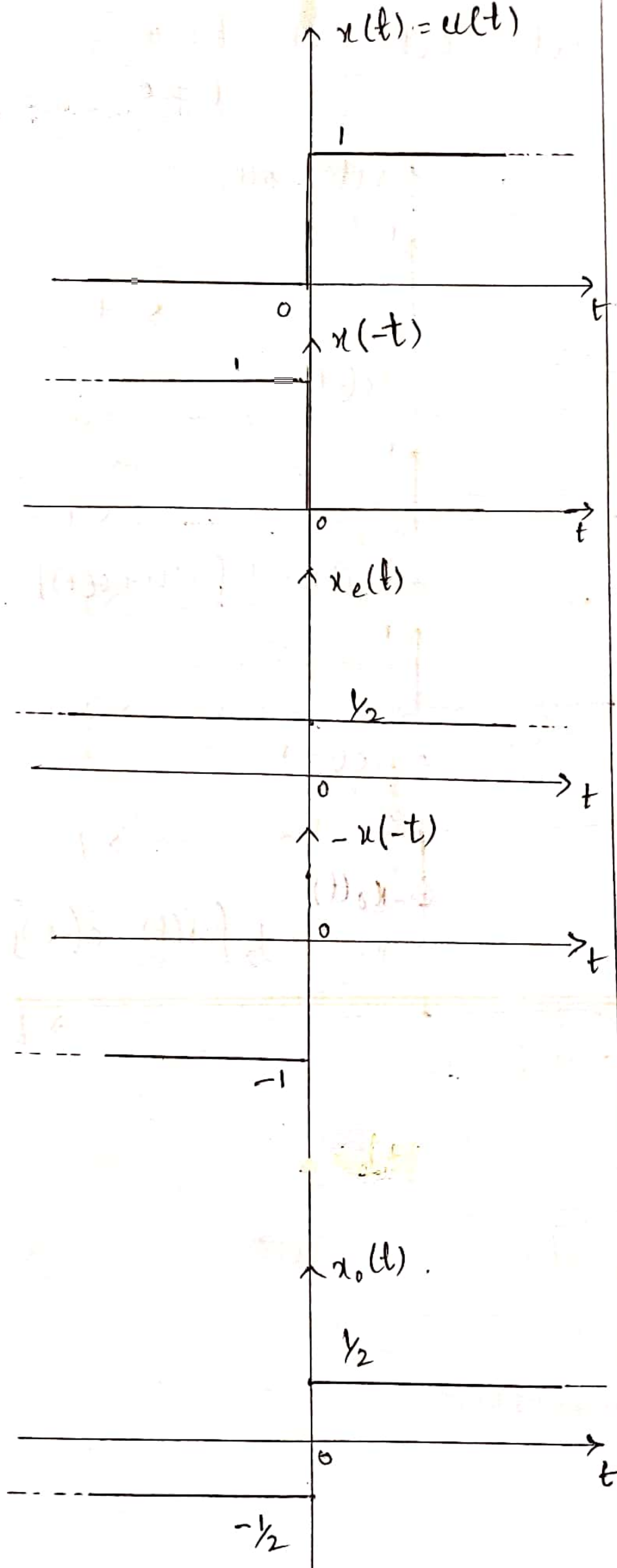
$x(-2) = (0.5)^{-2} u(-2) = (0.5)^{-2} \times (0) = 0$

& so on.



7 Sketch Even & odd part of $x(t) = u(t)$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

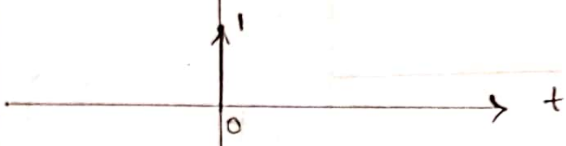


(8) Sketch even & odd component of

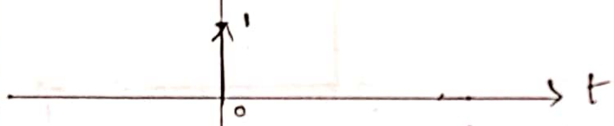
$x(t) = \delta(t)$

$$x(t) = \delta(t) = \begin{cases} 1 & t = 0 \\ 0 & t \neq 0 \end{cases}$$

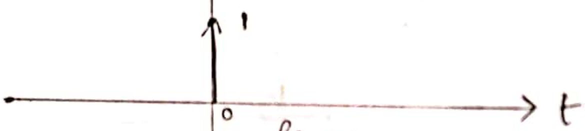
$x(t) = \delta(t)$



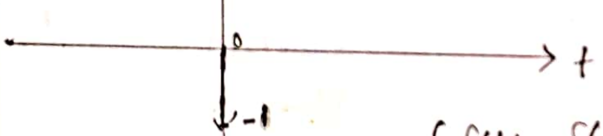
$\delta(-t)$



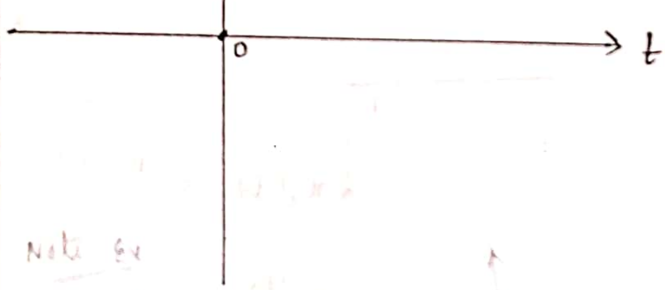
$x_e(t) = \frac{1}{2}[\delta(t) + \delta(-t)]$



$-\delta(-t)$

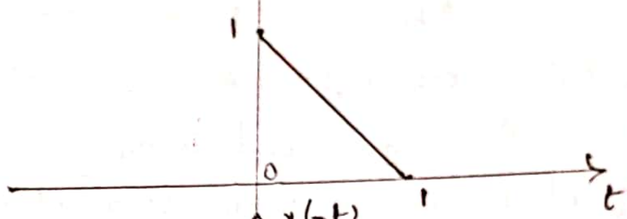


$x_o(t) = \frac{1}{2}[\delta(t) - \delta(-t)]$

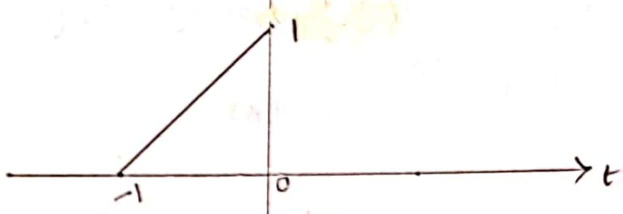


Sketch even & odd component for given signal.

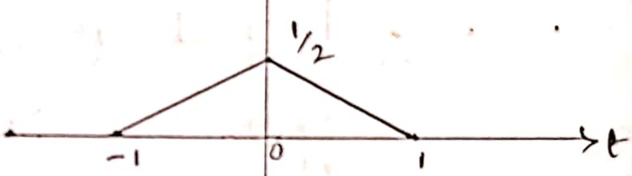
(9) $x(t)$



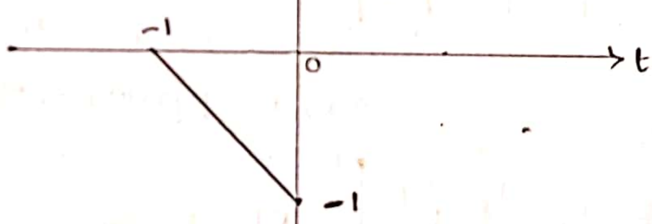
$x(-t)$



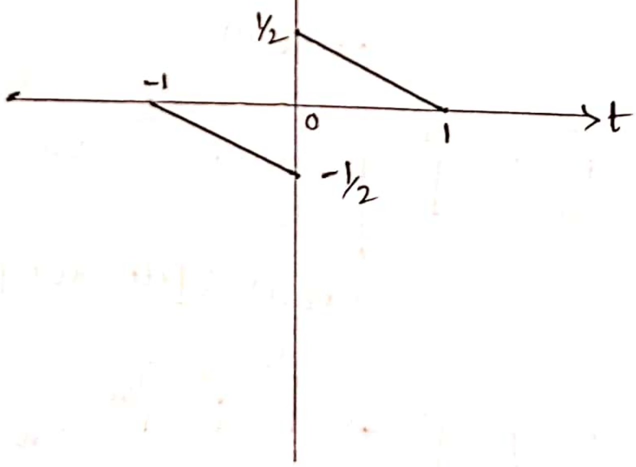
$x_e(t)$



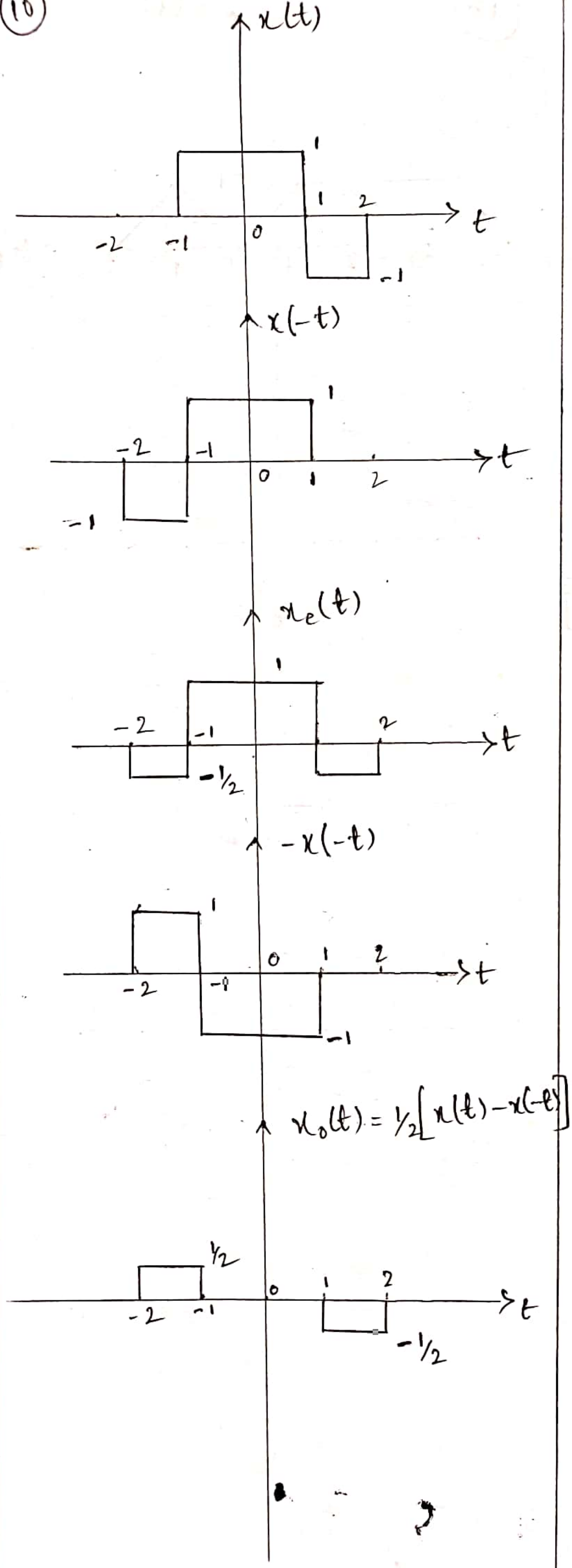
$-x(-t)$



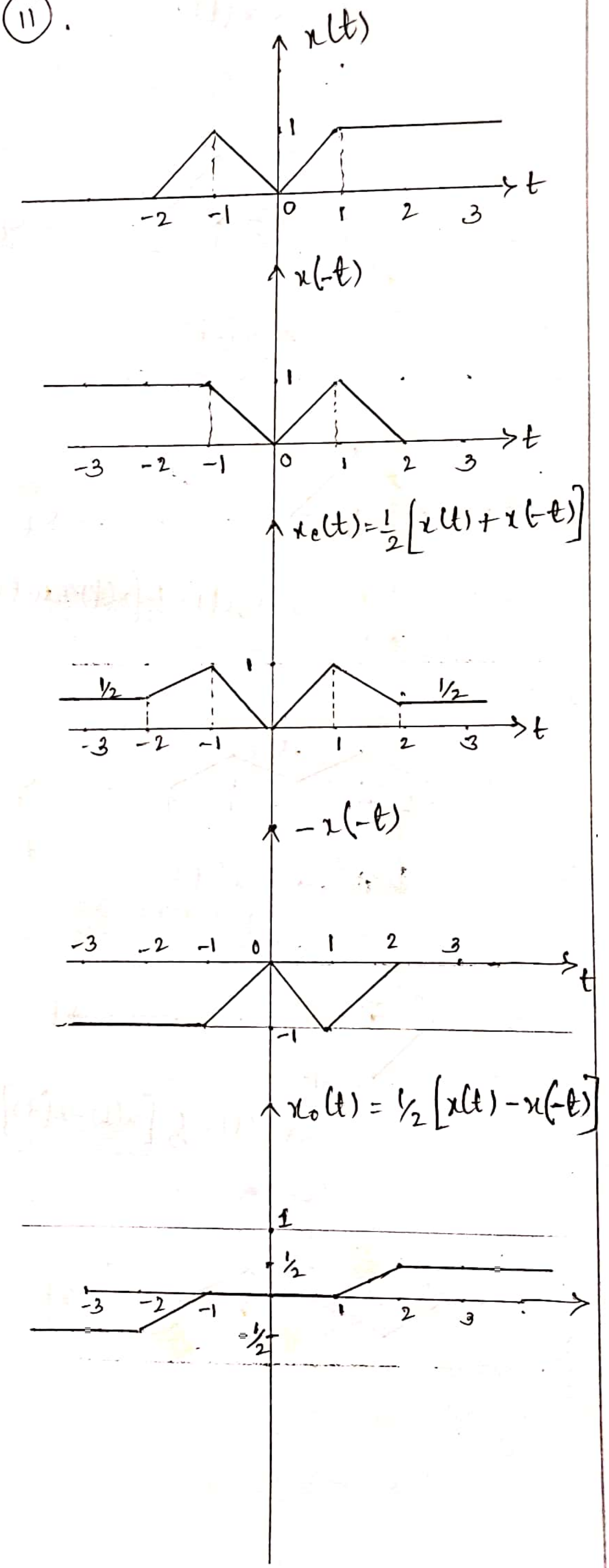
$x_o(t)$



(10)


x(t)
x(-t)
x_e(t)
-x(-t)
x_o(t) = 1/2[x(t) - x(-t)]
t
1/2
-1/2

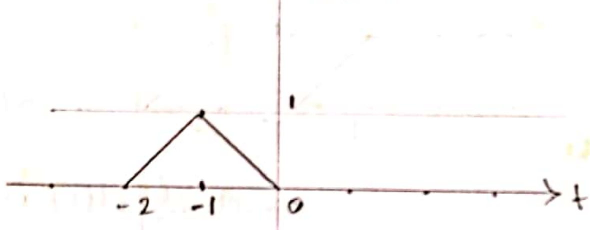
(11).


x(t)
x(-t)
x_e(t) = 1/2[x(t) + x(-t)]
-x(-t)
x_o(t) = 1/2[x(t) - x(-t)]
t
1/2
-1/2

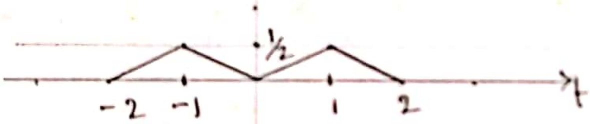
12

$x(t)$

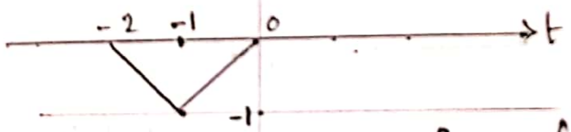
$x(-t)$



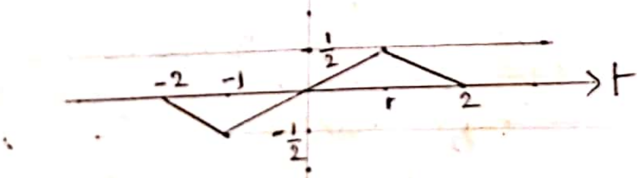
$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$



$-x(-t)$

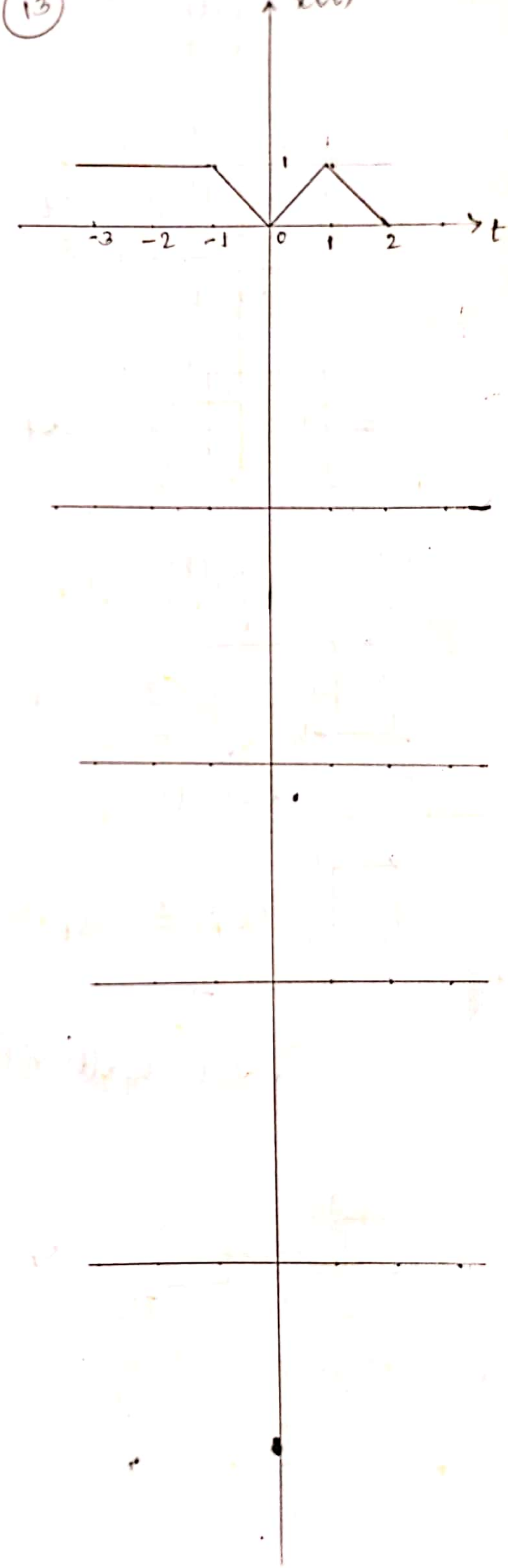


$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$



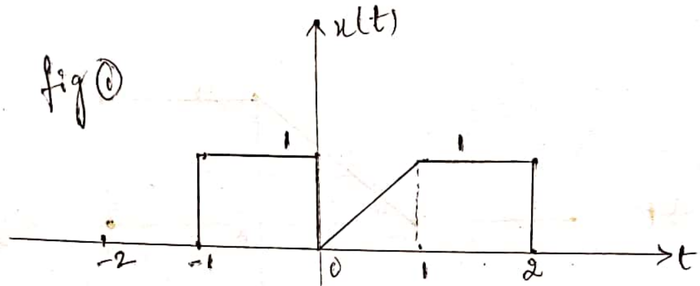
13

$x(t)$



Sketch even & odd

fig ①



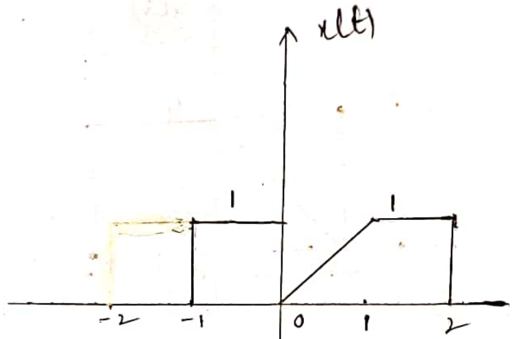
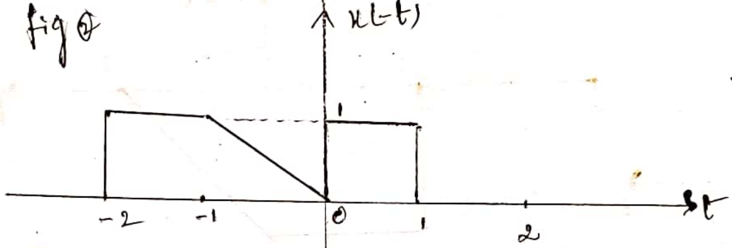


fig ②



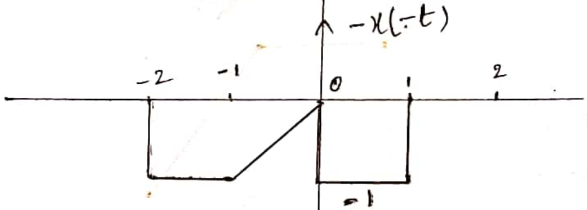
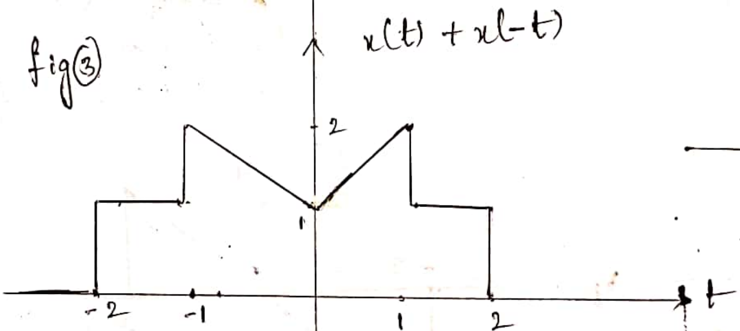


fig ③



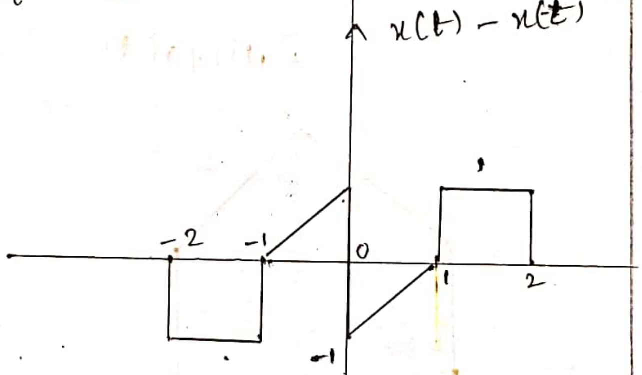
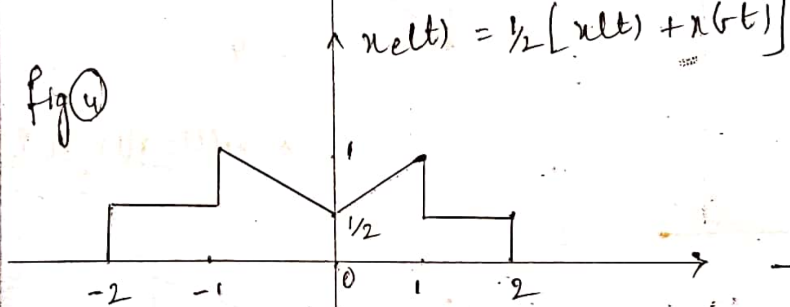
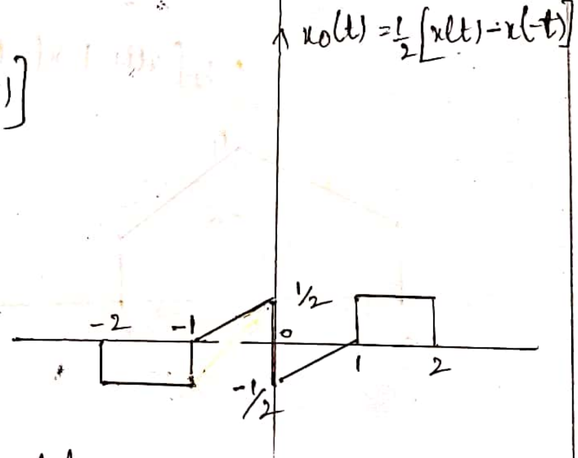


fig ④





Even:

step 1: sketch $x(t)$

2: sketch $x(-t)$

3) Add $x(t)$ & $x(-t)$

4) Divide $x(t) + x(-t)$ by 2

odd:

step 1: sketch $x(t)$

2: sketch $-x(-t)$

3: Add $x(t)$ with $-x(-t)$

4: Divide $x(t) - x(-t)$ by 2

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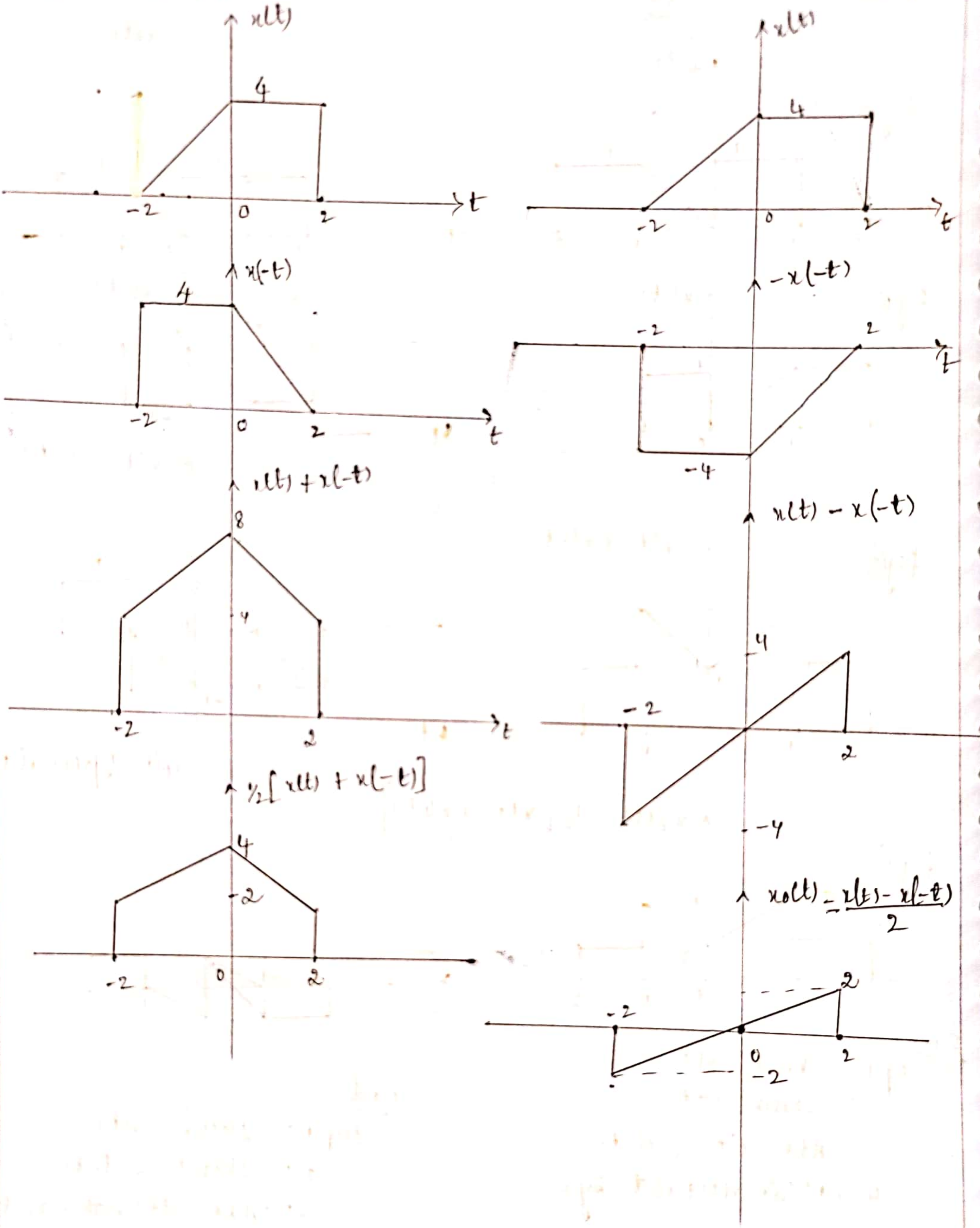
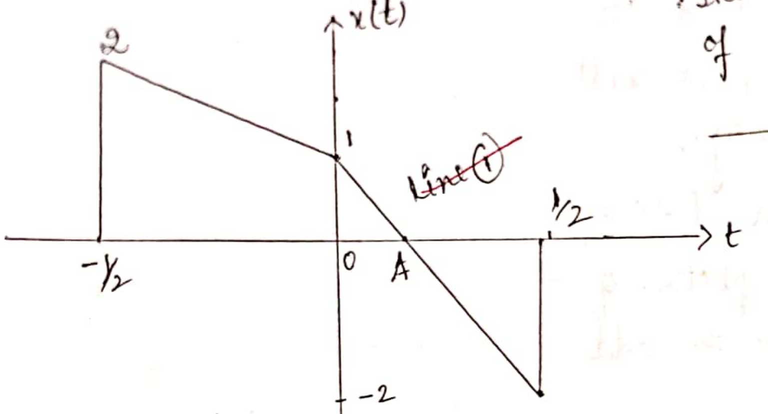
Sketch Even & odd



Sketch even & odd component of $x(t)$. given.



Let $x(t) = y$
$t = x$

for line ①

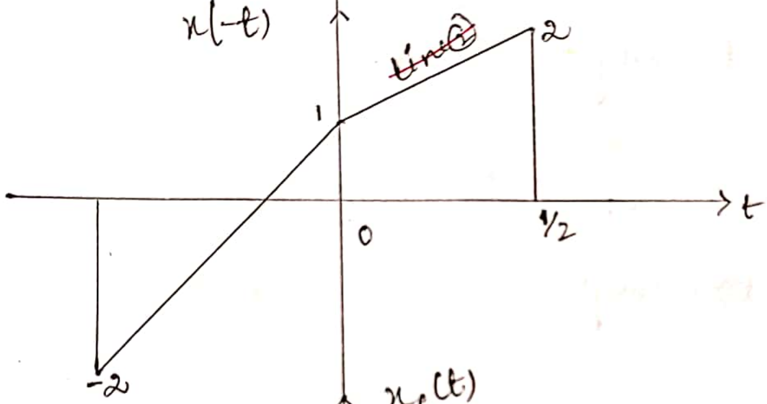
$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{y-1}{x-0} = \frac{-2-1}{\frac{1}{2}-0}$$

$$\frac{y-1}{x} = \frac{-3}{\frac{1}{2}} = -6$$

$\therefore\ x(t) = -6t + 1$

~~At point A~~ $t = \frac{1}{6}$



line ②

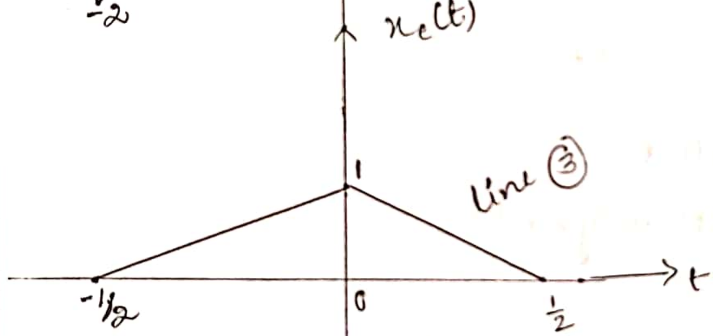
$$\frac{y-2}{x-\frac{1}{2}} = \frac{1-2}{0-\frac{1}{2}}$$

$$\frac{y-2}{2x-1} = \frac{-1}{-1} = 1$$

$x(-t) = 2t + 1$

At point A $t = 1$

Not required.

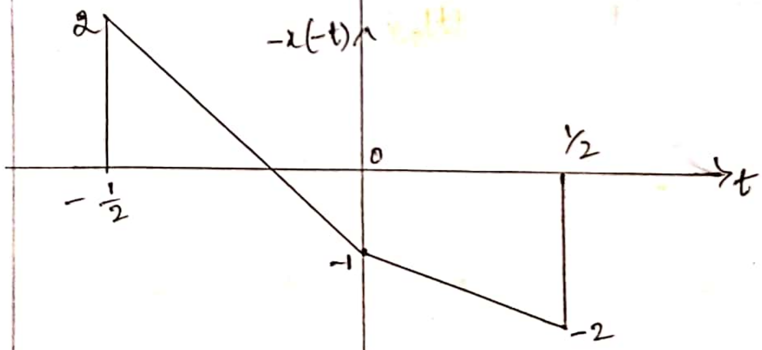


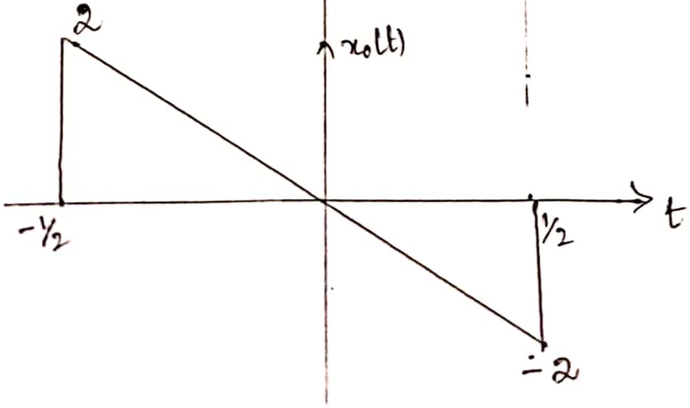
$x_e(t)$; b/w $t = 0$ & $\frac{1}{2}$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

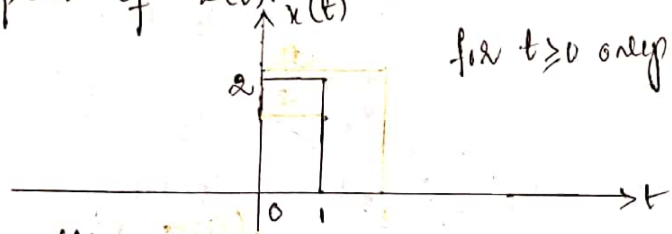
$$x_e(t) = \frac{-4t+2}{2}$$

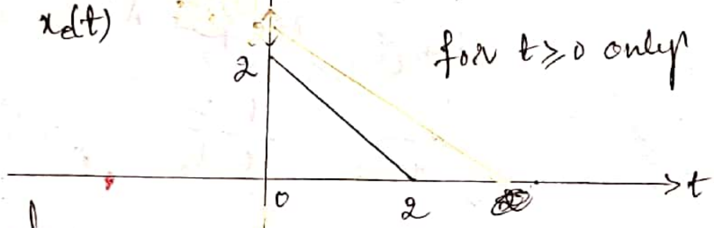
$x_e(t) = -2t + 1 \leftarrow$ line ③



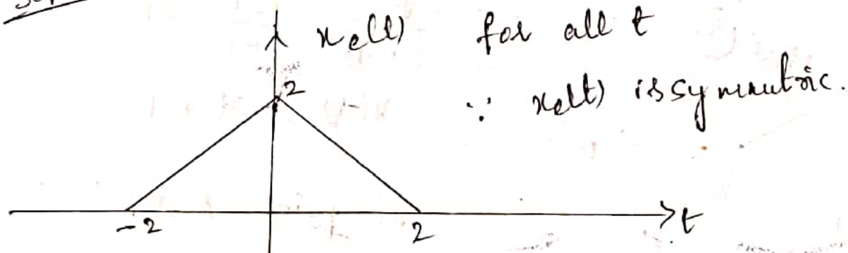


Below figure shows a part of the signal $x(t)$ & its even part $x_e(t)$ respectively, for $t \geq 0$ only.
$x(t)$ & even part $x_e(t)$ for $t < 0$ is not shown. Complete the plots of $x(t)$ & $x_e(t)$. Also draw the odd part of $x(t)$.





Soln



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PERIODIC AND NON-PERIODIC SIGNALS

A CTS $x(t)$ is said to be periodic, if it satisfies the condition

$$x(t) = x(t+T) \quad \forall t$$

(OR) $$x(t) = x(t+mT) \quad m = 1, 2, 3 \ldots$$

m is an integer

The smallest value of 'T' which satisfies the above condition is called FUNDAMENTAL PERIOD & it is a duration of one complete cycle.

'T' takes real value.

The angular frequency (ω) is determined as follows.

$$\omega = 2\pi f = \frac{2\pi}{T} \text{ rad/sec.} \qquad \because f = \frac{1}{T}$$

$$\boxed{\omega = \frac{2\pi}{T}} \text{ rad/sec.}$$

$$\boxed{T = \frac{2\pi}{\omega}} \text{ sec}$$

If 'T' can't be determined, then $x(t)$ is a Nonperiodic.

A CDTS $x(n)$ is said to be periodic, if it satisfies the condition

$$x(n) = x(n+N) \quad \forall n = \ldots, -2, -1, 0, 1, 2, 3, \ldots$$

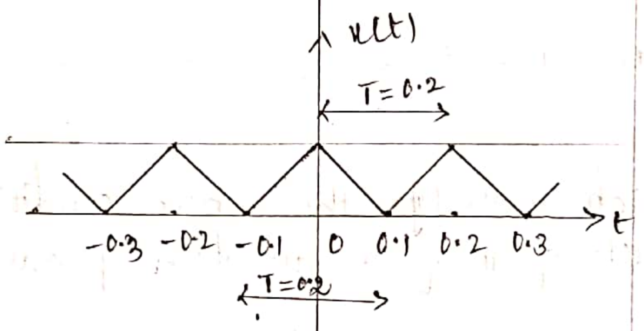
where N is the fundamental period which is an integer.

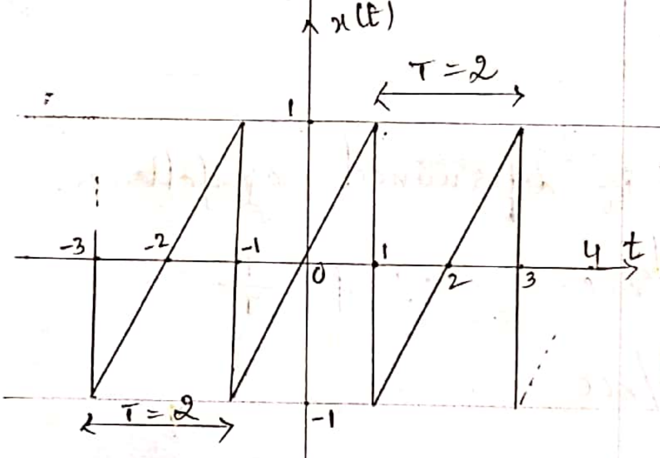
The angular frequency of $x(n)$ is given by

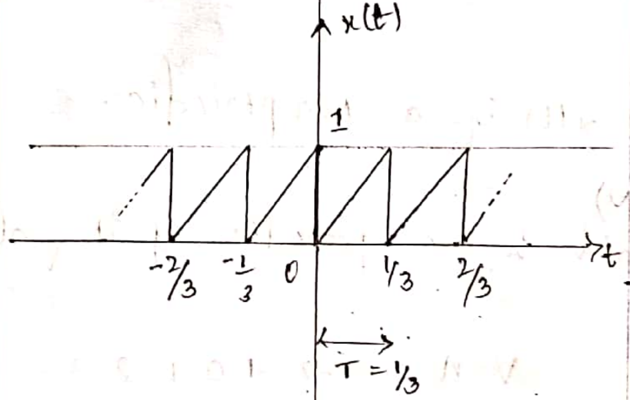
$$\boxed{\Omega_0 = \frac{2\pi}{N}} \text{ rad/sample.}$$

If $x(n)$ is not equal to $x(n+N)$ [i.e., $x(n) \neq x(n+N)$], then $x(n)$ is a nonperiod signal.

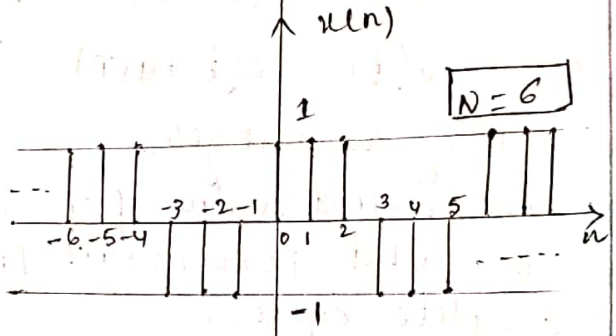
Examples of periodic CTS

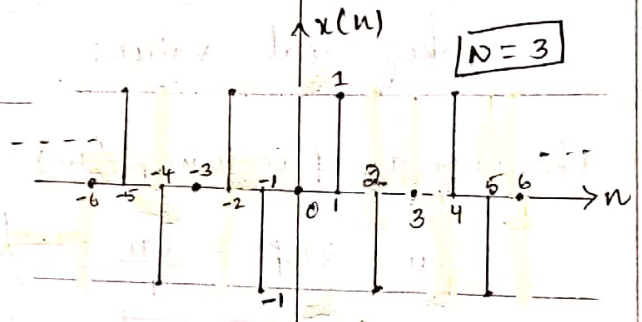


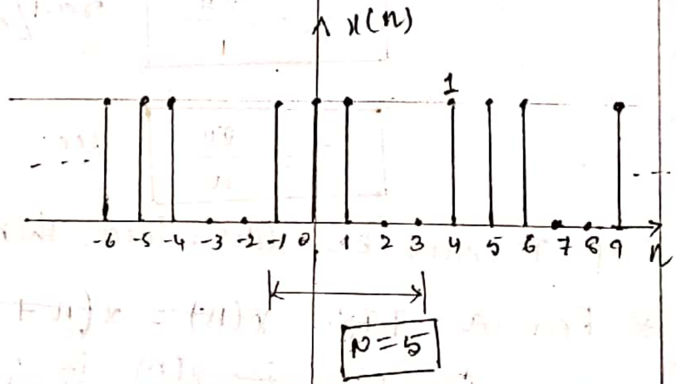




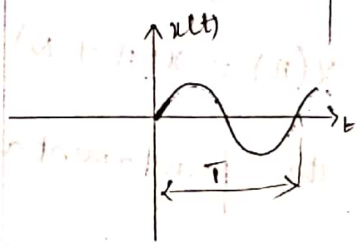
Examples of periodic DTS.







ex: $x(t) = \sin t$



$T = \frac{2\pi}{1}$

$T = 2\pi$ sec

Verification:

$t \to t+T$ in ①

$x(t) = \sin t \to$ ①

$= \sin \omega t$

$x(t+T) = \sin(t+T)$

$x(t+T) = \sin(t+2\pi)$

$= \sin t$

$\omega = 1$

$\omega = 2\pi f = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

$x(t+T) = x(t)$

Note: A periodic signal is one which repeats for every T seconds

For a DTS given below derive the condition to be periodic.

$$x(n) = e^{j\Omega_0 n} \longrightarrow ①$$

Soln For $x(n)$ to be periodic it must satisfy the condition $x(n) = x(n+N)$ where N is an integer.

$\therefore$ Put $n = n+N$ in ①

$$x(n+N) = e^{j\Omega_0(n+N)}$$

$$x(n+N) = \underbrace{e^{j\Omega_0 n}} \cdot e^{j\Omega_0 N}.$$

$$x(n+N) = x(n) \quad e^{j\Omega_0 N}.$$

For $x(n)$ to be periodic; $e^{j\Omega_0 N}$ must be equal to 1

i.e., $e^{j\Omega_0 N} = \cos \Omega_0 N + j\sin \Omega_0 N = 1 \longrightarrow ②$

The above condition (Equ 2) is true only if

$$\Omega_0 N = 2\pi m \; ; \quad m = 1 \;\; 2 \;\; 3 \; \ldots$$

$\Rightarrow$

For $x(n)$ to be periodic.

$$\Omega_0 N = 2\pi m$$

$$\boxed{\frac{\Omega_0}{2\pi} = \frac{m}{N}}$$; A Rational number (Ratio of integers)

Consider the following signals, determine whether each $x(n)$ is periodic & if it is periodic find its fundamental period.

1) $x(n) = 5\sin(2n)$

2) $x(n) = 5\cos(0.2\pi n)$

3) $x(n) = 5\cos(6\pi n)$

4) $x(n) = 5\sin\left(\frac{6\pi}{35} n\right)$

Soln

1) $x(n) = 5\sin(2n)$

It is of the form

$x(n) = 5\sin\Omega_0 n$

$\Omega_0 = 2$

Condition for periodicity

$$\frac{\Omega_0}{2\pi} = \frac{m}{N}$$

$$\frac{m}{N} = \frac{2}{2\pi} = \frac{1}{\pi}$$

which is not a ratio of integer. ∴ $x(n)$ is not a periodic.

2) $x(n) = 5\cos(0.2\pi n)$

$x(n) = 5\cos\Omega_0 n$

$\Omega_0 = 0.2\pi$

$$\frac{m}{N} = \frac{\Omega_0}{2\pi}$$

$$\frac{m}{N} = \frac{0.2\pi}{2\pi} = \frac{1}{5}\frac{1}{2}$$

$$\frac{m}{N} = \frac{1}{10}$$ $N = 10$ samples

Note: $N = 10m$

for $m = 1\ 2\ 3 \ldots$

The smallest value of N is the fundamental period obtained with $m=1$

∴ $N = 10(1)$

$\boxed{N = 10}$ samples.

3) $x(n) = 5\cos 6\pi n$

$= 5\cos\Omega_0 n$

$$\frac{\Omega_0}{2\pi} = \frac{m}{N} = \frac{6\pi}{2\pi} = \frac{6}{2} = \frac{3}{1}$$

$$\frac{\Omega_0}{2\pi} = \frac{3}{1} = \frac{m}{N}$$ Ratio of integers.

∴ $x(n)$ is periodic. $N=1$

Note $3N = m$

$N = \frac{m}{3}$

with $m = 3, 6\ 9 \ldots$

$N = 1\ 2\ 3 \ldots$

Fundamental period is smallest value of N

$\boxed{N = 1}$ sample.

4) $x(n) = 5\sin\left(\frac{6\pi n}{35}\right)$

$= 5\sin(\Omega_0 n)$

$\Omega_0 = \frac{6\pi}{35}$

$\frac{\Omega_0}{2\pi} = \frac{m}{N}$

$\frac{m}{N} = \frac{6\pi}{35} \cdot \frac{1}{2\pi} = \frac{3}{35}$

; it is a ratio of integers

$\therefore$ $x(n)$ is periodic.

$\frac{m}{N} = \frac{3}{35}$

$\boxed{N = 35}$ samples.

Note:

$N = \frac{35m}{3}$

For $m = 3, 6, 9, 12$ ...

$N = 35, 70, 105$ ...

Fundamental period is smallest value of N i.e. 35.

5) $x(n) = (-1)^n$ is periodic?

soln: sketch the signal.

$n = 0$ $x(0) = (-1)^0 = 1$

1 $x(1) = (-1)^1 = -1$

2 $x(2) = (-1)^2 = 1$

3 $x(3) = (-1)^3 = -1$

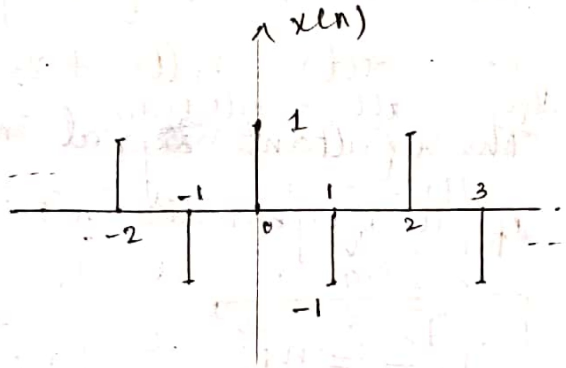
⋮

$n = -1$ $x(-1) = (-1)^{-1} = -1$

-2 $x(-2) = (-1)^{-2} = 1$

-3 $x(-3) = (-1)^{-3} = -1$

⋮



The signal $x(n)$ is repeating $\therefore$ it is periodic with fundamental period $N = 2$ samples.

6) $x(n) = (-1)^{n^2}$

$n = 0$ $x(0) = (-1)^{0^2} = 1$

1 $x(1) = (-1)^{1^2} = -1$

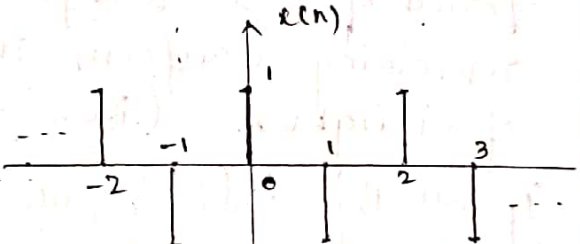
2 $x(2) = (-1)^{2^2} = 1$

3 $x(3) = (-1)^{3^2} = -1$

⋮

-1 $x(-1) = (-1)^{(-1)^2} = -1$

-2 $x(-2) = (-1)^{(-2)^2} = 1$



It is periodic $\boxed{N = 2}$ Samples.

CONDITION for SUM of 'm' periodic signals to be periodic

Consider 'm' periodic signals $x_1(t), x_2(t), \ldots x_m(t)$ with fundamental period $T_1, T_2 \ldots T_m$.

Let $x(t)$ be the sum of all those 'm' signals.

i.e., $x(t) = x_1(t) + x_2(t) + \ldots + x_m(t)$.

Since $x_1(t), x_2(t) \ldots x_m(t)$ are periodic, we can write

$x(t) = x_1(t+n_1T_1) + x_2(t+n_2T_2) + \ldots + x_m(t+n_mT_m)$

The sum signal $x(t)$ is periodic with fundamental period T_0 only if its fundamental period is equal to that of individual signals i.e.,

$\boxed{n_1\ n_2 \ldots n_m \text{ are integers}}$

$$T_0 = T_1 n_1 = T_2 n_2 = \ldots = T_m n_m$$

OR

$$\boxed{\frac{T_1}{T_2} = \frac{n_2}{n_1}};\quad \boxed{\frac{T_1}{T_3} = \frac{n_3}{n_1}};\quad \ldots \quad \boxed{\frac{T_1}{T_m} = \frac{n_m}{n_1}}$$

$$\boxed{\frac{T_1}{T_i} = \frac{n_i}{n_1}} \qquad 2 \le i \le m$$

The fundamental period of sum signal is

$$\boxed{T_0 = T_i n_i}$$

For $x(t)$ to be periodic the ratio of fundamental period of first waveform to the fundamental period of remaining waveform in the sum must be ratio of integers. (i.e., rational number)

If all the ratio result in ratio of integers, then the fundamental period of resulting signal is equal to

$$\boxed{T_0 = T_i n_i}$$

Determine whether or not the following signals are periodic.

1) $x(t) = \cos t + \sin\sqrt{2}t$

2) $x(n) = \cos\left(\frac{n\pi}{12}\right) + \sin\left(\frac{n\pi}{18}\right)$

3) $x(n) = \cos\left(\frac{n\pi}{12}\right)\sin\left(\frac{n\pi}{18}\right)$

Soln

$x(t) = x_1(t) + x_2(t)$

$x_1(t) = \cos t = \cos\omega_1 t$

$x_2(t) = \sin\sqrt{2}t = \sin\omega_2 t$

$\omega_1 = 1$; $\omega_1 = 2\pi f_1$

$\omega_1 = \frac{2\pi}{T_1}$; $\boxed{T_1 = \frac{2\pi}{\omega_1}}$

$\omega_2 = \sqrt{2}$ $\omega_2 = 2\pi f_2$

$\omega_2 = \frac{2\pi}{T_{02}}$ $\boxed{T_2 = \frac{2\pi}{\omega_2}}$

$T_1 = \frac{2\pi}{1} = 2\pi$ sec

$T_2 = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ sec.

$$\frac{T_1}{T_2} = \frac{2\pi}{\sqrt{2}\pi} = \frac{\sqrt{2}}{1} = \frac{n_2}{n_1}$$

which is not a ratio of integers

$\therefore$ $x(t)$ is not periodic.

Note

For DTS

$$\boxed{\frac{N_1}{N_i} = \frac{n_i}{n_1}}$$

$2 \le i \le m$

Fundamental period is given as

$N_0 = N_i n_i$

2) $x(n) = x_1(n) + x_2(n)$

$x_1(n) = \cos\frac{n\pi}{12}$

$= \cos\Omega_1 n$

$\Omega_1 = \frac{\pi}{12}$

$x_2(n) = \sin\frac{n\pi}{18}$

$= \sin\Omega_2 n$

$\Omega_2 = \frac{\pi}{18}$

For $x_1(n)$ to be periodic

$\frac{\Omega_1}{2\pi}$ must be rational

$$\frac{\Omega_1}{2\pi} = \frac{\pi}{12}\cdot\frac{1}{2\pi} = \frac{1}{24}$$

$$= \frac{m}{N_1}$$

$$\frac{m}{N_1} = \frac{1}{24} \qquad \boxed{N_1 = 24}$$

Note:

$N_1 = 24\,m$

For $m = 1, 2, 3 \ldots$

$N_1 = 24, 48 \ldots$

$\boxed{N_1 = 24}$ with $m = 1$

$$\frac{\Omega_2}{2\pi} = \frac{\pi}{18}\,\frac{1}{2\pi} = \frac{1}{36}$$

$$\frac{1}{36} = \frac{m}{N_2}$$

$\boxed{N_2 = 36}$

$N_2 = 36\,m \qquad m = 1\ 2\ 3 \ldots$

$N_2 = 36, 72, \ldots$

Now For $x(n)$ to be periodic $\frac{N_1}{N_2}$ must be rational

$$\frac{N_1}{N_2} = \frac{24}{36} = \frac{2}{3} = \frac{n_2}{n_1}$$

The fundamental period of $x(n)$

$N_0 = N_1 n_1 = N_2 n_2$

$N_0 = 24 \times 3$ or 36×2

$\boxed{N_0 = 72}$

$x(n)$ is periodic with fundamental period 72 samples.

3) $x(n) = \cos\left(\frac{n\pi}{12}\right)\sin\left(\frac{n\pi}{18}\right)$

Soln

Expressing as a sum.

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos\left(\frac{n\pi}{12}\right)\sin\left(\frac{n\pi}{18}\right)$$

$$= \frac{1}{2}\left[\sin n\pi\left(\frac{1}{12}+\frac{1}{18}\right) - \sin n\pi\left(\frac{1}{12}-\frac{1}{18}\right)\right]$$

$$= \frac{1}{2}\left[\sin\left(n\pi\,\frac{5}{36}\right) - \sin\left(\frac{n\pi}{36}\right)\right]$$

$$x(n) = \frac{1}{2}\left[\sin\left(\frac{5}{36}n\pi\right)\right] - \frac{1}{2}\sin\left(\frac{n\pi}{36}\right)$$

$$= x_1(n) - x_2(n)$$

$$\Omega_1 = \frac{5\pi}{36} \qquad \Omega_2 = \frac{\pi}{36}$$

For $x_1(n)$ to be periodic

$$\frac{\Omega_1}{2\pi} = \frac{5\pi}{36} \times \frac{1}{2\pi}$$

$$\frac{\Omega_1}{2\pi} = \frac{5}{72} \qquad \frac{m}{N_1}$$

$\boxed{N_1 = 72}$ samples

$$\frac{\Omega_2}{2\pi} = \frac{\pi}{36} \times \frac{1}{2\pi}$$

$$= \frac{1}{72} = \frac{m}{N_2}$$

$x_2(n)$ is periodic with fundamental period 72

$\boxed{N_2 = 72}$ samples

$$\frac{N_1}{N_2} = \frac{72}{72} = \frac{1}{1} = \frac{n_2}{n_1}$$

$$N_0 = N_1 n_1 = N_2 n_2$$
$$72 \times 1 \quad 72 \times 1$$

$N_0 = 1 \times 72$

$N_0 = 72$ samples

Qu (4)
$x(t)$ is sum of $x_1(t)$, $x_2(t)$ & $x_3(t)$ which periodic with fundamental period 1.2, 0.8 & 1.04. Determine whether $x(t)$ is periodic or not. If it is periodic find the fundamental period.

Soln. $T_1 = 1.2$ $T_2 = 0.8$ $T_3 = 1.04$

$$\frac{T_1}{T_i} = \frac{n_i}{n_1} \quad 2 \leq i \leq m$$

$m = 3$

$$\frac{T_1}{T_2} = \frac{n_2}{n_1}$$

$$\frac{1.2}{0.8} = \frac{n_2}{n_1}$$

$$\frac{12}{8} = \frac{n_2}{n_1}$$

$$\frac{T_1}{T_2} = \frac{3}{2} = \frac{n_2}{n_1} = \text{rational.}$$

$$\frac{T_1}{T_3} = \frac{1.2}{1.04} = \frac{120}{104}$$

$$\frac{T_1}{T_3} = \frac{30}{26} = \frac{15}{13} = \frac{n_3}{n_1}$$

Both the ratios are rational number ∴ the sum signal is periodic.

To find fundamental period

$T_0 = T_{01} n_1$ OR ~~$T_0 = T_{02} n_2$~~ ~~OR~~ $T_0 = T_{03} n_3$

$T_0 = T_1 n_1$ OR $T_0 = T_2 n_2$ OR $T_0 = T_3 n_3$

$n_1 = \text{LCM}(2, 13)$ $n_2 = \text{LCM}(3, 13)$ $n_3 = \text{LCM}(10, 15)$

$n_1 = 26$ $n_2 = 39$ $n_3 = 30$

$T_0 = 1.2 \times 26$ $T_0 = 0.8 \times 39$ $T_0 = 1.04 \times 30$

$T_0 = 31.2$ sec $T_0 = 31.2$ $T_0 = 31.2$

5) Determine whether the CTS $x(t) = (\cos 2\pi t)^2$ is periodic or not. If periodic, find the fundamental period.

Soln

$x(t) = \cos^2 2\pi t$

$= \frac{1}{2}[1 + \cos 4\pi t]$

Recall: $\cos^2\theta = \frac{1}{2}[1 + \cos 2\theta]$

$x(t) = \frac{1}{2} + \frac{1}{2}\cos 4\pi t$

Generally sinusoidal signals are periodic in nature.

$T = \frac{2\pi}{\omega}$ sec

$T = \frac{2\pi}{4\pi} = \frac{1}{2}$ sec

$\boxed{T = 0.5 \text{ sec}}$

Verification:

$x(t) = \cos^2 2\pi t$

$t \rightarrow t + T = t + 0.5$

$x(t+0.5) = \cos^2[2\pi(t+0.5)]$

$= \cos^2[2\pi t + \pi]$

$= \frac{1}{2}[1 + \cos(4\pi t + 2\pi)]$

$= \frac{1}{2}[1 + \cos 4\pi t] = x(t)$

$\therefore$ it is periodic.

6) a) $x(t) = \cos(t + \pi/4)$

It is periodic with $\omega = 1$ rad/sec.

$T = \frac{2\pi}{\omega} = \frac{2\pi}{1}$

$T = 2\pi$ sec

$\Rightarrow x(t) = \cos(\frac{\pi}{3}t) + \sin(\frac{\pi}{4}t)$

$= x_1(t) + x_2(t)$

$\omega_1 = \pi/3$ $\qquad \omega_2 = \frac{\pi}{4}$

$T_1 = \frac{2\pi}{\omega_1}$ $\qquad T_2 = \frac{2\pi}{\omega_2}$

$T_1 = \frac{2\pi}{\frac{\pi}{3}}$ $\qquad T_2 = \frac{2\pi}{\pi/4}$

$T_1 = 6$ sec $\qquad T_2 = 8$ sec.

$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4} = \frac{n_2}{n_1}$

$n_2 = 3 \qquad n_1 = 4$

$\therefore x(t)$ is periodic.

Fundamental period is

LCM (T_1, T_2) OR

LCM $(6, 8)$

$\boxed{T = 24 \text{ sec}}$

Note

$T = T_1 n_1$
$= 6 \times 4$
$= 24$ sec

OR

$T = T_2 n_2$
$= 8 \times 3$
$= 24$ sec

6(b)

$x(n) = \cos^2\left(\frac{\pi}{8} n\right)$

$x(n) = \frac{1}{2}\left[1 + \cos 2 \frac{\pi}{8} n\right]$

$= \frac{1}{2} + \frac{1}{2} \cos \frac{\pi}{4} n$

$\frac{\Omega}{2\pi} = \frac{\frac{\pi}{4}}{2\pi} = \frac{1}{8} = \frac{m}{N}$

$\therefore$ it is periodic.

$N = 8$ samples.

---

(6) c)

$\Rightarrow x(n) = \sin\left(\frac{\pi n}{3}\right) \cdot \cos\left(\frac{\pi}{5} n\right)$

Soln

$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$x(n) = \frac{1}{2}\left[\sin\left(\frac{1}{3}+\frac{1}{5}\right)\pi n + \sin\left(\frac{1}{3}-\frac{1}{5}\right)\pi n\right]$

$= \frac{1}{2}\left[\sin \frac{8\pi}{15} n + \sin \frac{2\pi}{15} n\right]$

$= \frac{1}{2} \sin \frac{8\pi}{15} n + \frac{1}{2} \sin \frac{2\pi}{15} n$

$x_1(n)$ | $x_2(n)$

$\Omega_1 = \frac{8\pi}{15}$ | $\Omega_2 = \frac{2\pi}{15}$

$\frac{\Omega_1}{2\pi} = \frac{8\pi}{15} \frac{1}{2\pi}$ | $\frac{\Omega_2}{2\pi} = \frac{2\pi}{15} \frac{1}{2\pi}$

$= \frac{4}{15} = \frac{m}{N_1}$ | $\frac{1}{15} = \frac{m}{N_2}$

$x_1(n)$ is periodic | $x_2(n)$ is periodic.

$N_1 = 15$ samples | $N_2 = 15$ samples

Now

$\frac{N_1}{N_2} = \frac{15}{15} = \frac{1}{1}$; Rational

$\therefore x(n)$ is periodic with fundamental period $N = \text{LCM}(N_1, N_2)$

$N = \text{LCM}(15, 15)$

$N = 15$ samples.

$\Rightarrow x(n) = 5 \cos 0.2\pi n$

$\Omega = 0.2\pi$

$\frac{\Omega}{2\pi} = \frac{0.2\pi}{2\pi} = \frac{1}{5} \frac{1}{2} = \frac{1}{10}$

a Rational.

$\frac{1}{10} = \frac{m}{N}$

$N = 10$ sec

$\Rightarrow$ (7)

$x(n) = \cos\left(\frac{\pi}{8} n\right) \cos\left(\frac{\pi}{3} n\right)$

Recall:

$\cos A \cos B =$

$\frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

$$x(n) = \frac{1}{2}\left[\cos\left(\frac{\pi}{8}+\frac{\pi}{3}\right)n + \cos\left(\frac{\pi}{8}-\frac{\pi}{3}\right)n\right]$$

$$= \frac{1}{2}\left[\cos\left(\frac{11\pi}{24}n\right) + \cos\left(\frac{-5}{24}\right)\pi n\right]$$

$$= \frac{1}{2}\cos\left(\frac{11\pi}{24}n\right) + \cos\frac{5\pi}{24}n$$

$$= x_1(n) + x_2(n)$$

$$\Omega_1 = \frac{11\pi}{24} \qquad \Omega_2 = \frac{5\pi}{24}$$

$$\frac{\Omega_1}{2\pi} = \frac{11\pi}{24}\cdot\frac{1}{2\pi} = \frac{11}{48} = \frac{m}{N_1}$$

$\therefore$ $x_1(n)$ periodic

$N_1 = 48$ samples.

$$\frac{\Omega_2}{2\pi} = \frac{5\pi}{24}\cdot\frac{1}{2\pi} = \frac{5}{48} = \frac{m}{N_2}$$

$\therefore$ $x_2(n)$ is periodic.

$N_2 = 48$ samples.

$$\frac{N_1}{N_2} = \frac{48}{48} = \frac{1}{1}$$ a Rational.

$\therefore$ $x(n)$ is periodic with fundamental period.

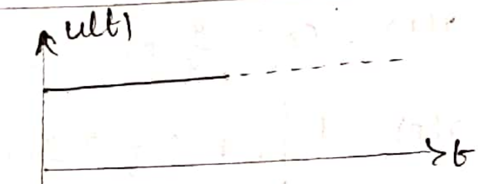
$N = LCM(N_1, N_2)$

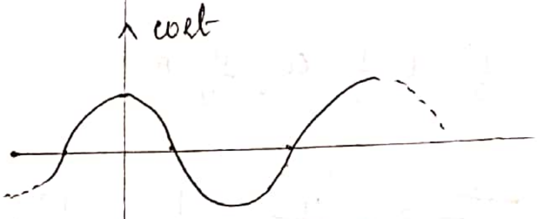
$N = LCM(48, 48)$

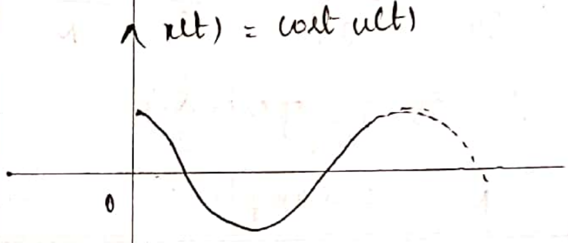
$N = 48$ samples.

(8) $\Rightarrow$ check if periodic.

$x(t) = \cos t \, u(t)$



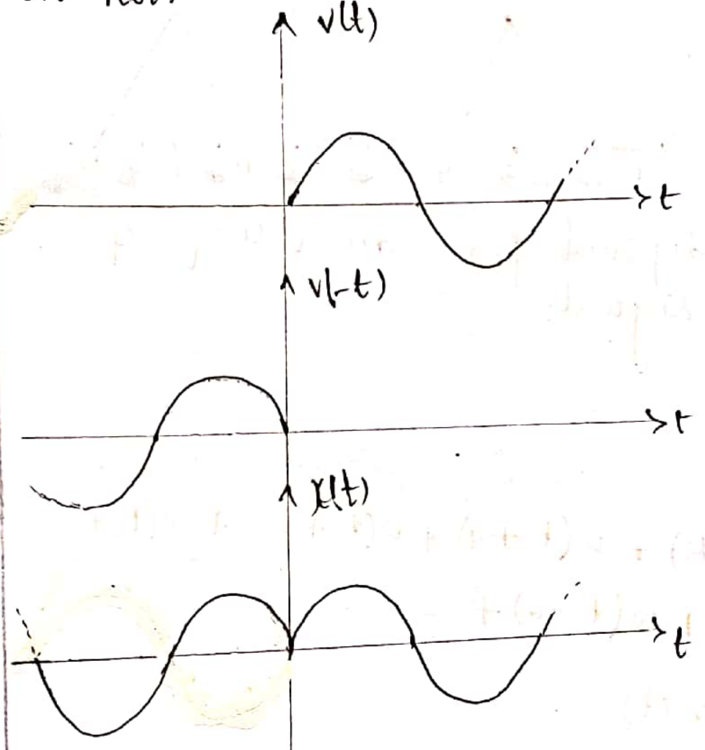




Since the signal $x(t)$ is discontinuous, it is not periodic. But, as the signal exists for +ve values of t, we can say that it is conditionally periodic.

(9) $\Rightarrow$ $x(t) = \sin t \, u(t)$. check if periodic.

Given $v(t) = \sin t\, u(t)$. check $x(t) = v(t) + v(-t)$ is periodic or not.



As there is a discontinuity at origin $x(t)$ is not periodic.

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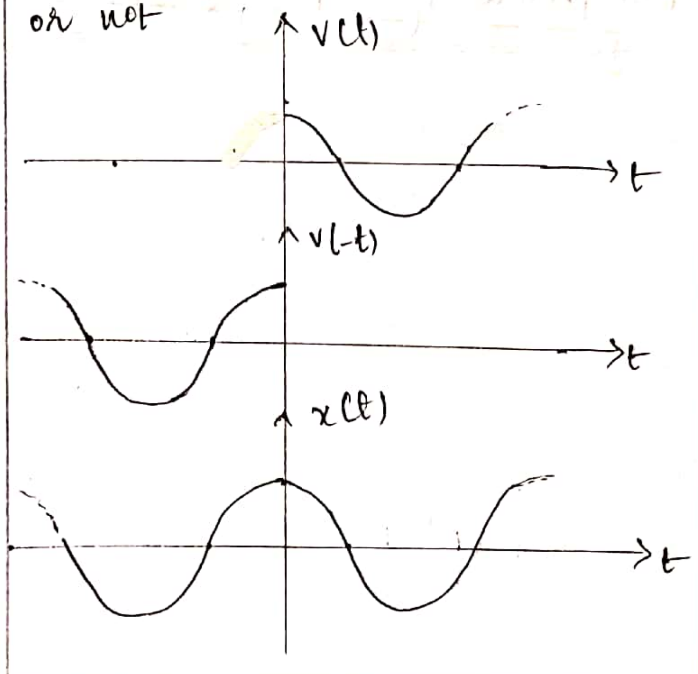
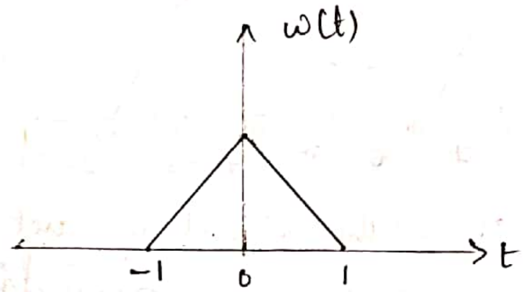
(10) => Given $v(t) = \cos t\, u(t)$ check $x(t) = v(t) + v(-t)$ is periodic or not



The signal $x(t)$ is periodic

=> (11) Given



check if the following signals are periodic.

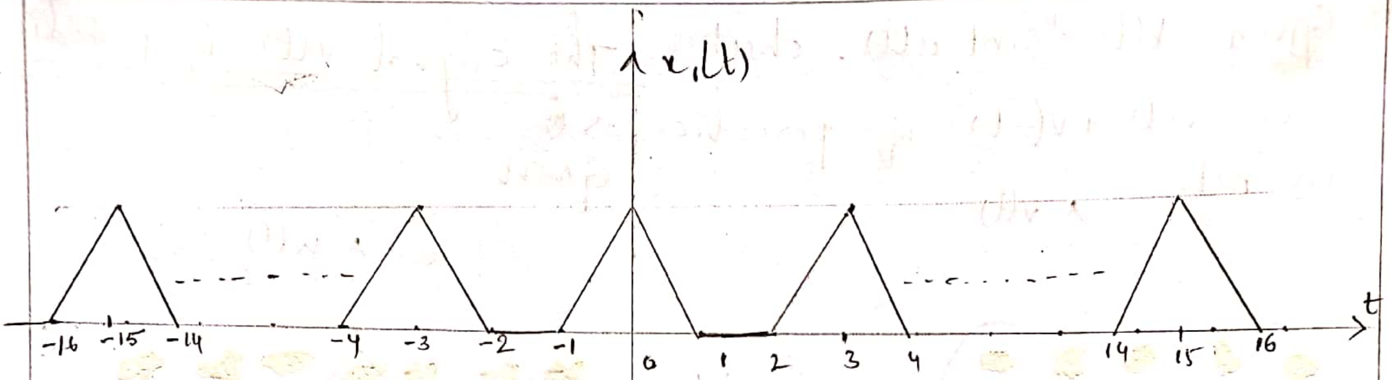
a) $x_1(t) = \sum_{k=-5}^{5} w(t-3k)$

b) $x_2(t) = \sum_{k=-10}^{10} w(t-2k)$

soln:

a) $x_1(t) = \sum_{k=-5}^{5} w(t-3k)$

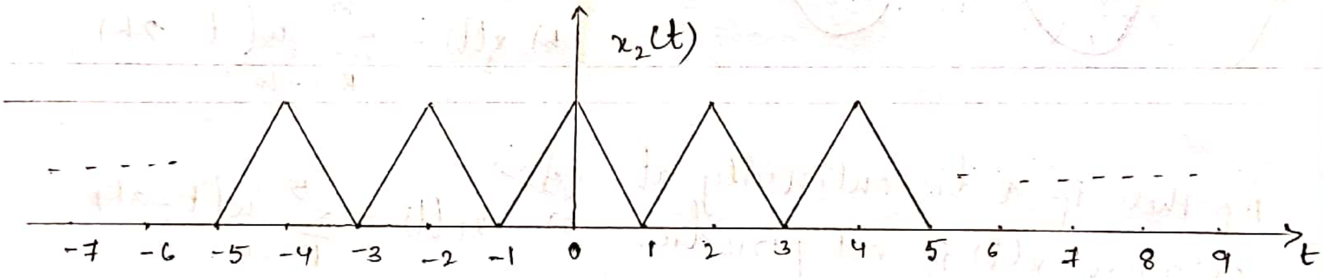
$= w(t+15) + w(t+12) + w(t+9) + w(t+6) + w(t+3) + w(t) + w(t-3) + \dots + w(t-15)$



Since the signal is not defined for all values of t it is not a periodic signal.

b) $x_2(t) = \sum_{k=-\infty}^{\infty} w(t-2k)$

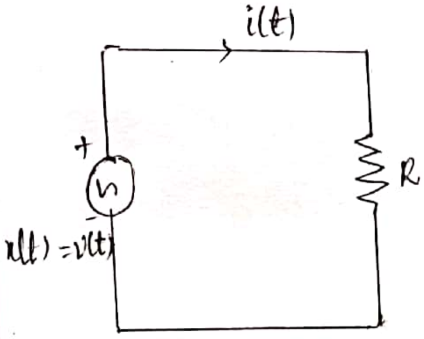
$= \text{------} + w(t+6) + w(t+4) + w(t+2) + w(t) + w(t-2) + w(t-4) + w(t-6) + \text{------}$



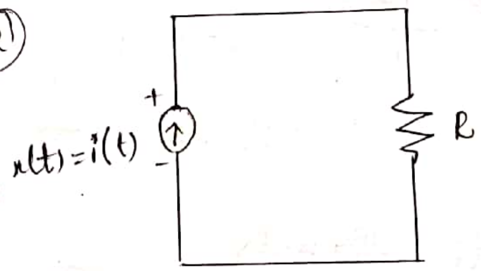
The signal is periodic as it is defined for all values of t. & the fundamental period is $T = 2$ sec.

$\Rightarrow$

ENERGY AND POWER SIGNALS



OR



Power dissipated in the resistor R is

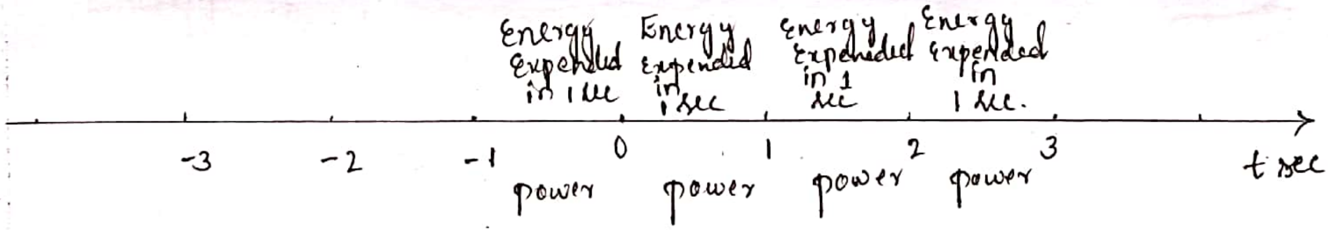
$$p(t) = \frac{v^2(t)}{R} = \frac{x^2(t)}{R}$$

$$p(t) = i^2(t) R = x^2(t) R$$

on normalizing the resistor $R = 1$

$$p(t) = x^2(t)$$

$$p(t) = x^2(t)$$



$$\text{Power} = \frac{\text{Energy expended in 1 sec}}{1 \text{ sec.}}$$

OR

$$\left.\begin{array}{c}\text{instantaneous}\\ \text{Power}\end{array}\right\} = \frac{\text{Energy expended at time instant } t}{\text{time instant } 't'} = p(t)$$

Adding all the power values from $t = -\infty$ to ∞ we get total Energy.

ENERGY & POWER FORMULAE

ENERGY	Non-periodic	CTS	$E = \int_{t=-\infty}^{\infty} x^2(t)\,dt$ OR $E = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)\,dt$
		DTS	$E = \sum_{n=-\infty}^{\infty} \lvert x(n)\rvert^2$.
	Periodic	CTS	— — —
		DTS	— — —
POWER	Periodic	CTS	$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$ $P = \frac{1}{T} \int_{0}^{T} x^2(t)\,dt$
		DTS	$P = \frac{1}{N} \sum_{n=0}^{N-1} \lvert x(n)\rvert^2$
	Non-periodic	CTS	$P = \lim_{T\to\infty} \frac{1}{T} \int_{t=-T/2}^{T/2} x^2(t)\,dt$
		DTS	$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \lvert x(n)\rvert^2$

Note: Power $= \frac{\text{Energy}}{\text{time}}$

ENERGY AND POWER SIGNAL

A signal is referred to as energy signal if & only if it satisfies the condition

$0 < E < \infty$ i.e, finite value.

$E \rightarrow$ calculated value of energy.

A signal is referred to as power signal if and only if the average power of the signal satisfies the condition $\boxed{0 < P < \infty}$ a finite value.

Formulae concerned with Energy Signal

CTS:

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

DTS:

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$ If complex take mo

Formulae concerned with power signal

For periodic & conditionally periodic signal

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(n)$$

For periodic signal

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x^2(n)$$

Usually all periodic & random signals are power signals where as deterministic & non periodic signals Energy signals.

The Energy & power signals are mutually exclusive.

In particular an Energy signal has zero avg power where as a power signal has infinite Energy.

Categorize the following signals as Energy or power signals & find the energy or power of the signal.

1)

$$x(t) = \begin{cases} t & 0 \le t \le 1 \\ 2-t & 1 < t < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Soln.

Since the signal is non periodic. it is an Energy signal.

To find energy;

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_0^1 t^2\,dt + \int_1^2 (2-t)^2\,dt + 0$$

$$= \left[\frac{t^3}{3}\right]_0^1 + \left[\frac{(2-t)^3}{-3}\right]_1^2$$

$$= \frac{1}{3}(1^3-0) - \frac{1}{3}\left[(2-2)^3-(2-1)^3\right]$$

$$= \frac{1}{3} - \frac{1}{3}(0-1)$$

$$= \frac{2}{3} \text{ joule}.$$

$0 < E < \infty$.

(2)

$$x(n) = \begin{cases} n & 0 \le n \le 5 \\ 10-n & 6 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$$

Since the signal is non periodic, it is an energy signal.

To find energy

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=0}^{5} n^2 + \sum_{n=6}^{10} (10-n)^2 + 0$$

$$= 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2$$

$$= 1 + 4 + 9 + 16 + 25 + 16 + 9 + 4 + 1$$

$= \underline{85}$ joule.

3) $$x(t) = \begin{cases} 5\cos\pi t & -1 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Since the signal is non periodic, it is a energy signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-1}^{1} (5\cos\pi t)^2\,dt$$

$$= 25 \int_{-1}^{1} \frac{1}{2}(1+\cos 2\pi t)\,dt$$

$$= \frac{25}{2}\left[t + \frac{\sin 2\pi t}{2\pi}\right]_{-1}^{1}$$

$$= \frac{25}{2}\left[1-(-1) + \frac{\sin 2\pi}{2\pi} - \frac{\sin(-2\pi)}{2\pi}\right]$$

$$= \frac{25}{2}[2]$$

$E = 25$ joules.

Since the signal energy is finite, it is energy signal.

4) $$x(n) = \begin{cases} \cos\pi n & -4 < n < 4 \\ 0 & \text{elsewhere.} \end{cases}$$

Since the signal is non periodic

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$E = \sum_{-4}^{4} \cos^2 \pi n$$

$\cos\pi = -1$

we can write

$$\cos\pi n = (-1)^n$$

$$E = \sum_{-4}^{4}\left[(-1)^n\right]^2$$

$$= \sum_{-4}^{4}\left[(-1)^2\right]^n$$

$$= \sum_{-4}^{4} 1^n$$

$$= 1^{-4} + 1^{-3} + 1^{-2} + 1^{-1} + 1^0 + 1^1 + 1^2 + 1^3 + 1^4$$

$= 9$ joule.

5) $x(n) = 2^n u(n)$

soln: It is nonperiodic.

$$E = \sum_{n=0}^{\infty} (2^n)^2$$

$$= \sum_{n=0}^{\infty} (2^2)^n = \sum_{n=0}^{\infty} 4^n$$

$= \infty$ so it is not energy signal.

6) $x(n) = 2^n u(-n)$

$$E = \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [2^n u(-n)]^2$$

$$= \sum_{n=-\infty}^{0} [2^n u(-n)]^2 + \underbrace{\sum_{n=1}^{\infty} [2^n u(-n)]^2}_{0}$$

$$= \sum_{n=-\infty}^{0} (2^2)^n$$

$$= \sum_{n=-\infty}^{0} 4^n$$

Note

i) $\sum_{k=0}^{N} 1^k = N+1$ samples

ii) $\sum_{k=1}^{N} 1^k = N$ samples.

iii) $\sum_{k=-N}^{N} A = A(2N+1)$ samples.

iv) $\sum_{k=0}^{n} a^k = \frac{1-a^{n+1}}{1-a}$ $a \neq 1$

v) $\sum_{k=0}^{\infty} a^k = \frac{1}{1-a}$ $|a| < 1$

vi) $\sum_{k=0}^{\infty} a^k =$ diverges $|a| > 1$

vii) $\sum_{k=0}^{N} k = \frac{N(N+1)}{2}$

viii) $\sum_{k=0}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}$

ix) $\sum_{i=m}^{n} a^i = \frac{a^{n+1} - a^m}{a-1}$ $a \neq 1$.

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put $n = -n$

$n = -\infty \rightarrow n' = \infty$

$n = 0 \rightarrow n = 0$

$$E = \sum_{n=\infty}^{0} 4^{-n} = \sum_{n=0}^{\infty} 4^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n \qquad |a| < 1$$

$$= \frac{1}{1-\frac{1}{4}}$$

$$= \frac{1}{\frac{4-1}{4}} = \frac{4}{3} \text{ joule.}$$

since $0 < E < \infty$, it is Energy signal.

---

(7) $\Rightarrow x(n) = 8(0.5)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=-\infty}^{\infty} [8(0.5)^n u(n)]^2$$

$$= 64 \sum_{n=0}^{\infty} (0.5)^{2n} 1^2$$

$$= 64 \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= 64 \cdot \frac{1}{1-\frac{1}{4}} = 64 \frac{4}{4-1}$$

$$E = 64 \times \frac{4}{3}$$

$$E = 85.333 \text{ joules.}$$

---

(8) $\Rightarrow x(n) = \cos \pi n \quad -\infty < n < \infty$

Soln: First check if periodic.

$$\frac{\Omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2} = \frac{m}{N}; \text{ a}$$

rational. so it is periodic with period $N = 2$

To find power

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2(n)$$

$$= \frac{1}{2} \sum_{n=0}^{2-1} \cos^2 \pi n$$

$$= \frac{1}{2} \sum_{n=0}^{1} [(-1)^n]^2$$

$$= \frac{1}{2} \sum_{n=0}^{1} 1^n$$

$$= \frac{1}{2}[1+1] = 1$$

$$P = 1 \text{ watt.}$$

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(9) $x(n) = e^{jn\pi}$

$$\frac{\Omega}{2\pi} = \frac{\pi}{2\pi} = \frac{1}{2}\text{; Rational.}$$

It is periodic with period $N = 2$ samples.

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x^2(n)$$

Since $x(n)$ given is a complex take magnitude.

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$P = \frac{1}{N}\sum |e^{jn\pi}|^2$$

$$= \frac{1}{N}\sum_{0}^{N-1} 1^2$$

$\therefore e^{jn\pi} = \cos n\pi + j\sin n\pi$

$$|e^{jn\pi}| = \sqrt{\cos^2 n\pi + \sin^2 n\pi}$$

$$|e^{jn\pi}| = 1$$

$$P = \frac{1}{2}\sum_{0}^{2-1} 1 = \frac{1}{2}\sum_{0}^{1} 1$$

$$= \frac{1}{2}(2)$$

$P = 1$ watt; which is finite $\therefore$ it is power signal.

$\because \sum_{k=0}^{N} = N+1$

(10) $\Rightarrow x(t) = u(t)$

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{-\infty}^{\infty} 1^2\,dt = \int_{0}^{\infty} 1\,dt$$

$$= [t]_{0}^{\infty} = \infty - 0 = \infty$$

$\therefore$ it is not an Energy signal.

Now finding power.

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 1^2\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}[t]_{-T/2}^{T/2}$$

$$= \lim_{T\to\infty} \frac{1}{T}\left[\frac{T}{2} - \left(-\frac{T}{2}\right)\right]$$

$$= \lim_{T\to\infty} \frac{1}{T}\,T$$

$P = 1$ watt

$\therefore$ ~~Unit step function is a power signal.~~

(11) $\Rightarrow$ $x(t) = \left[2\cos^2\left(\frac{\pi t}{2}\right) - 1\right] \sin \pi t \cos \pi t$ is periodic ? Find fundamental period if it is periodic.

$$x(t) = \underbrace{\left[2\cos^2\left(\frac{\pi t}{2}\right) - 1\right]} \sin \pi t \cos \pi t$$

$$x(t) = \cos \pi t \sin \pi t \cos \pi t$$

$$\because \cos\theta = 2\cos^2(\theta/2) - 1$$

$$x(t) = \frac{1}{2}\cos \pi t\left[\sin(\pi t + \pi t) + \sin(\pi t - \pi t)\right]$$

$$x(t) = \frac{1}{2}\cos \pi t\left[\sin 2\pi t\right]$$

$$= \frac{1}{2}\,\frac{1}{2}\left[\sin(2\pi + \pi)t + \sin(2\pi - \pi)t\right]$$

$$= \frac{1}{4}\left[\sin 3\pi t + \sin \pi t\right]$$

$$= \frac{1}{4}\sin 3\pi t + \frac{1}{4}\sin \pi t$$

$$x_1(t) + x_2(t)$$

$\omega_1 = 3\pi$ $\qquad$ $\omega_2 = \pi$

$T_1 = \frac{2\pi}{\omega_1}$ $\qquad$ $T_2 = \frac{2\pi}{\omega_2}$

$T_1 = \frac{2\pi}{3\pi}$ $\qquad$ $T_2 = \frac{2\pi}{\pi}$

$T_1 = \frac{2}{3}$ sec $\qquad$ $T_2 = 2$ sec

$$\frac{T_1}{T_2} = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3} = \frac{n_2}{n_1}$$

Recall:

$\sin A \cos B =$

$= \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$

$\therefore$ it is periodic

Fundamental period.

$n_2 = 3$, $n_2 = 1$

$T = T_1 n_1$, OR $T = T_2 n_2$

$T = \frac{2}{3} \times 3$ $\qquad$ $T = 2 \times 1$

$= \underline{\underline{2}}$ $\qquad$ $T = 2$

$T = \underline{2 \text{ sec}}$ OR

$T = LCM\left(\frac{2}{3}, 2\right)$

$\frac{2}{3}$ $\frac{4}{3}$ $\frac{6}{3}$ $\frac{8}{3}$ --- $LCM = 2$

2 4 6 8 10 12 14 16 18 ---

(12) $\Rightarrow$ ramp signal

$x(t) = r(t) = t\,u(t)$

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{0}^{\infty} t^2 \cdot 1^2\,dt$$

$$= \left[\frac{t^3}{3}\right]_0^{\infty}$$

$$= \underline{\underline{\infty}}$$

$\therefore$ it is not a Energy signal.

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} t^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{0}^{T/2} t^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[\frac{t^3}{3} \right]_0^{T/2}$$

$$= \lim_{T \to \infty} \frac{1}{T} \frac{1}{3} \left[\frac{T^3}{8} - 0 \right]$$

$$= \lim_{T \to \infty} \frac{T^2}{24}$$

$$= \infty$$

$\therefore$ It is neither Energy signal nor power signal.

(13) $\Rightarrow x(n) = r(n)$

$$= n\,u(n)$$

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$E = \sum_{n=0}^{\infty} n^2$$

$E = \infty$

$\therefore$ it is not a Energy signal.

To find power:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(n)$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{0}^{N} n^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \frac{N(N+1)(2N+1)}{6}$$

$$= \lim_{N \to \infty} \frac{N(N+1)}{6}$$

$P = \infty$

$\therefore$ it is neither Energy signal nor power signal.

Note:

$$\sum_{n=0}^{N} n^2 = \frac{N(N+1)(2N+1)}{6}$$

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⇒ 14

$x(t) = 5\cos\pi t + \sin\pi t \qquad -\infty < t < \infty$

Soln

checking for periodicity

$\omega_1 = \pi \qquad \omega_2 = \pi$

$T_1 = \frac{2\pi}{\omega_1} \qquad T_2 = \frac{2\pi}{\omega_2}$

$T_1 = \frac{2\pi}{\pi} = 2\text{ sec} \qquad T_2 = \frac{2\pi}{\pi} = 2\text{ sec.}$

$T = LCM(T_1, T_2)$

$\frac{T_1}{T_2} = \frac{1}{1}$ Rational ; it is periodic.

$\boxed{T = 2\text{ sec}}$

$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{2}\int_0^2 [5\cos\pi t + \sin\pi t]^2\,dt$$

$$= \frac{1}{2}\int_0^2 (25\cos^2\pi t + \sin^2\pi t + 10\cos\pi t\sin\pi t)\,dt$$

$$= \frac{25}{2}\int_0^2 \frac{1}{2}(1+\cos 2\pi t)\,dt + \frac{1}{2}\int_0^2 \frac{1}{2}(1-\cos 2\pi t)\,dt + \frac{10}{2}\int_0^2 \frac{1}{2}(\sin 2\pi t)\,dt$$

$$= \frac{1}{2}\frac{25}{2}\left[t + \frac{\sin 2\pi t}{2\pi}\right]_0^2 + \frac{1}{2}\frac{1}{2}\left[t - \frac{\sin 2\pi t}{2\pi}\right]_0^2 + \frac{5}{2}\left(-\frac{\cos 2\pi t}{2\pi}\right)_0^2$$

$$= \frac{25}{2}\frac{1}{2}[2+0-0-0] + \frac{1}{4}[2-0-0+0] + \frac{5}{2}\left(\frac{1-1}{2\pi}\right)$$

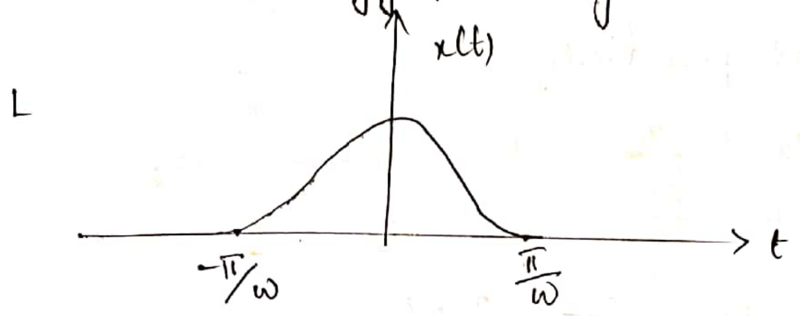
$$= \frac{25}{2} + \frac{1}{2} + 0$$

$$\frac{25+1}{2} = \frac{26}{2}$$

$\boxed{P = 13\text{ watt}}$

(15)

⇒ The raised consine function is as shown in figure. Determine the Energy of the signal.



$$x(t) = \begin{cases} \frac{1}{2}(\cos \omega t + 1) & -\pi/\omega \le t \le \pi/\omega \\ 0 & \text{otherwise.} \end{cases}$$

$$E = \int_{t=-\infty}^{t=\infty} x^2(t)\, dt$$

$$= \int_{-\pi/\omega}^{\pi/\omega} \frac{1}{4} (\cos \omega t + 1)^2\, dt$$

$$= \frac{1}{4} \int_{-\pi/\omega}^{\pi/\omega} [\cos^2 \omega t + 1 + 2\cos \omega t]\, dt$$

$$= \frac{1}{4} \int_{-\pi/\omega}^{\frac{\pi}{\omega}} \left\{ \frac{1}{2}(1 + \cos 2\omega t) + 1 + 2\cos \omega t \right\} dt$$

$$= \frac{1}{4} \frac{1}{2} \left(t + \frac{\sin 2\omega t}{2\omega} \right)_{-\pi/\omega}^{\pi/\omega} + \left(\frac{1}{4} t \right)_{-\pi/\omega}^{\pi/\omega} + \frac{1}{4} 2 \left(\frac{\sin \omega t}{\omega} \right)_{-\pi/\omega}^{\pi/\omega}$$

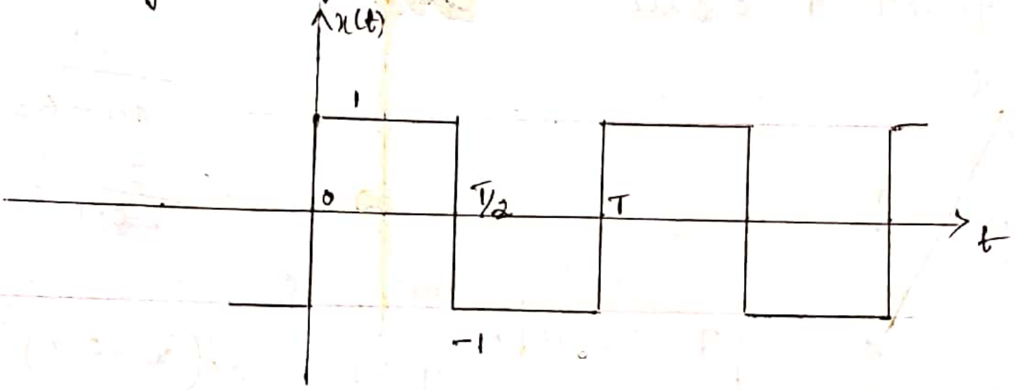
$$= \frac{1}{8}\left[\frac{\pi}{\omega} - \left(\frac{\pi}{\omega}\right) + \frac{\sin 2\omega\frac{\pi}{\omega} - \sin(-2\omega\frac{\pi}{\omega})}{2\omega}\right] + \frac{1}{4}\left[\frac{\pi}{\omega} - \left(\frac{\pi}{\omega}\right)\right]$$

$$+ \frac{1}{2}\left[\frac{\sin \omega\frac{\pi}{\omega} - \sin(-\omega\pi/\omega)}{\omega}\right] = \frac{1}{8}\left(\frac{2\pi}{\omega}\right) + \frac{2\pi}{4\omega} + 0 = \frac{3}{4}\frac{\pi}{\omega}$$

$$\boxed{E = \frac{3\pi}{4\omega}} \text{ watt}$$

Scanned by CamScanner

16. For the given signal calculate the average power.



Soln: From the given waveform

$$x(t) = \begin{cases} 1 & 0 < t \leq T/2 \\ -1 & T/2 \leq t \leq T \end{cases}$$

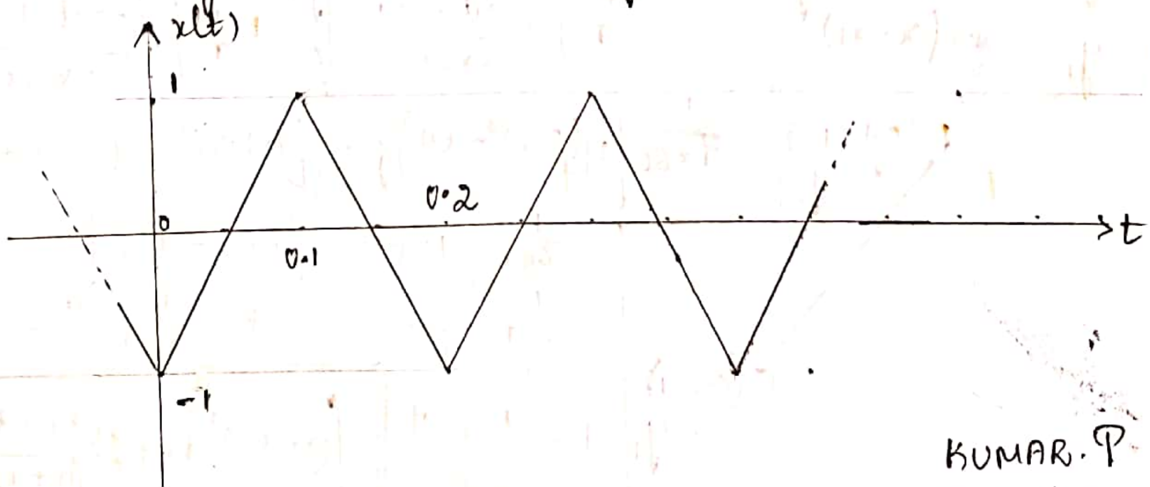
$$P = \frac{1}{T}\int_0^T x^2(t)\,dt$$

$$= \frac{1}{T}\int_0^{T/2} 1^2\,dt + \frac{1}{T}\int_{T/2}^{T} (-1)^2\,dt$$

$$= \frac{1}{T}\left[t\right]_0^{T/2} + \frac{1}{T}\left[t\right]_{T/2}^{T} = \frac{1}{T}\left[\frac{T}{2}\right] + \frac{1}{T}\left[T - \frac{T}{2}\right]$$

$$= \frac{1}{2} + \frac{1}{\cancel{T}}\left[\frac{\cancel{T}}{2}\right] = 1 \text{ watt} \qquad \boxed{P = 1\text{ watt}}$$

17. Find the average power of the signal.



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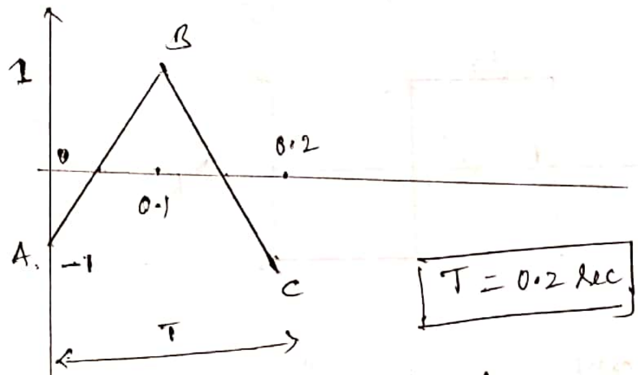
Soln

Consider one complete cycle.
Fundamental period $T = 0.2$ sec.



$\boxed{T = 0.2 \text{ sec}}$

Consider line segment AB.
It is written in slop intercept form as.

$y = mx + c$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$B(0.1, 1)$ (x_2 y_2)

$A(0, -1)$ (x_1 y_1)

$m = \dfrac{1+1}{0.1 - 0}$

$m = 20$; the slope.

$c = -1$; the y intercept

$y = 20x - 1$ (OR)

$x(t) = 20t - 1 \longrightarrow (1)$

Consider line segment BC

$y - y_1 = m(x - x_1)$

$B(0.1, 1)$ (x_2 y_2)

$C(0.2, -1)$ (x_1 y_1)

Slope, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{1+1}{0.1 - 0.2}$

$= \dfrac{2}{-0.1} = \underline{\underline{-20}}$

$m = -20$

$y + 1 = -20(x - 0.2)$

$y + 1 = -20x + 4$

$y = -20x + 3$

$x(t) = -20t + 3 \longrightarrow (2)$

from (1) & (2)

$$x(t) = \begin{cases} 20t - 1 & 0 \le t \le 0.1 \\ -20t + 3 & 0.1 \le t \le 0.2 \end{cases}$$

Power of one complete cycle

$$P = \frac{1}{T}\int_0^T x^2(t)\, dt$$

$$= \frac{1}{T}\int_0^{0.1} (20t-1)^2 dt + \frac{1}{T}\int_{0.1}^{0.2} (-20t+3)^2 dt$$

$$= \frac{1}{T}\left[\frac{(20t-1)^3}{3\times 20}\right]_0^{0.1} + \frac{1}{T}\left[\frac{(-20t+3)^3}{3\times -20}\right]_{0.1}^{0.2}$$

$$= \frac{1}{T\times 60}\left\{\left[(2-1)^3 - (-1)^3\right] - \left[(-4+3)^3 - (-2+3)^3\right]\right\}$$

$$= \frac{1}{60T}\left\{[1-(-1)] - [(-1)^3 - 1^3]\right\}$$

$$= \frac{1}{60T}\left\{2 - (-2)\right\} = \frac{1}{60T}4 = \frac{4}{60\times 0.2}$$

$P = \frac{1}{3}$ watt

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a}$$

with $n \ne -1$

(18) Find Energy of the signal.

1) $x(t) = e^{2t}\, u(-t)$

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} e^{4t} \left[u(-t)\right]^2 dt$$

$$= \int_{-\infty}^{0} e^{4t} \cdot 1\, dt$$

$$= \left[\frac{e^{4t}}{4}\right]_{-\infty}^{0}$$

$$= \frac{1}{4}\left[1 - e^{-\infty}\right]$$

$$= \frac{1}{4} \text{ joule.}$$

(19)

2) $x(t) = e^{(1+t)}\, u(-t)$

$$E = \int_{-\infty}^{\infty} e^{2(1+t)} \left[u(-t)\right]^2 dt$$

$$= \int_{-\infty}^{0} e^{2} \cdot e^{2t}\, dt$$

$$= e^2 \left[\frac{e^{2t}}{2}\right]_{-\infty}^{0}$$

$$= \frac{e^2}{2}\left[1 - e^{-\infty}\right]$$

$$E = \frac{e^2}{2} \text{ joule.}$$

3) $x(t) = \frac{1}{t} \quad t \geq 1$

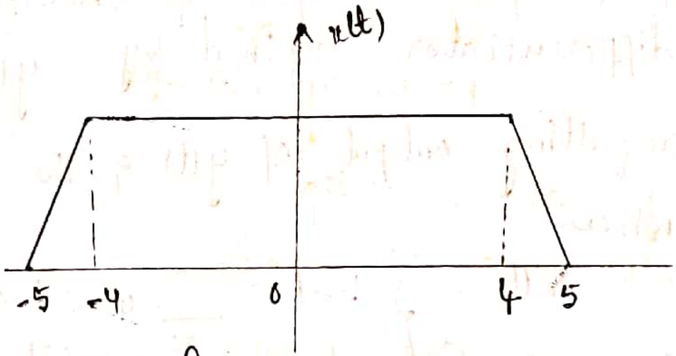
$$E = \int_{-\infty}^{\infty} x^2(t)\, dt = \int_{-\infty}^{\infty} \left(\frac{1}{t}\right)^2 dt \quad t \geq 1$$

$$= \int_{1}^{\infty} \frac{1}{t^2}\, dt = \left[-\frac{1}{t}\right]_{1}^{\infty}$$

$$= \frac{1}{1} - \frac{1}{\infty}$$

$$= 1 \text{ joule.}$$

(20) A trapizoidal pulse $x(t)$ is as shown in figure.



& defined as

$$x(t) = \begin{cases} 5-t & 4 \leq t \leq 5 \\ 1 & -4 \leq t \leq 4 \\ t+5 & -5 \leq t \leq -4 \\ 0 & \text{other wise.} \end{cases}$$

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-5}^{-4} (t+5)^2 dt + \int_{-4}^{4} 1\, dt + \int_{4}^{5} (5-t)^2\, dt$$

$$= \left[\frac{(t+5)^3}{3}\right]_{-5}^{-4} + \left[t\right]_{-4}^{4} + \left[\frac{(5-t)^3}{-3}\right]_{4}^{5}$$

$$= \frac{1}{3}\left[(-4+5)^3 - (-5+5)^3\right] + \left[4-(-4)\right] + \left(-\frac{1}{3}\right)\left[0^3 - 1^3\right]$$

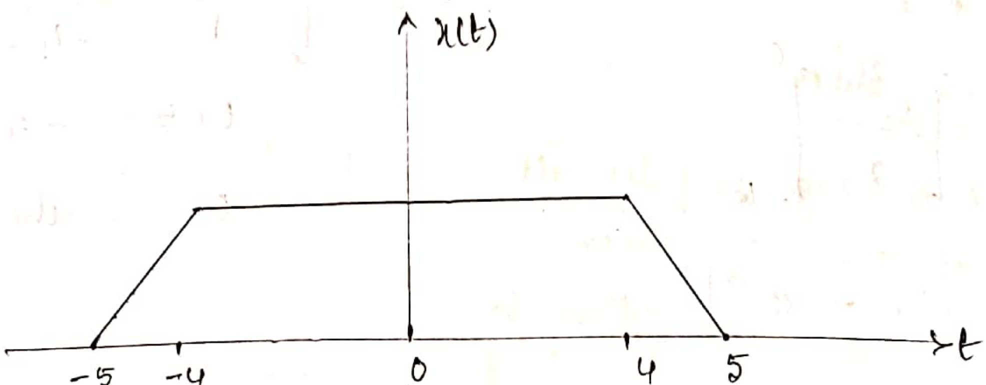
$$= \frac{1}{3}\left[(+1)^3 - 0\right] + 8 + \frac{1}{3} = \frac{1}{3} + 8 + \frac{1}{3} = \frac{2}{3} + 8$$

$$= \underline{\underline{\frac{26}{3}}}\ J \qquad P = \lim_{T\to\infty} \frac{1}{T}\int x^2(t)\,dt = 0$$

---

② A trapezoidal pulse is shown in figure, applied to a differentiator defined by $y(t) = \frac{dx(t)}{dt}$. Determine the resulting output of $y(t)$ & the total energy of $y(t)$ where

$$x(t) = \begin{cases} 5-t & 4 < t < 5 \\ 1 & -4 < t < 4 \\ t+5 & -5 < t < -4 \end{cases}$$

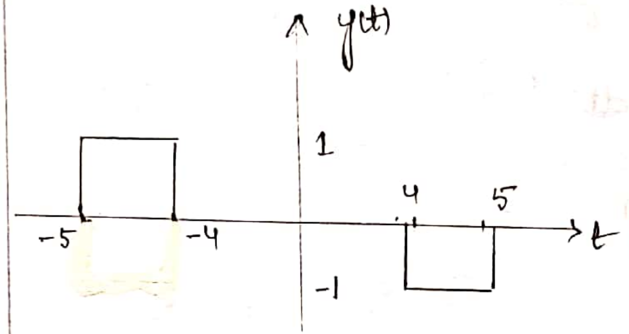


soln

i) To find $y(t)$:

Differentiate $x(t)$ w.r.t t.

$$y(t) = \begin{cases} -1 & 4 < t < 5 \\ 0 & -4 < t < 4 \\ 1 & -5 < t < -4 \end{cases}$$



ii)

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-5}^{-4} 1^2\, dt + \int_{4}^{5} (-1)^2\, dt$$

$$= [t]_{-5}^{-4} + [t]_{4}^{5}$$

$$= (-4+5) + (5-4)$$

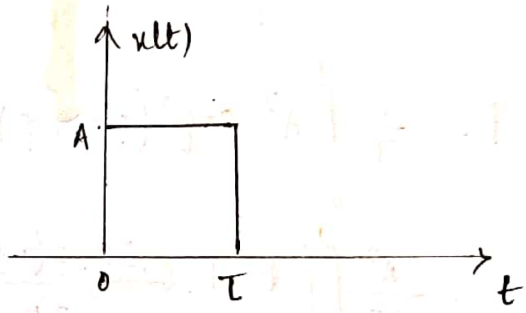
$$= 1 + 1$$

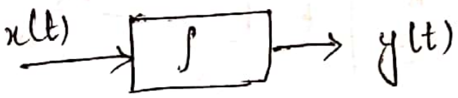
$$= 2 \text{ joules.}$$

22) A rectangular pulse $x(t)$ is defined as below. The pulse $x(t)$ is applied to an integrator defined by

$$y(t) = \int_0^t x(\tau)\, d\tau$$

$$x(t) = \begin{cases} A & 0 < t < T \\ 0 & \text{otherwise} \end{cases}$$





$$y(t) = \int_0^t x(\tau)\, d\tau$$

$$y(t) = \int_0^t A\, d\tau$$

$$y(t) = A[\tau]_0^t$$

$$y(t) = At$$

$$E = \int_0^T (At)^2\, dt$$

$$= A^2 \left[\frac{t^3}{3}\right]_0^T$$

$$E = \frac{A^2 T^3}{3} \text{ joule.}$$

23 Consider a sinusoidal signal $x(t)$ given, determine the average power of $x(t)$. $x(t) = A\cos(\omega t + \phi)$

Soln: To find fundamental period; $T = \frac{2\pi}{\omega}$.

For a periodic signal $x(t)$ power is given by

$$P = \frac{1}{T}\int_0^T x^2(t)\, dt$$

$$P = \frac{1}{T}\int_0^T A^2\cos^2(\omega t + \phi)\, dt = \frac{A^2}{T}\int_0^T \frac{1}{2}\left[1 + \cos 2(\omega t + \phi)\right] dt$$

$$= \frac{A^2}{2T}\int_0^T \left[1 + \cos(2\omega t + 2\phi)\right] dt$$

$$= \frac{A^2}{2T}\left[t + \frac{\sin(2\omega t + 2\phi)}{2\omega}\right]_0^T$$

$$= \frac{A^2}{2T}\left[T + \frac{\sin(2\omega T + 2\phi)}{2\omega} - 0 - \frac{\sin 2\phi}{2\omega}\right]$$

$$= \frac{A^2}{2T}\left[T + \frac{\sin(2\cdot\frac{2\pi}{T}\cdot T + 2\phi)}{2\omega} - \frac{\sin 2\phi}{2\omega}\right]$$

$$= \frac{A^2}{2T}\left[T + \frac{\sin(4\pi + 2\phi)}{2\omega} - \frac{\sin 2\phi}{2\omega}\right] = \frac{A^2}{2T}\left[T + \cancel{\frac{\sin 2\phi}{2\omega}} - \cancel{\frac{\sin 2\phi}{2\omega}}\right]$$

$$= \frac{A^2}{2T}\cdot T$$

$$\boxed{P = \frac{A^2}{2}} \text{ watt.}$$

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24

$\Rightarrow x(n) = \begin{cases} n & 0 \le n \le 5 \\ 10-n & 5 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

$x(n)$ is non periodic $\therefore$ it is Energy signal.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

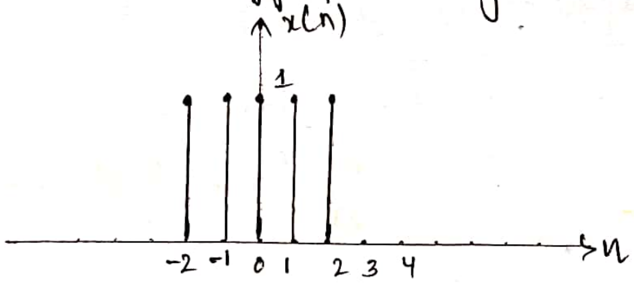
$$= \sum_{n=-\infty}^{-1} |x(n)|^2 + \sum_{n=0}^{4} |x(n)|^2 + \sum_{n=5}^{10} |x(n)|^2$$

$$= 0 + \sum_{n=0}^{4} n^2 + \sum_{n=5}^{10} (10-n)^2$$

$$= 0^2 + 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 + 0^2$$

$$E = 85 \text{ J}$$

$\Rightarrow$ Find Energy of the signal.



$x(n) = \begin{cases} 0 & n \le -3 \\ 1 & -2 \le n \le 2 \\ 0 & n > 2 \end{cases}$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=-2}^{2} 1^2$$

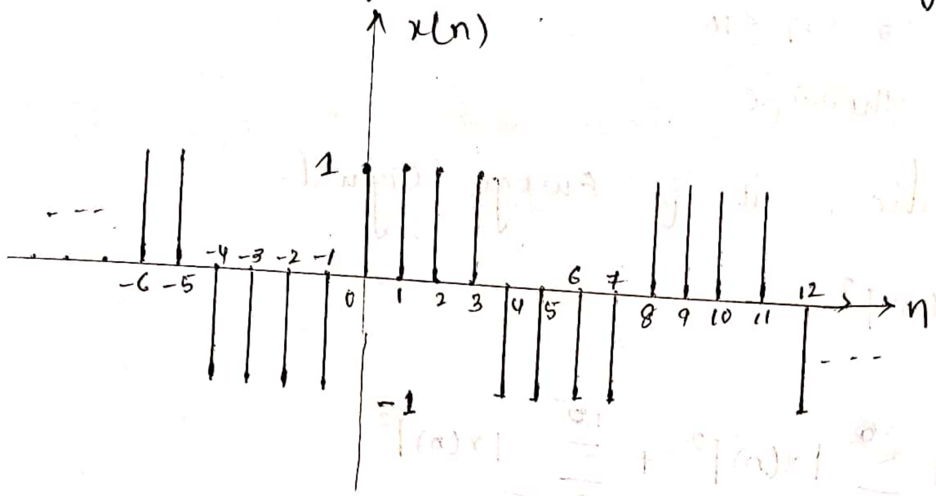
$$= \sum_{n=-2}^{2} 1$$

Note:

$$\sum_{n=N_1}^{N_2} A = A(N_2 - N_1 + 1)$$

$$E = 1(2-(-2)+1) = 5 \text{ J}$$

(25) Find the average power of discrete signal.



Soln: The given signal is periodic with fundamental period. N = 8 samples.

Consider one complete cycle.

$$x(n) = \begin{cases} 1 & 0 \leq n \leq 3 \\ -1 & 4 \leq n \leq 7 \end{cases}$$

Average power of signal is given as

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

$$= \frac{1}{8} \sum_{n=0}^{3} (1)^2 + \frac{1}{8} \sum_{n=4}^{7} |-1|^2$$

$$= \frac{1}{8} \sum_{n=0}^{3} 1 + \frac{1}{8} \sum_{4}^{7} 1$$

$$= \frac{1}{8} (3-0+1) + \frac{1}{8} (7-4+1)$$

$$= \frac{1}{8} 4 + \frac{4}{8}$$

$$\boxed{P = 1 \text{ watt}}$$

OPERATION PERFORMED ON SIGNAL

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A. Operation performed on dependent variable.
B. Operation performed on independent variable.

The different operations performed on dependent variable.

1) Amplitude or Magnitude Scaling
2) Addition or Subtraction
3) Multiplication
4) Differentiation
5) Integration

The first three operations can be performed on both continuous time & discrete time signal.
The last 2 operations are performed on CTS only.

Integration for CTS is equivalent to summation for DTS.

Differentiation for CTS is equivalent to differencing for DTS.

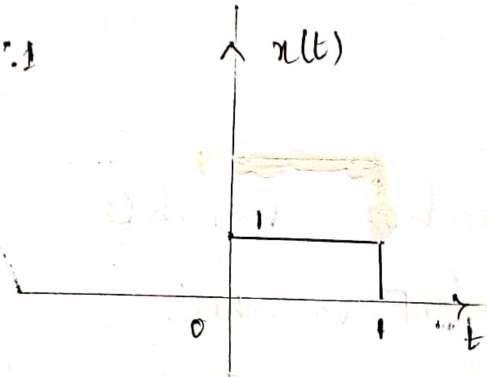
1) Amplitude Scaling

Let $y(t)$ be a CTS which is obtained by performing amplitude scaling on $x(t)$

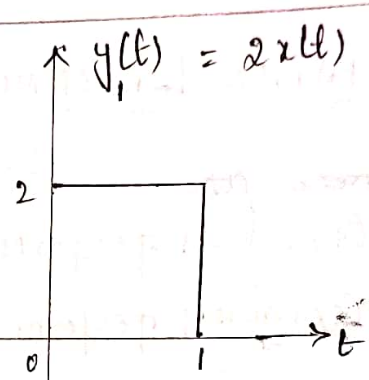
$$y(t) = a\, x(t)$$

If $a > 1$ then $y(t)$ is amplified version of $x(t)$
If $a < 1$; $y(t)$ is attenuated version of $x(t)$.

Ex: 1



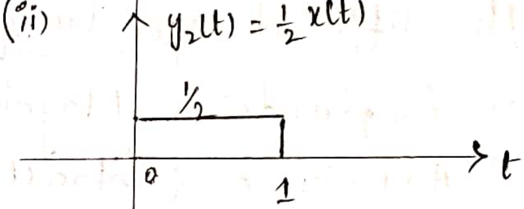
(i)



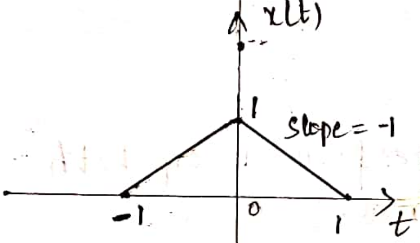
sketch (i) $y_1(t) = 2x(t)$

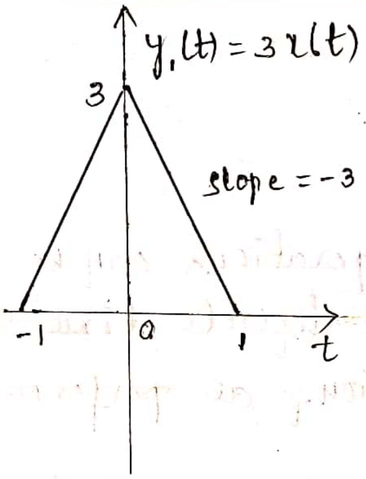
(ii) $y_2(t) = \frac{1}{2}x(t)$

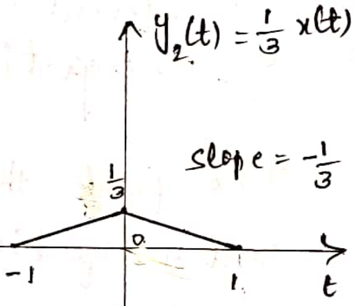
(ii)



Ex 2:







sketch (i) $y_1(t) = 3x(t)$

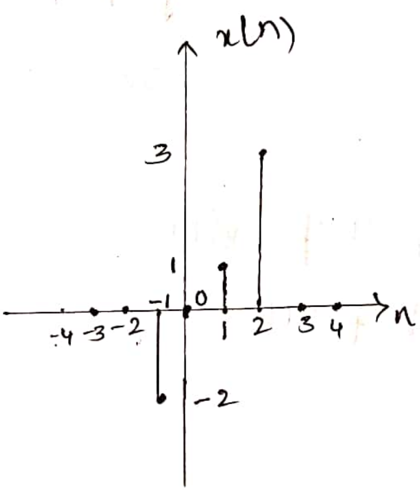
(ii) $y_2(t) = \frac{1}{3}x(t)$

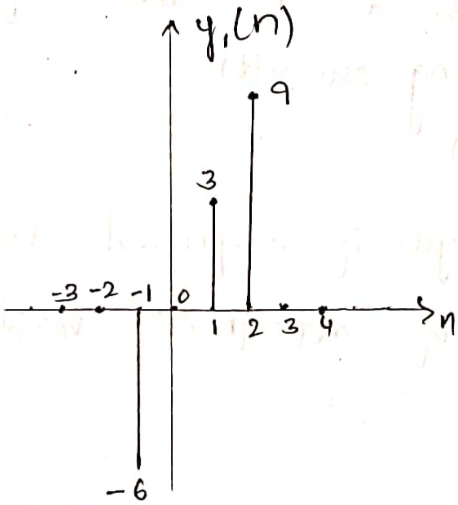
Ex:

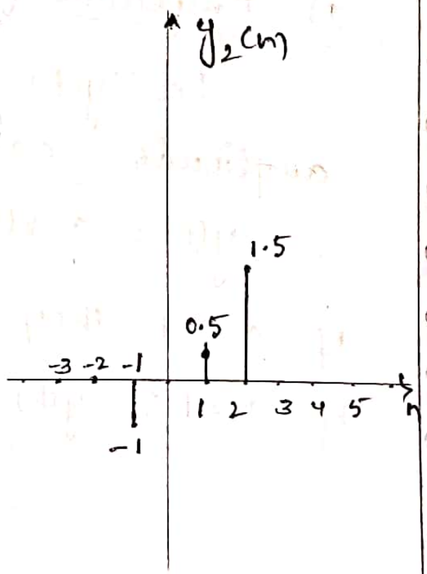
3) sketch

i) $y_1(n) = 3x(n)$

$y_2(n) = \frac{1}{2}x(n)$





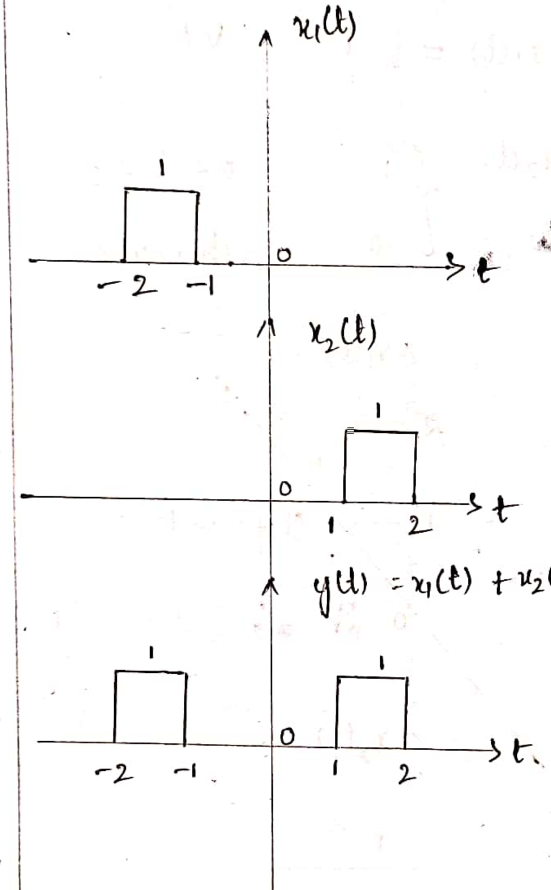


Addition :

Ex ① ⇒

$x_1(t)$ & $x_2(t)$ be two CTS & their sum be

$$y(t) = x_1(t) + x_2(t)$$

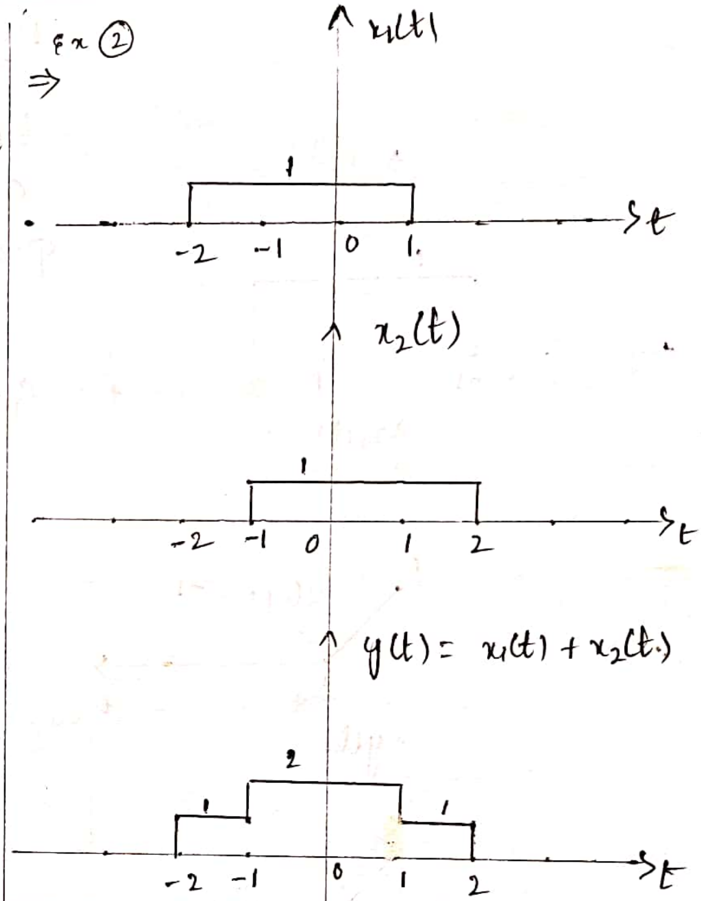


$$x_1(t) = \begin{cases} 1 & -2 \le t \le -1 \\ 0 & \text{otherwise.} \end{cases}$$

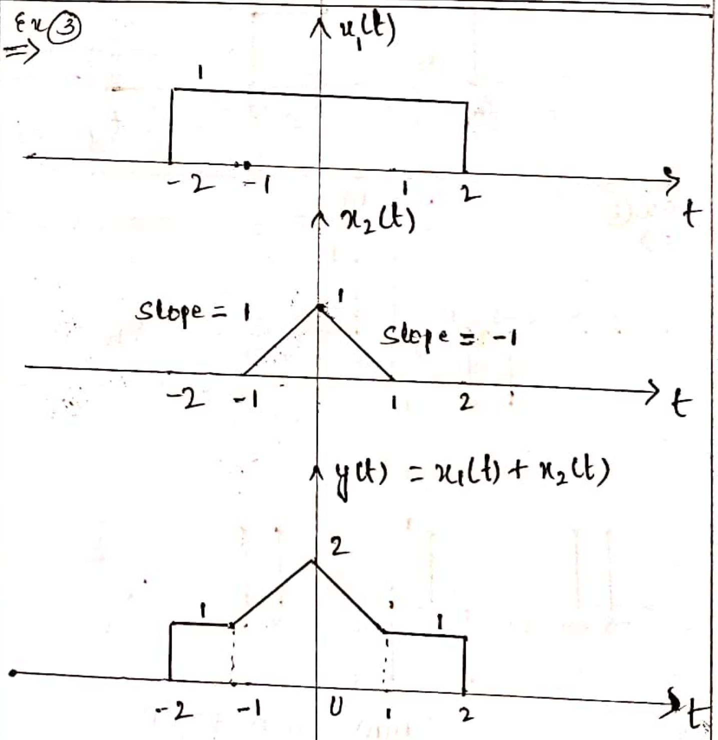
$$x_2(t) = \begin{cases} 1 & 1 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

$$y(t) = \begin{cases} 1 & -2 \le t \le -1 \\ 1 & 1 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

Ex ② ⇒

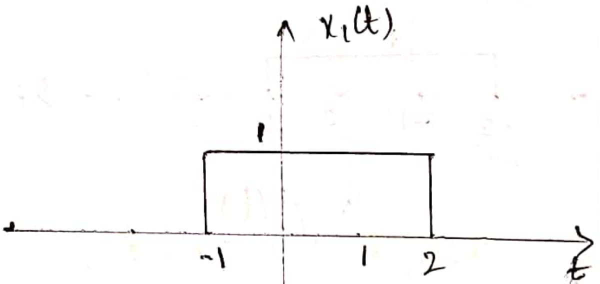


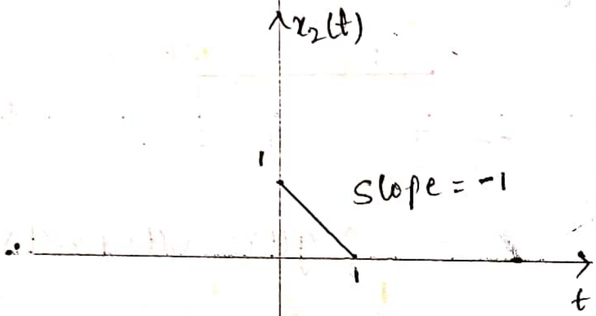
Ex ③ ⇒

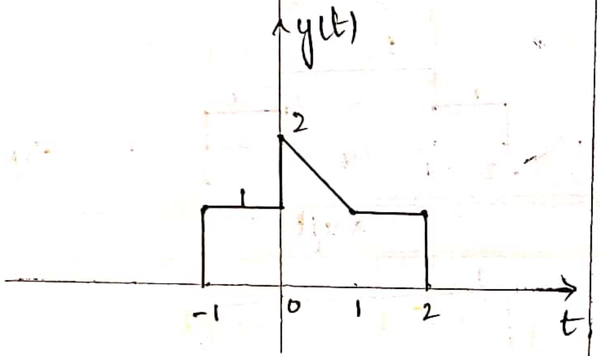


Ex 4

$\Rightarrow$

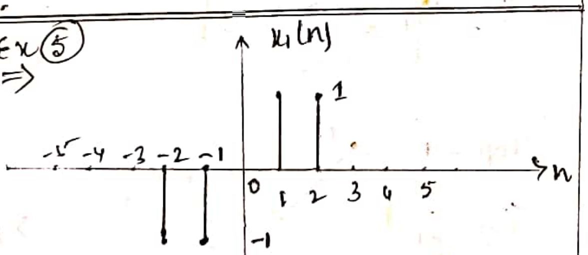


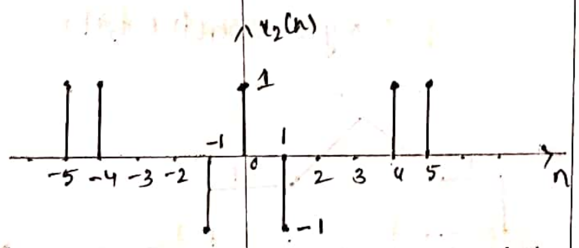


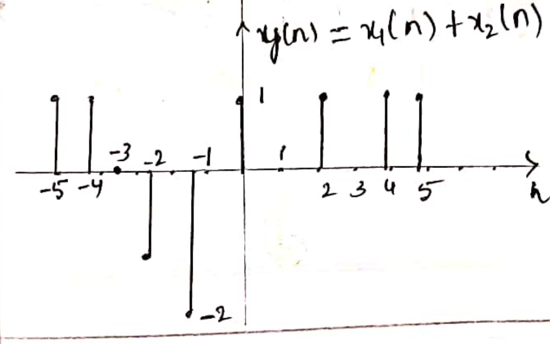


Ex 5

$\Rightarrow$







Multiplication

Ex 1

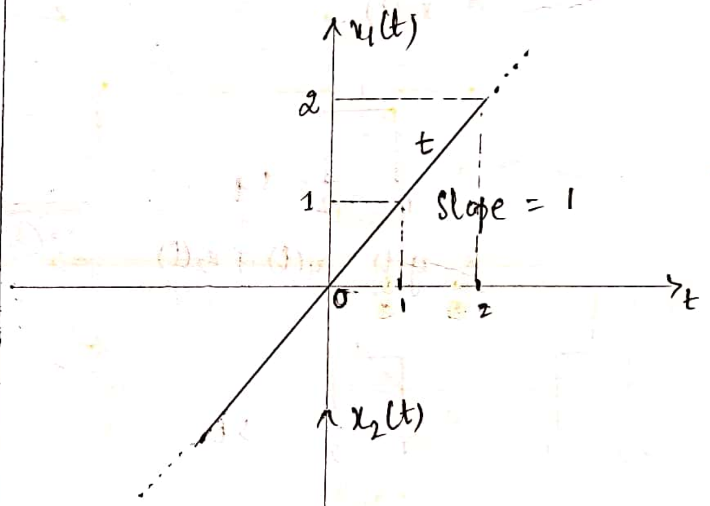
Let $x_1(t)$ & $x_2(t)$ be two CTS. Let $y(t)$ be their product.

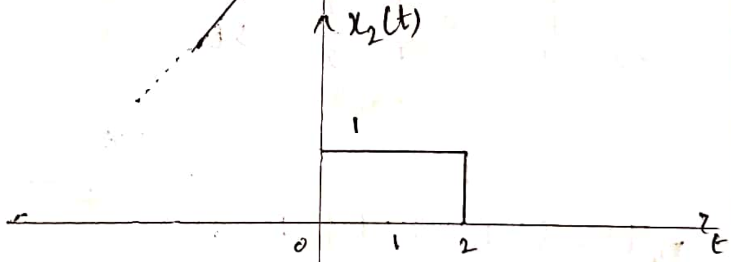
$$y(t) = x_1(t)\, x_2(t)$$

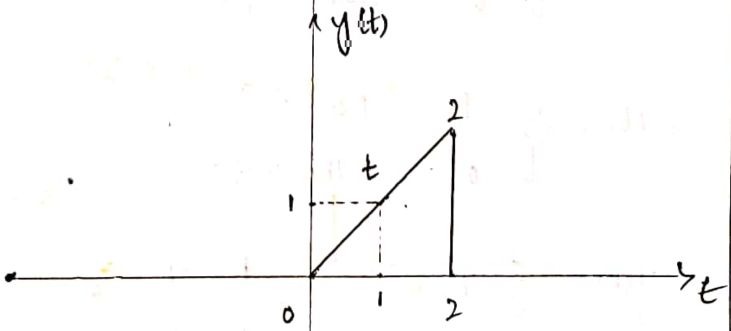
Given

$$x_1(t) = t \quad \forall t$$

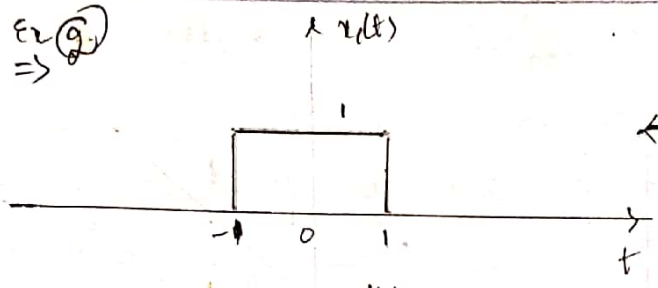
$$x_2(t) = \begin{cases} 1 & 0 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$



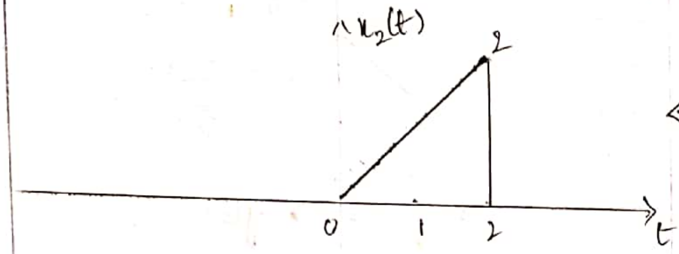




Ex (2)
=>

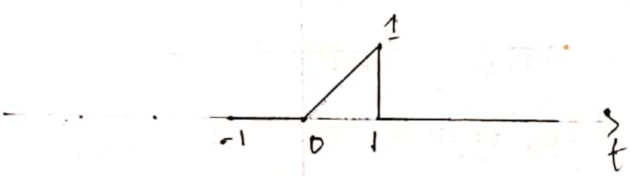


$$x_1(t) = \begin{cases} 1 & -1 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$



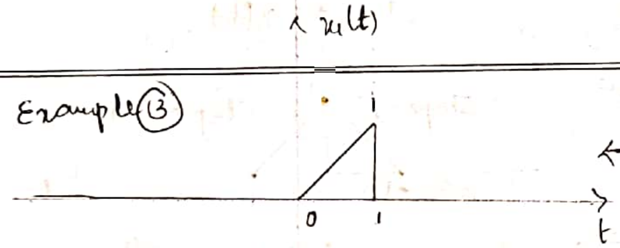
$$x_2(t) = \begin{cases} t & 0 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

$y(t) = x_1(t)\, x_2(t)$

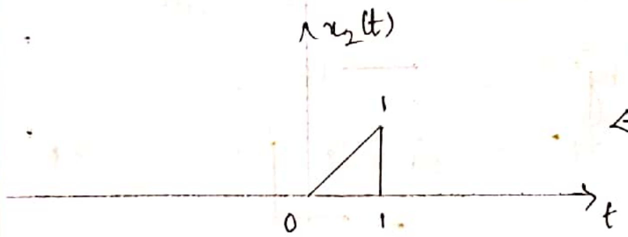


$$y(t) = \begin{cases} t & 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Example (3)

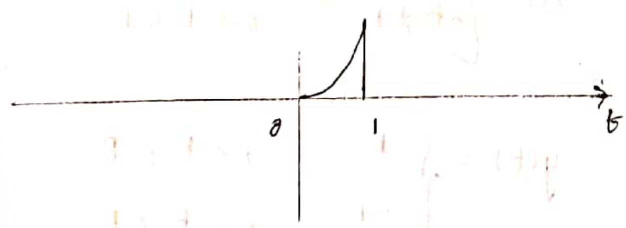


$$x_1(t) = \begin{cases} t & 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$



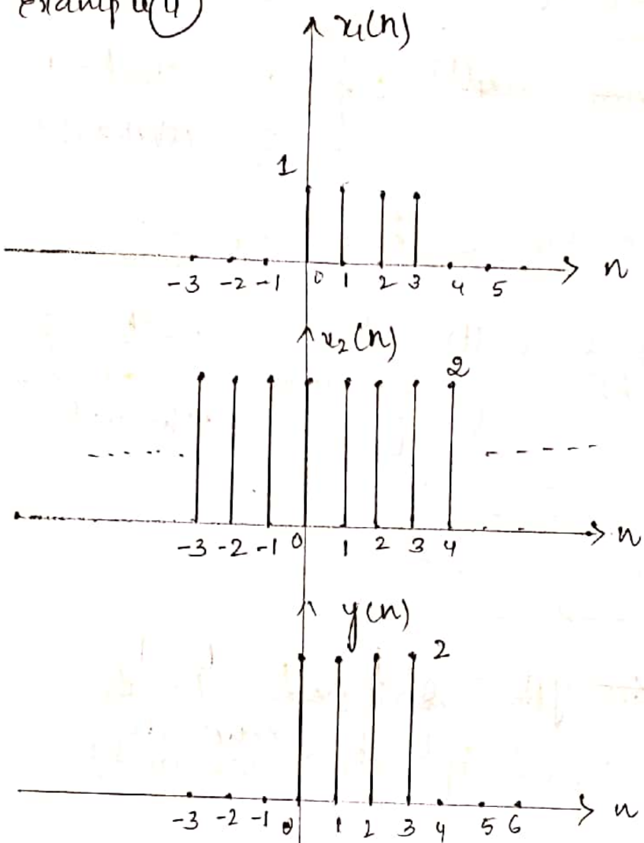
$$x_2(t) = \begin{cases} t & 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

$y(t) = x_1(t)\, x_2(t)$



$$y(t) = \begin{cases} t^2 & 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

Example 4 (ii)



Note: The corresponding amplitudes are multiplied at ~~given~~ each ~~in~~ time point.

Differentiation

Let y(t) be a CTS obtained by differentiating x(t)

$$\boxed{y(t) = \frac{d\,x(t)}{dt}}$$

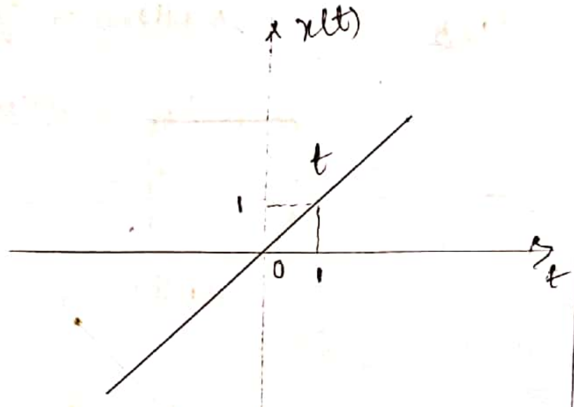
Ex 1: $x(t) = t \quad \forall t$

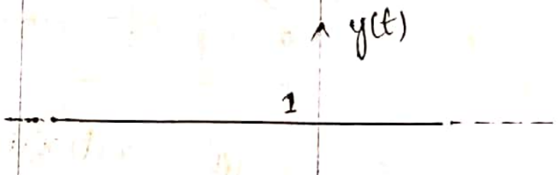
obtain $y(t) = \frac{d\,x(t)}{dt}$.

soln

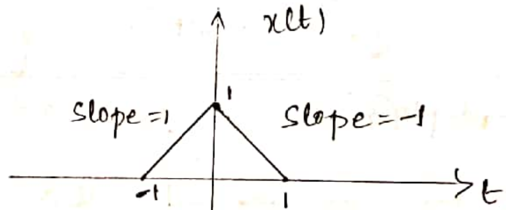
$$y(t) = \frac{d(t)}{dt}$$

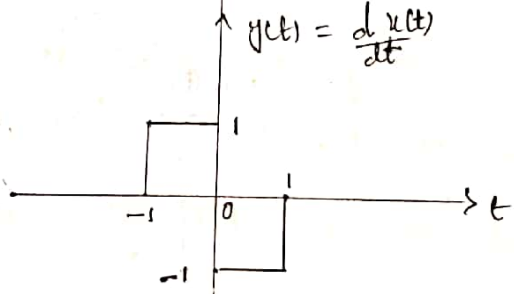
$$y(t) = 1 \quad \forall t.$$





Ex 2:





$$x(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ -t+1 & 0 \le t \le 1 \end{cases}$$

$$y(t) = \begin{cases} 1 & -1 \le t \le 0 \\ -1 & 0 \le t \le 1 \end{cases}$$

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OPERATON PERFORMED ON INDEPENDENT VARIABLE

The different operations performed on the independent variable are.

i) TIME SHIFTING 2) TIME SCALING 3) REFLECTION.

1) Time Shifting

Let $y(t)$ be a CTS which is obtained by performing time shifting on $x(t)$

$$\boxed{y(t) = x(t - t_0)}$$

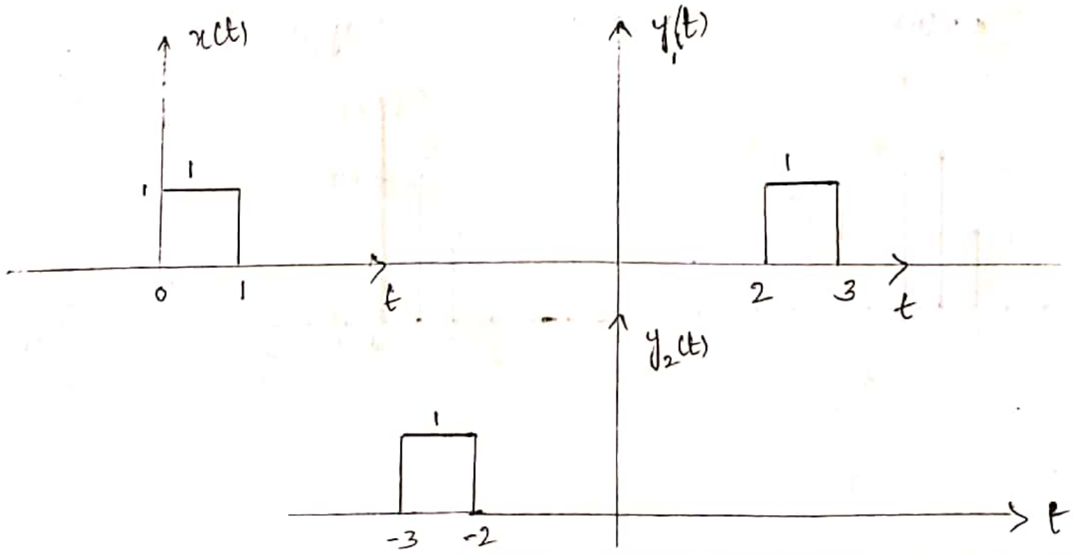
If $t_0 > 0$ then, $x(t)$ must be shifted to the right by t_0 unit (the signal is time delayed)

If $t_0 < 0$ then $x(t)$ is shifted to left by t_0 unit (the signal is time advanced).

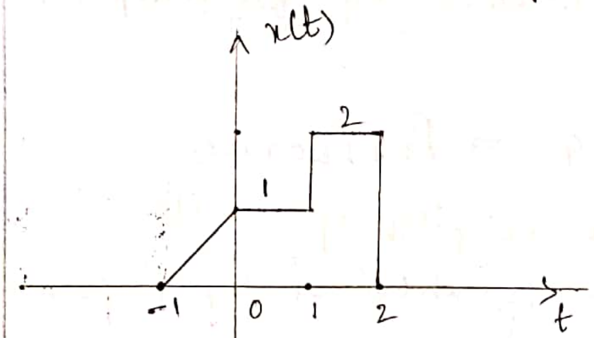
where t_0 is a real number.

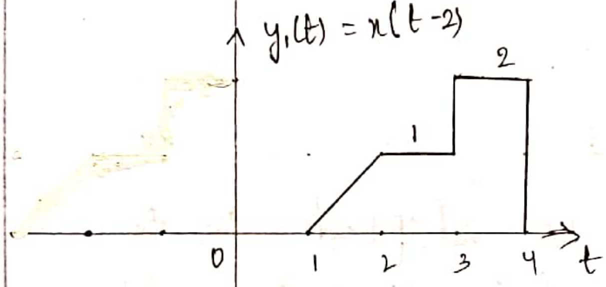
Similarly for discrete time signal $y(n) = x(n - n_0)$ where n_0 is an integer.

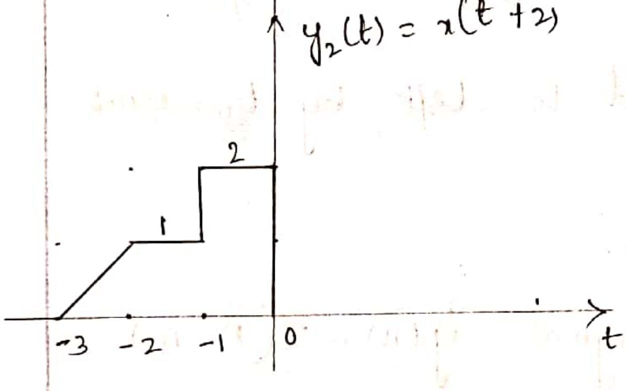
Example 01: For given $x(t)$, sketch $y_1(t) = x(t-2)$
$y_2(t) = x(t+3)$



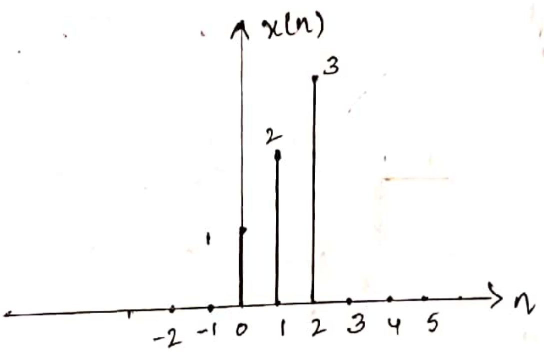
Example 02: Sketch i) $y_1(t) = x(t-2)$
ii) $y_2(t) = x(t+2)$







⇒ Sketch (i) $y_1(n) = x(n-3)$
(ii) $y_2(n) = x(n+2)$



i) $y_1(n) = x(n-3)$

$n=0$ $y_1(0) = x(-3) = 0$

$n=1$ $y_1(1) = x(-2) = 0$

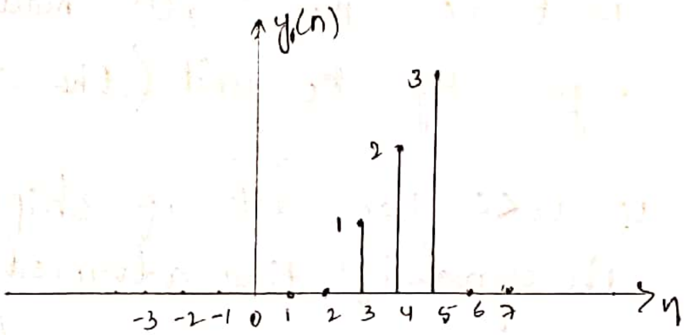
$n=2$ $y_1(2) = x(-1) = 0$

$n=3$ $y_1(3) = x(0) = 1$

$n=4$ $y_1(4) = x(1) = 2$

$n=5$ $y_1(5) = x(2) = 3$

6 $y_1(6) = x(3) = 0$



ii) $y_2(n) = x(n+2)$

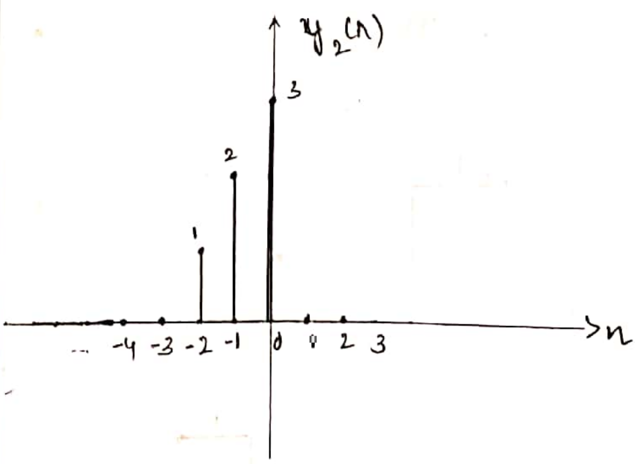
$n=0$ $y_2(0) = x(2) = 3$

$n=-1$ $y_2(-1) = x(1) = 2$

$n=-2$ $y_2(-2) = x(0) = 1$

$n=-3$ $y_2(-3) = x(-1) = 0$

$n=-4$ $y_2(-4) = x(-2) = 0$



Time Scaling

Let $y(t)$ be a CTS which is obtained by performing time scaling on $x(t)$.

$$y(t) = x(at)$$

If $|a| > 1$ $y(t)$ is compressed version of $x(t)$

If $|a| < 1$ $y(t)$ is Expanded (Enlarged) version of $x(t)$.

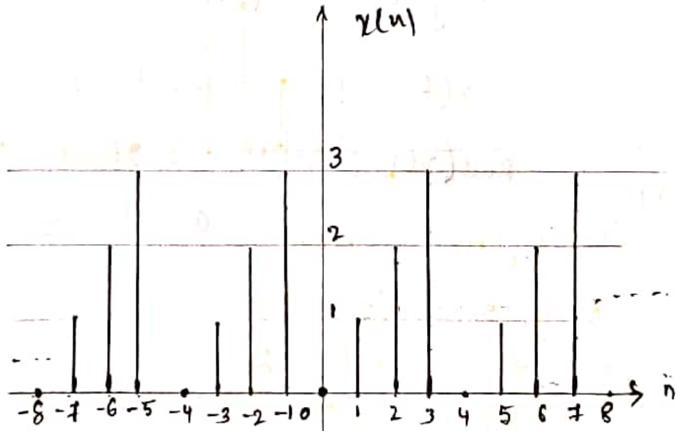
Similarly for DTS $y(n) = x(an)$

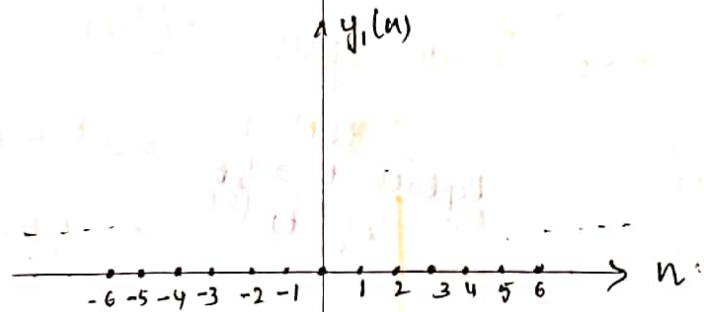
Here $a > 1$

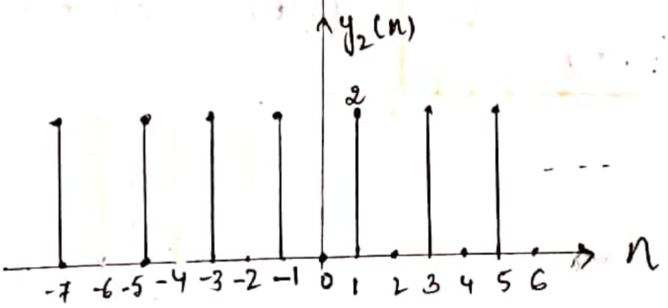
Ex 1: $x(n)$ is periodic with period 4. Sketch i) $y_1(n) = x(4n)$

ii) $y_2(n) = x(2n)$

iii) $y_3(n) = x(3n)$







i) $y_1(n) = x(4n)$

$y_1(0) = x(0) = 0$

$y_1(1) = x(4) = 0$

$y_1(2) = x(8) = 0$

$y_1(-1) = x(-4) = 0$

$y(-2) = x(-8) = 0$

ii) $y_2(n) = x(2n)$

$y_2(0) = x(0) = 0$

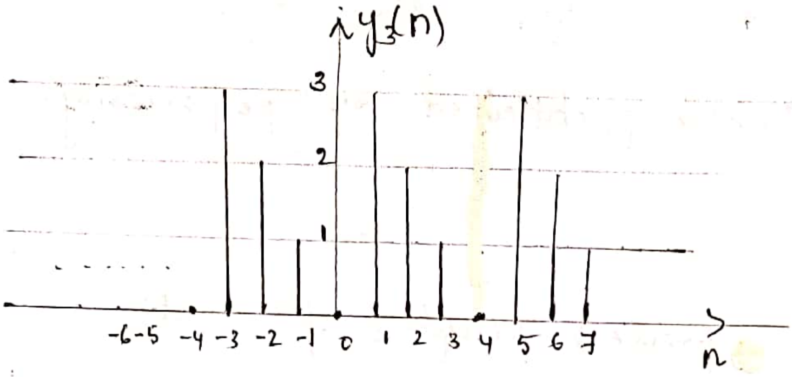
$y_2(1) = x(2) = 2$

$y_2(2) = x(4) = 0$

$y_2(3) = x(6) = 2$

$y_2(-1) = x(-2) = 2$

$y_2(-2) = x(-4) = 0$



$y_3(n) = x(3n)$

$y_3(0) = x(0) = 0$

$y_3(1) = x(3) = 3$

$y_3(2) = x(6) = 2$

$y_3(3) = x(9) = 1$

$y_3(-1) = x(-3) = 1$

$y_3(-2) = x(-6) = 2$

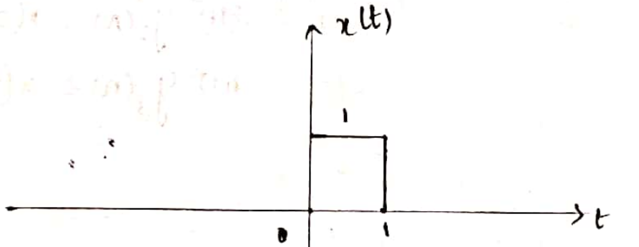
$y_3(-3) = x(-9) = 3$

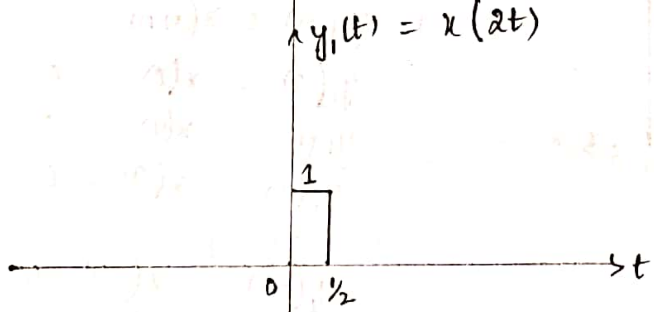
Example 2

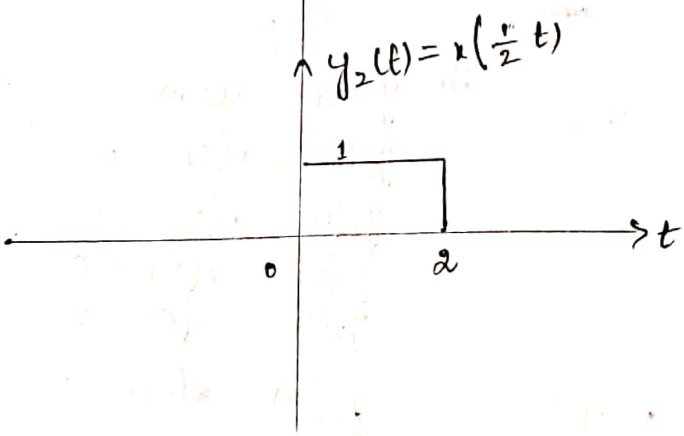
Given $x(t)$. Sketch

i) $y_1(t) = x(2t)$

ii) $y_2(t) = x(\frac{1}{2}t)$







Calculation:

i) $y_1(t) = x(2t)$

$x(t) = 1 \qquad 0 \leq t \leq 1$

But we need $x(2t)$ $\quad 0 \leq 2t \leq 1$

$0 \leq t \leq \frac{1}{2}$

ii) $y_2(t) = x(\frac{1}{2}t)$

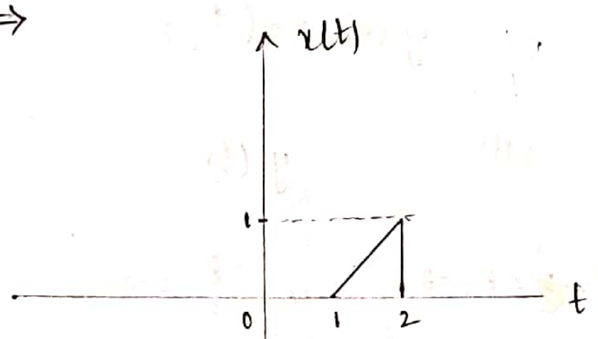
$x(t) = 1 \qquad 0 \leq t \leq 1$

But we need $x(\frac{1}{2}t)$ $\quad 0 \leq \frac{1}{2}t \leq 1$

so.

$0 \leq t \leq 2$

⇒



Given $x(t)$; Sketch

$y_1(t) = x(2t)$

$y_2(t) = x(t/3)$

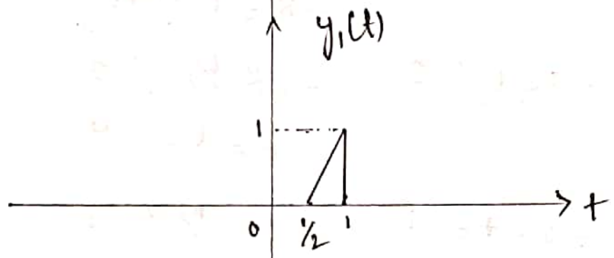
i) $y_1(t) = x(2t)$

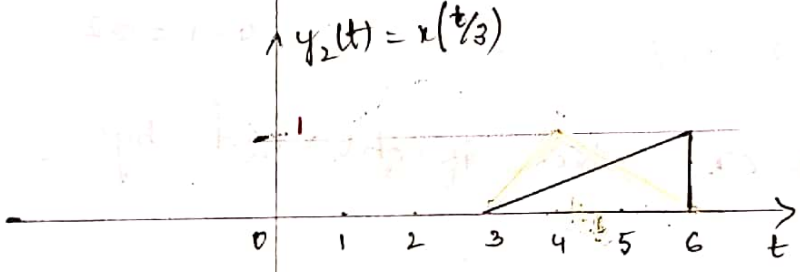
$x(t) = t \quad 1 \leq t \leq 2$

we need $x(2t)$

$\therefore 1 \leq 2t \leq 2$

$\frac{1}{2} \leq t \leq 1$





ii) $y_2(t) = x(t/3)$

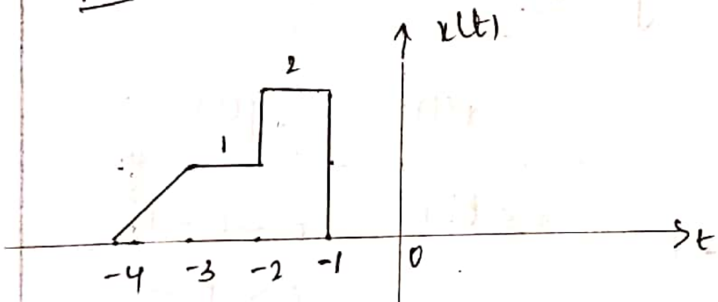
$x(t) = t \quad 1 < t < 2$

we need $x\left(\frac{t}{3}\right)$ so

$1 \leq \frac{t}{3} \leq 2$

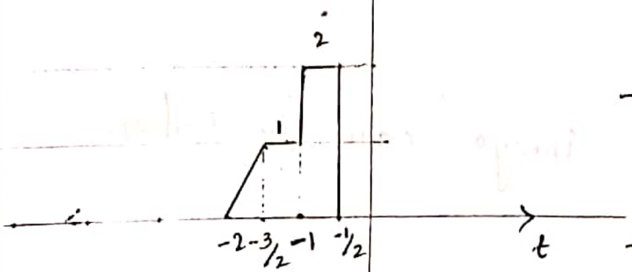
$3 \leq t \leq 6.$

Given:

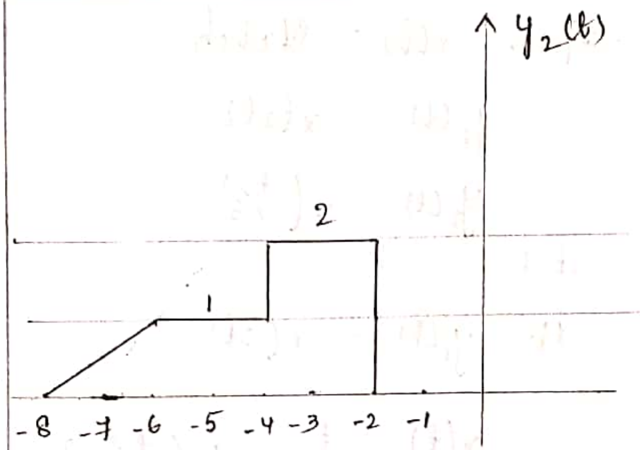


Sketch

$y_1(t) = x(2t)$



$x(t)$	$y_1(t)$
$-4 \leq t \leq -3$	$-4 \leq 2t \leq -3$ $-2 \leq t \leq -\frac{3}{2}$ ✓
$-3 \leq t \leq -2$	$-3 \leq 2t \leq -2$ $-3/2 \leq t \leq -1$ ✓
$-2 \leq t \leq -1$	$-2 \leq 2t \leq -1$ $-1 \leq t \leq -1/2$ ✓



$$y_2(t) = x(t/2)$$

$x(t)$	$y_2(t)$
$-4 \le t \le -3$	$-4 \le \frac{t}{2} \le -3$
	$-8 \le t \le -6$
$-3 < t \le -2$	$-3 \le t/2 \le -2$
	$-6 \le t \le -4$
$-2 \le t \le -1$	$-2 \le t/2 \le -1$
	$-4 \le t \le -2$

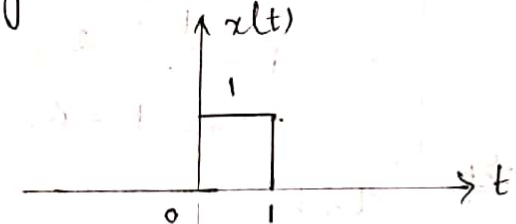
Reflection:

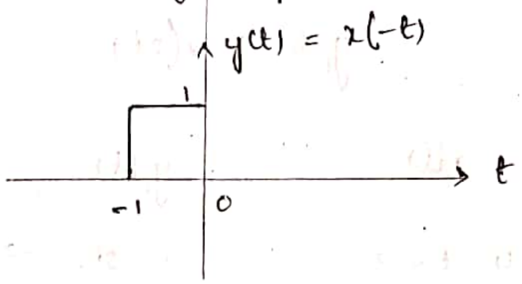
Let $y(t)$ be a CTS which is obtained by reflecting $x(t)$

$$y(t) = x(-t)$$

To get $y(t)$, replace t by $-t$ in $x(t)$.

Ex:

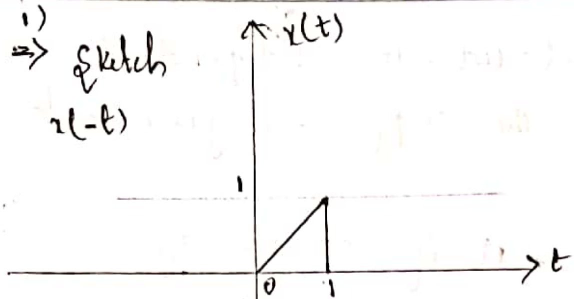




$x(t)$	$y(t)$
$0 \le t \le 1$	$0 \le -t \le 1$
	$0 \ge t \ge -1$
	$-1 \le t \le 0$

Note:
For reflection just mirror image can be taken.

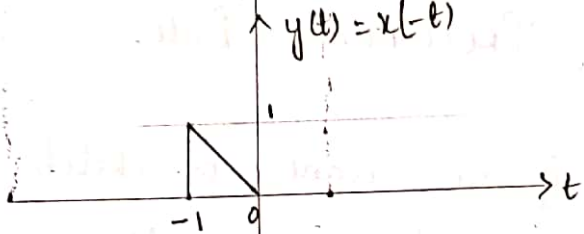
i)
⇒ Sketch $x(-t)$



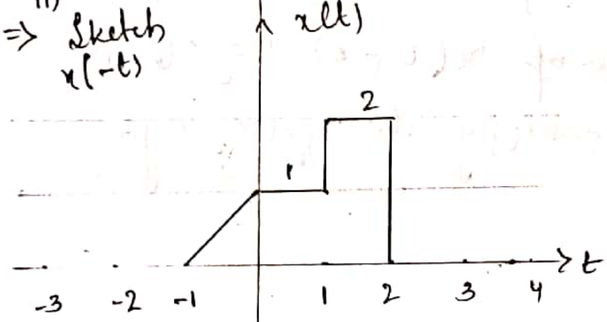
$$x(t) = \begin{cases} 0 & t < 0 \\ t & 0 \le t \le 1 \\ 0 & t > 1 \end{cases}$$

Replace all t by $-t$

$$x(-t) = 0 \quad t > 0$$
$$= -t \quad 0 \geq t \geq -1 \; [-1 \le t \le 0]$$
$$= 0 \quad t < -1$$



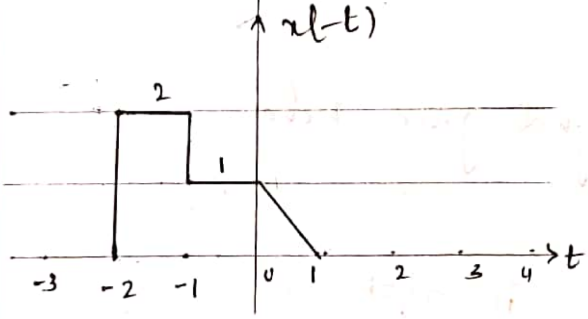
ii)
⇒ Sketch $x(-t)$



$$x(t) = 0 \quad t < -1$$
$$= t+1 \quad -1 < t < 0$$
$$= 1 \quad 0 \le t \le 1$$
$$= 2 \quad 1 \le t \le 2$$

Replace all t by $-t$

$$x(-t) = 0 \quad t > 1$$
$$= -t+1 \quad 1 > t > 0$$
$$= 1 \quad 0 \geq t \geq -1$$
$$= 2 \quad -1 \geq t \geq -2$$



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ECE Dept

When all the 3 operations related to independent variable (time) has to be performed, the steps are given below.

1) Time SHIFTING — First
2) Time SCALING — Second
3) Reflection — Third.

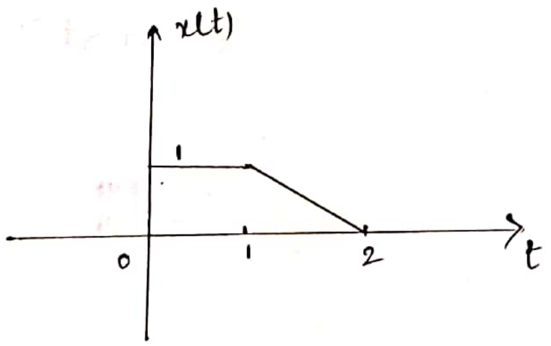
} It is called as PRECEDENCE Rule.

If $x(t)$ is a given CTS & if we want to sketch $y(t) = x(at-b)$, according to precedence rule, first we need to sketch $x(t-b)$ and then scale it by a factor 'a' to get $x(at-b)$. Then we can satisfy the following condition.

$$y(0) = x(-b)$$

$$y(b/a) = x(0).$$

Sketch the following for the signal given below.



i) $x(3t-2)$

ii) $x(\frac{t}{2}+2)$

iii) $x(-\frac{t}{2}+2)$

iv) $x(-t+4)$

Note

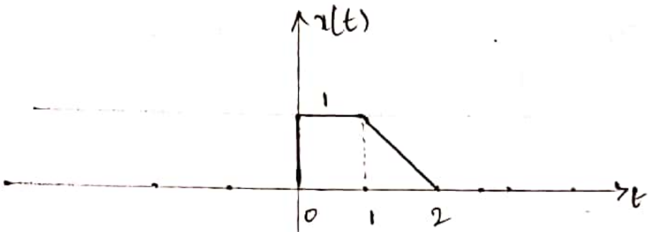
Consider $y(t) = x(at-b) \longrightarrow (1)$

put $t=0$ in (1)

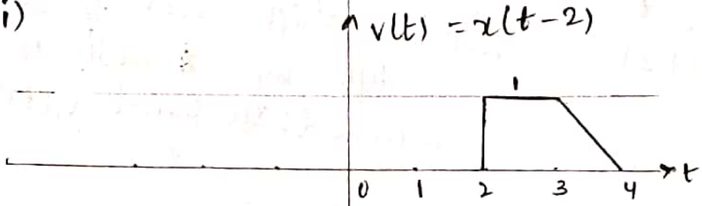
$$y(0) = x(-b) \longrightarrow (2)$$

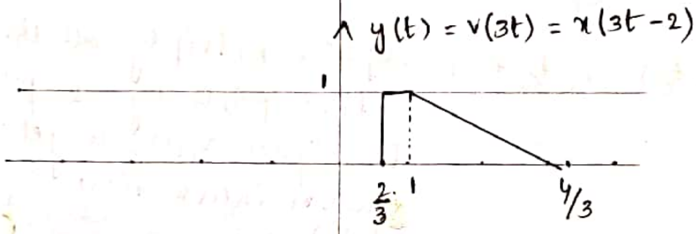
put $t = b/a$; $y(b/a) = x(0) \longrightarrow (3)$

$\Rightarrow$ (2) & (3) are satisfied only if precedence rule is followed.



i)





To sketch : $x(3t-2)$

step 1: shift $x(t)$ to right by 2 unit

step 2: time scaling

$v(t) = 1 \quad 2 < t \leq 3$

we need $3t$

$\therefore \quad 2 < 3t \leq 3$

$2/3 < t \leq 1$

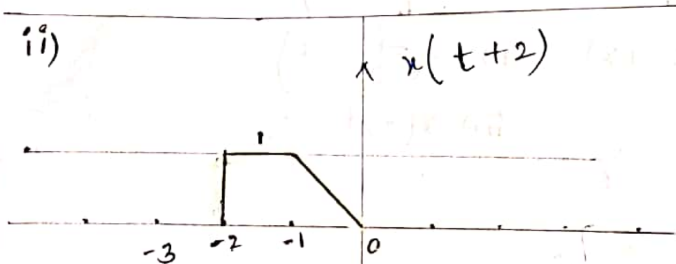
Now for the interval $\longrightarrow$ $3 < t \leq 4$

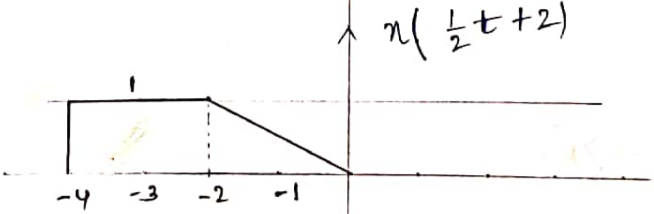
we need $3t$

$\therefore \quad 3 \leq 3t \leq 4$

$1 \leq t \leq 4/3$

ii)



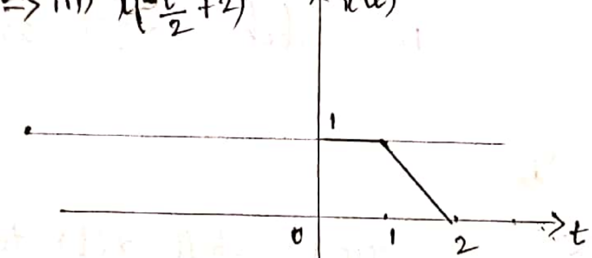


To sketch $x(t/2 + 2)$

step 1: shift $x(t)$ to left by 2 units to obtain $v(t)$

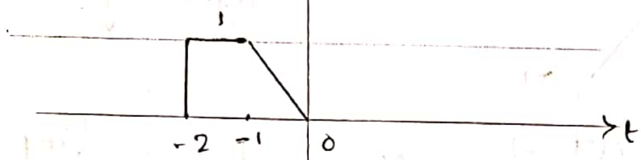
step 2: short cut: just mutiply all time point of shifted version of $x(t)$ i.e., $v(t)$ by 2

⇒ iii) $x\left(-\frac{t}{2}+2\right)$



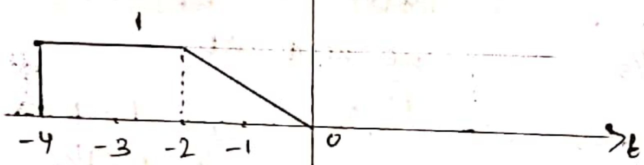
step 1: shift $x(t)$ towards left by 2 unit to get intermediate signal $v_1(t)$

$v_1(t) = x(t+2)$



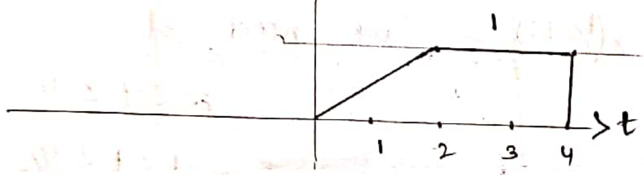
$v_2(t) = v_1(t/2) = x(t/2 + 2)$

step 2: Multiply all the time points by 2 for the signal $v_1(t)$ to get another intermediate signal $v_2(t)$.

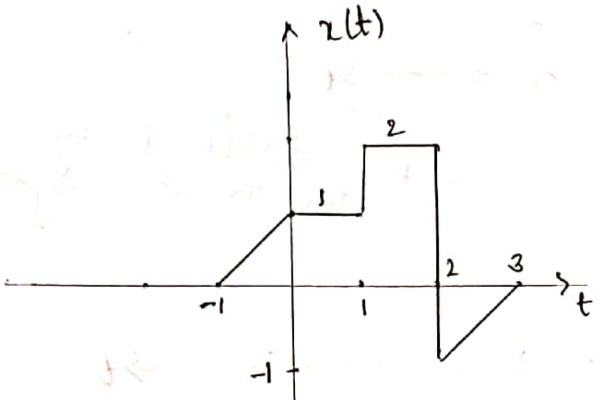


$v_3(t) = v_2(-t) = x\left(-\frac{t}{2}+2\right)$

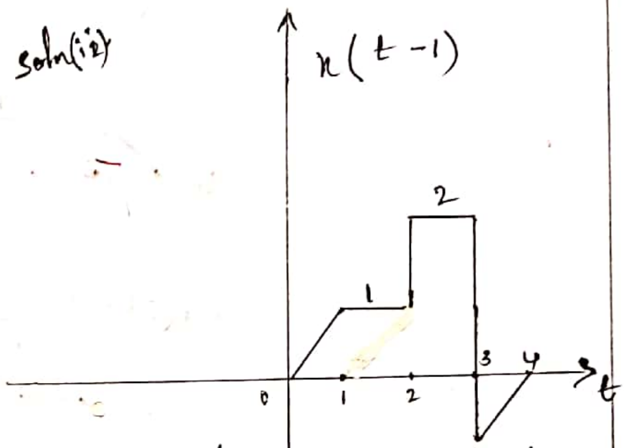
step 3: Take mirror image of $v_2(t)$, which gives $v_2(-t) = x\left(-\frac{t}{2}+2\right)$



Sketch i) $x(2t+2)$ ii) $x(-\frac{t}{2}-1)$ iii) $x(-2t-3)$ for given $x(t)$

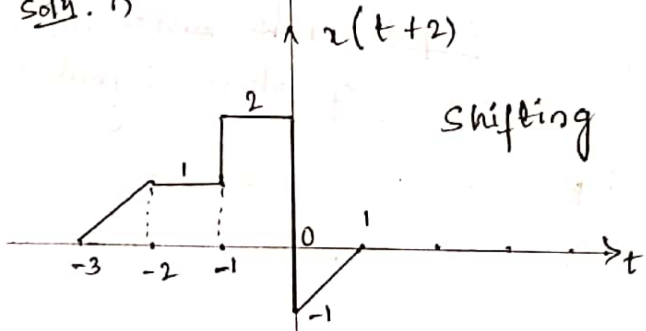


soln(ii)

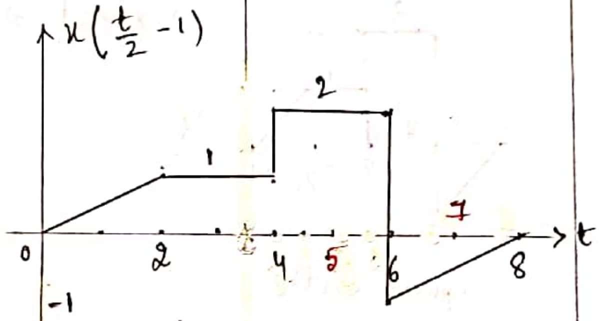


↑ shifted x(t) 1 unit right.

Soln: i)

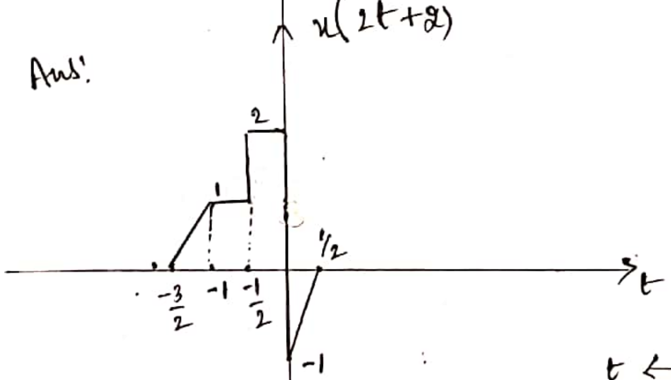


Shifting



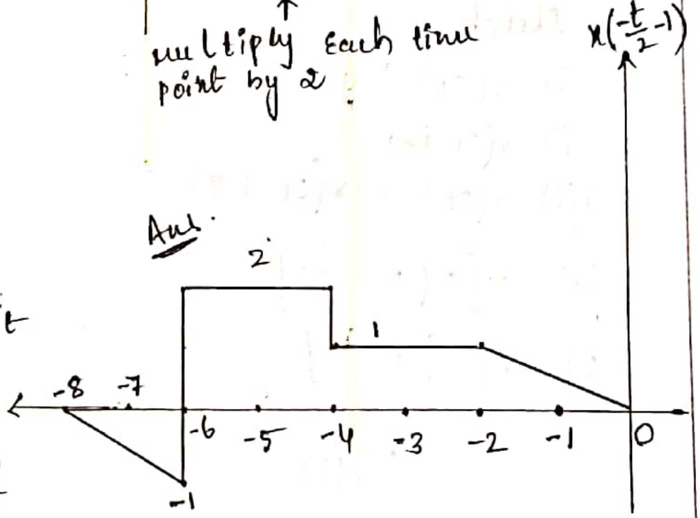
↑ multiply each time point by 2

Ans:



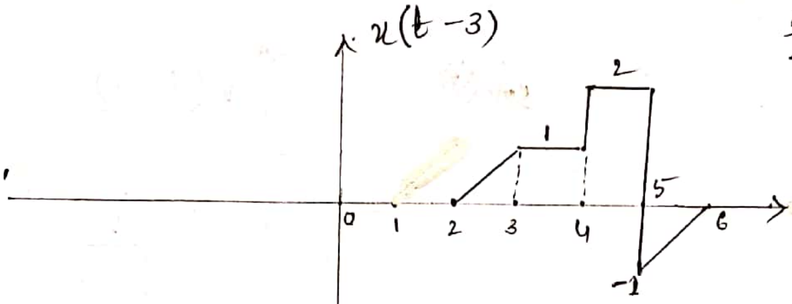
↑ we divided each time point of x(t+2) by 2. to get x(2t+2)

Ans.

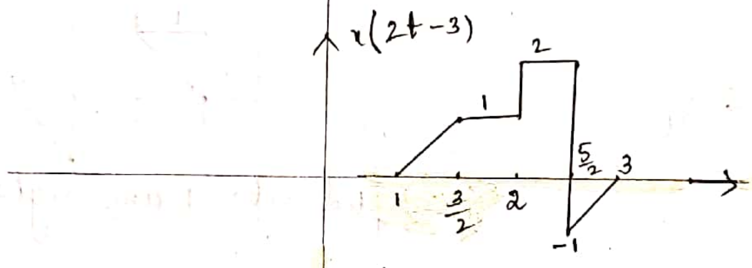


↑ Take mirror image.

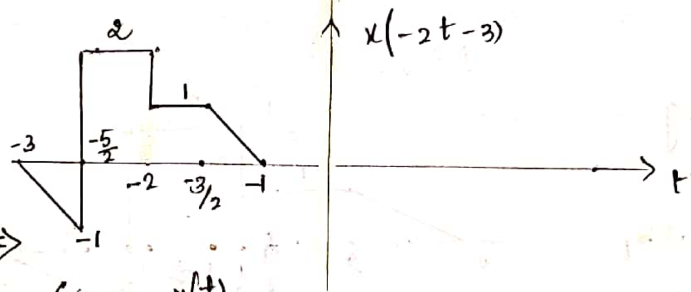
$\Rightarrow$ $x(-2t-3)$



step 1: shift right by 3 unit



step 2: divide t by 2 for the above signal.



step 3: Take mirror image of above signal.

$\Rightarrow$ Given $x(t)$

Sketch

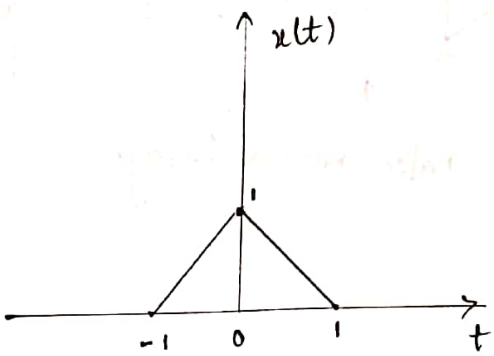
i) $x(3t)$

ii) $x(3t+2)$

iii) $x(3t) + x(3t+2)$

iv) $x[2(t+2)]$

v) $x[2(t-2)]$



i) soln:

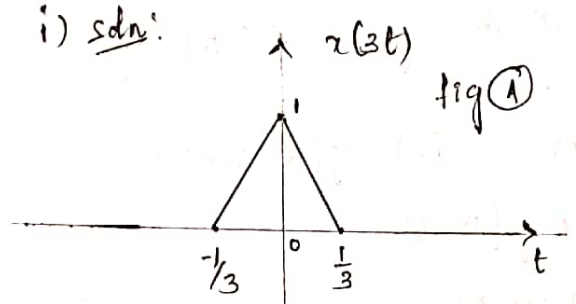
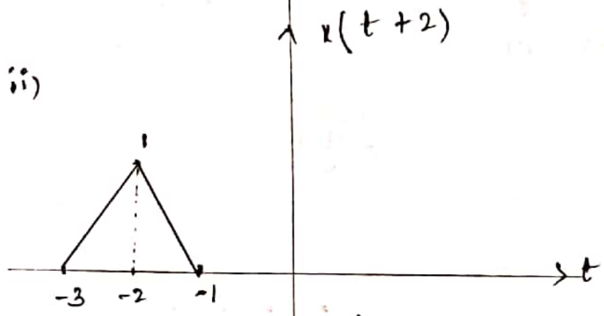


fig (A)

ii)



ii) Ans.

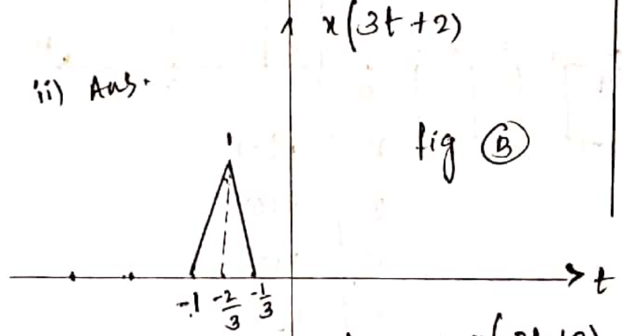


fig (B)

iii) Ans

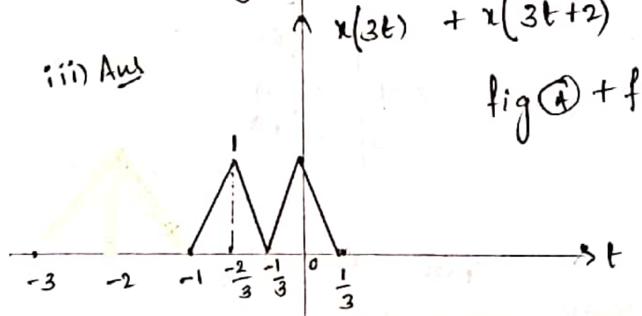
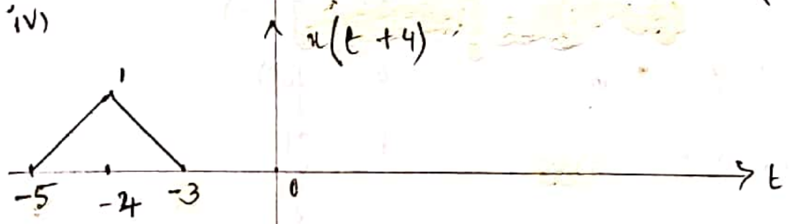


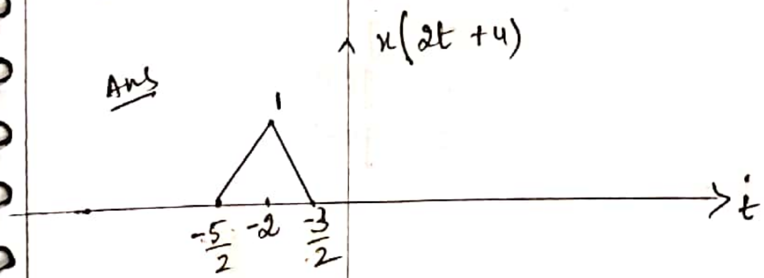
fig (A) + fig (B)

iv)



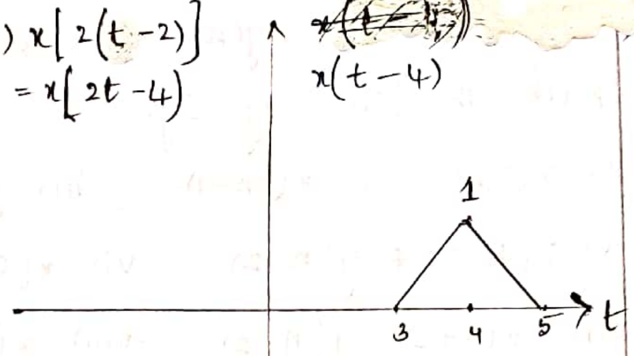
$\leftarrow x[2(t+2)] = x(2t+4)$

Ans

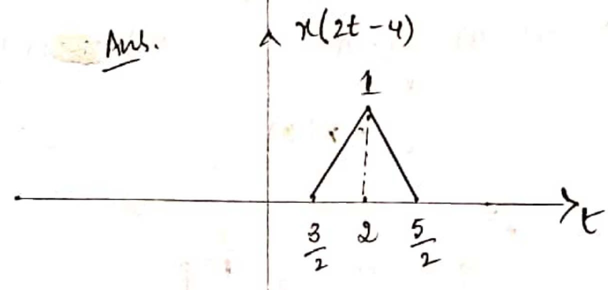


v) $x[2(t-2)]$

$= x[2t-4]$



Ans.



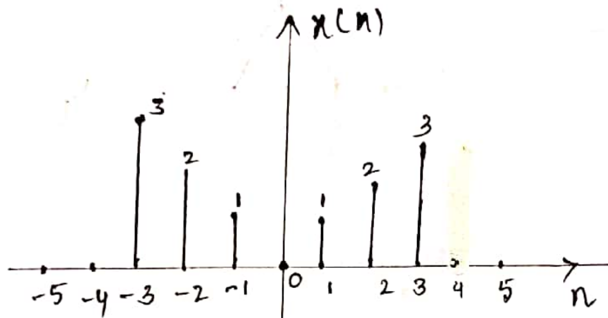
Let $x(n)$ & $y(n)$ are shown in the figure, carefully sketch the following signals.

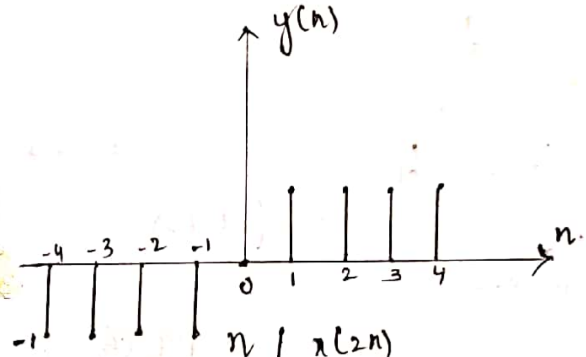
i) $x(2n)$ ii) $x(3n-1)$ iii) $y(1-n)$ iv) $y(2-2n)$ ✓

v) $x(n-2) + y(n-2)$ vi) $x(2n) + y(n-4)$

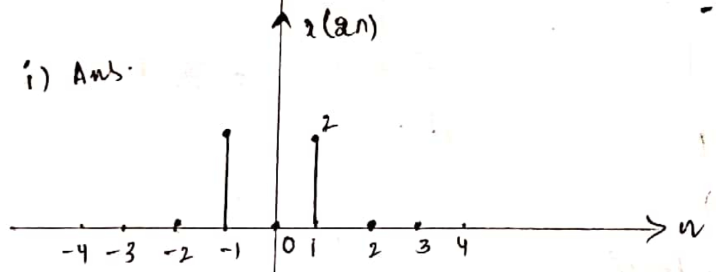
vii) $x(n+2) \cdot y(n-2)$ viii) $x(3-n) \cdot y(n)$

ix) $x(-n)\ y(-n)$ x) $x(n)\ y(-2-n)$



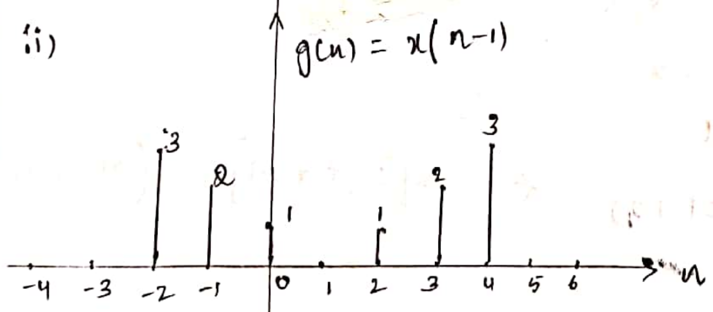


i) Ans.



n	$x(2n)$
0	$x(0)$
1	$x(2)$
2	$x(4)$
3	$x(6)$
-1	$x(-2)$
-2	$x(-4)$

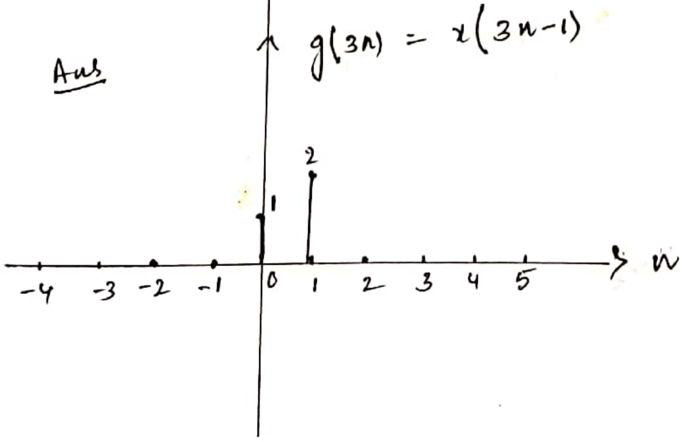
ii)



For $x(3n-1)$

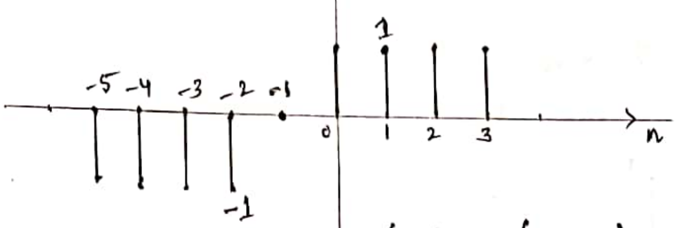
n	$g(3n)$
0	$g(0)$
1	$g(3)$
2	$g(6)$
3	$g(9)$
-1	$g(-3)$
-2	$g(-6)$

Ans



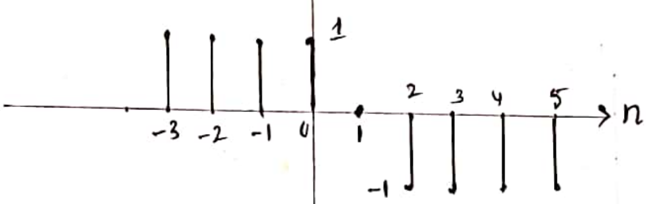
iii) $y(1-n) = y(-n+1)$

$g(n) = y(n+1)$



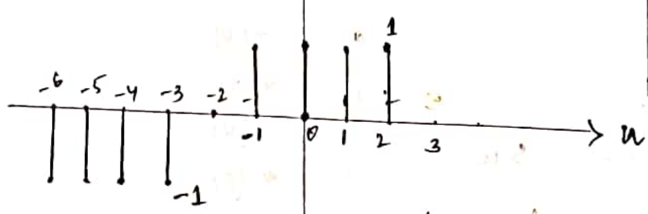
$g(-n) = y(-n+1)$

Ans:

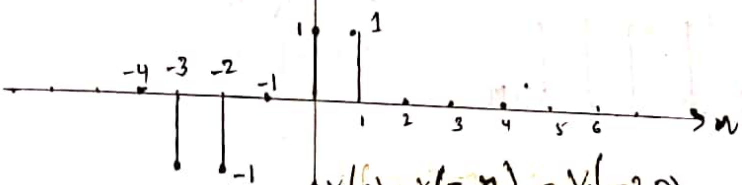


iv) $y(2-2n) = y(-2n+2)$

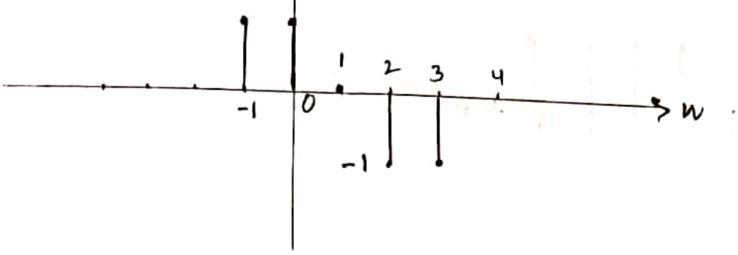
$v_1(n) = y(n+2)$



$v_2(n) = v_1(2n) = y(2n+2)$

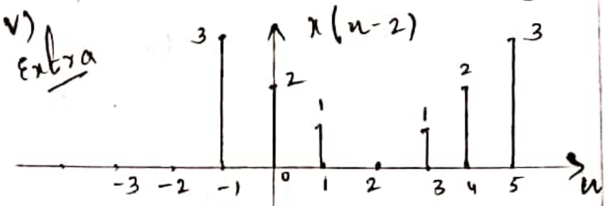


$v_3(n) = v_2(-n) = v_1(-2n) = y(-2n+2)$

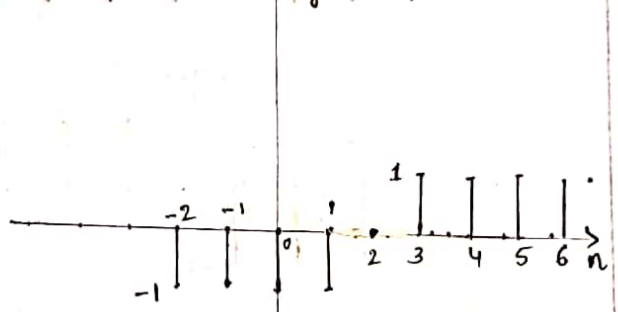


v) Extra

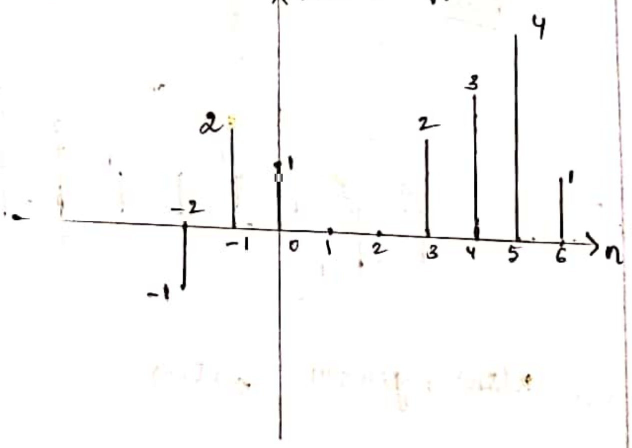
$x(n-2)$



$y(n-2)$



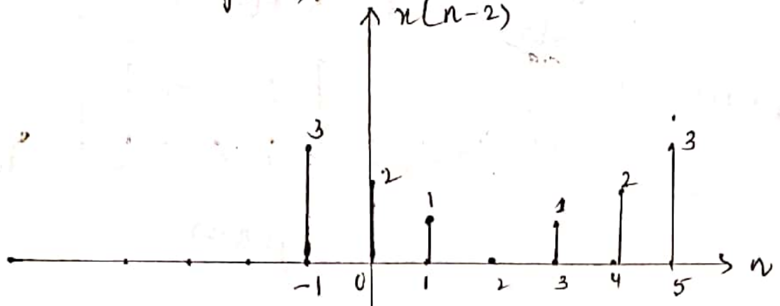
$x(n-2) + y(n-2)$

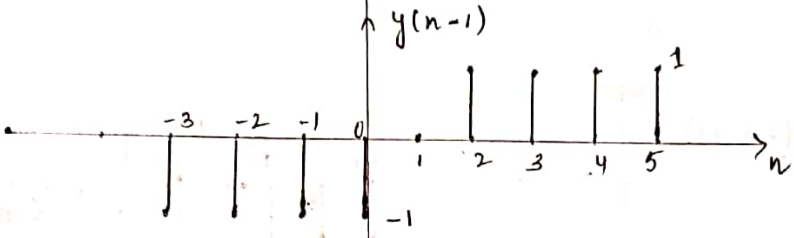


for $v(2n)$

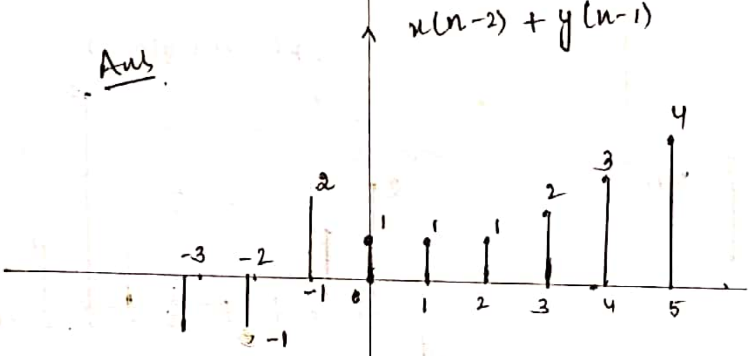
n	$v(2n)$
0	$v(0)$
1	$v(2)$
2	$v(4)$
3	$v(6)$
4	$v(8)$
-1	$v(-2)$
-2	$v(-4)$
-3	$v(-6)$
-4	$v(-8)$

v) = $x(n-2) + y(n-1)$.

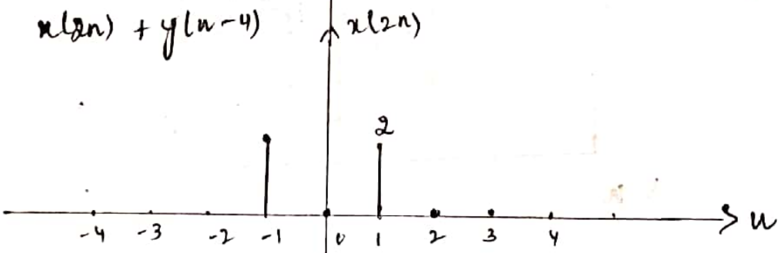


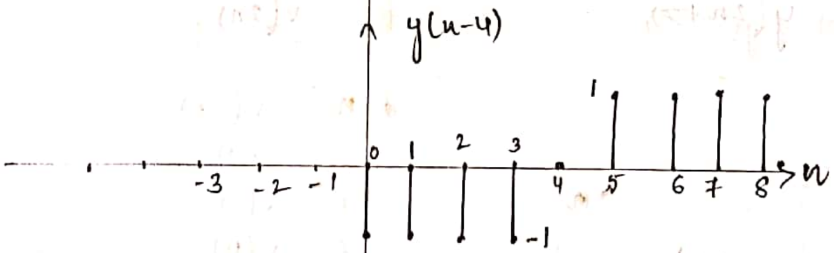


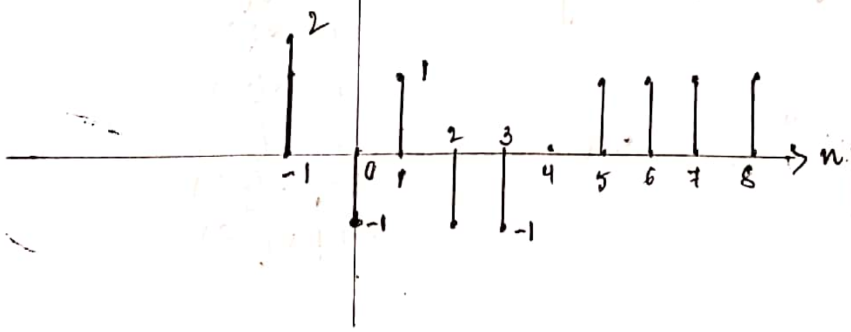
Ans.



vi) $x(2n) + y(n-4)$





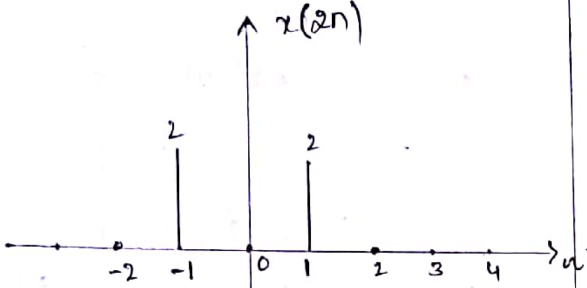


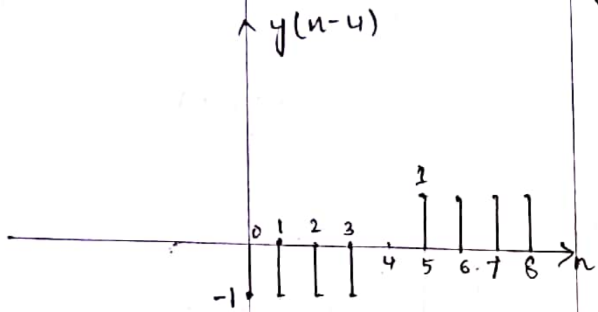
for $x(2n)$

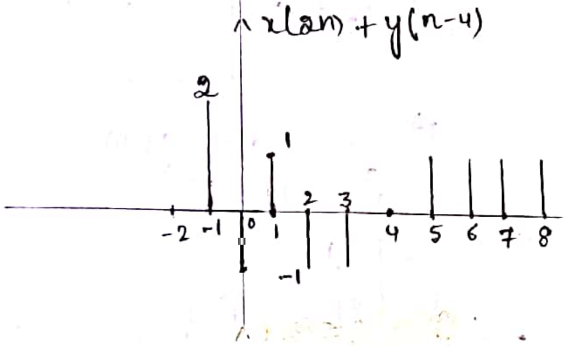
n	$x(2n)$
0	$x(0)$
1	$x(2)$
2	$x(4)$
3	$x(6)$
-1	$x(-2)$
-2	$x(-4)$
-3	$x(-6)$

vi) $x(2n) + y(n-4)$

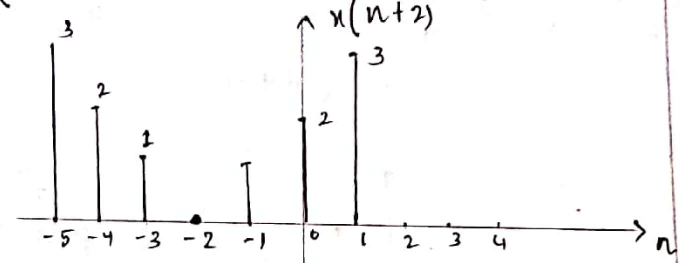
n	$x(2n)$	
0	$x(0)$	$= 0$
1	$x(2)$	$= 2$
2	$x(4)$	$= 0$
-1	$x(-2)$	$= 2$
-2	$x(-4)$	$= 0$

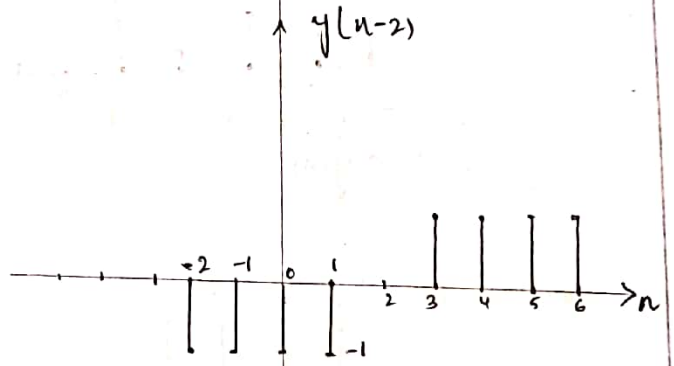


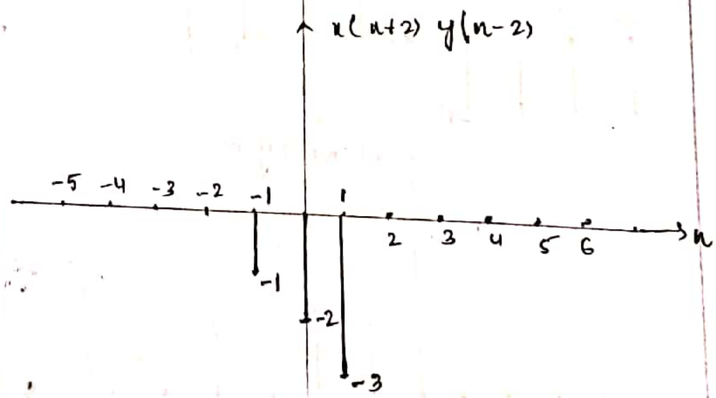




(vii)







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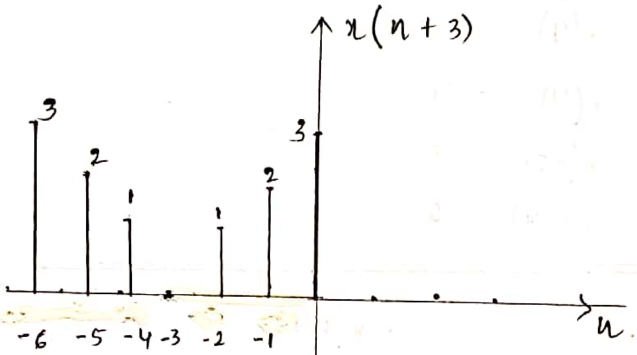
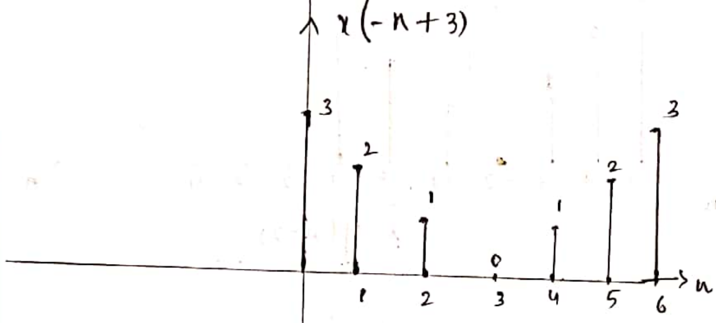
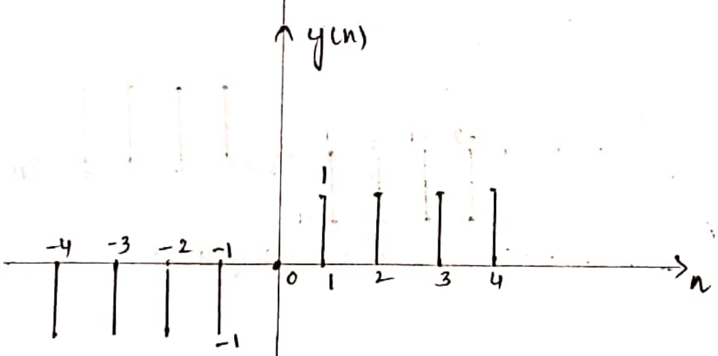
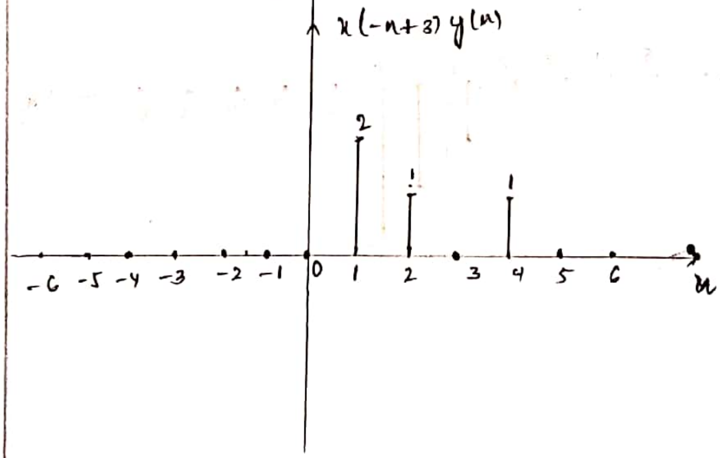
(viii) $x(3-n)\,y(n) = x(-n+3)\,y(n)$

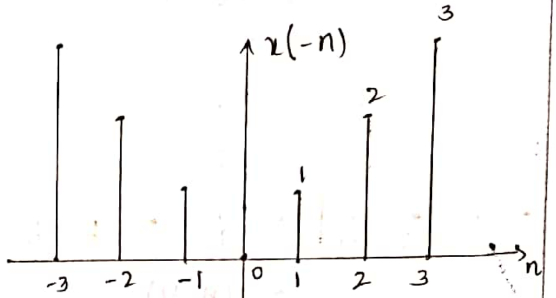


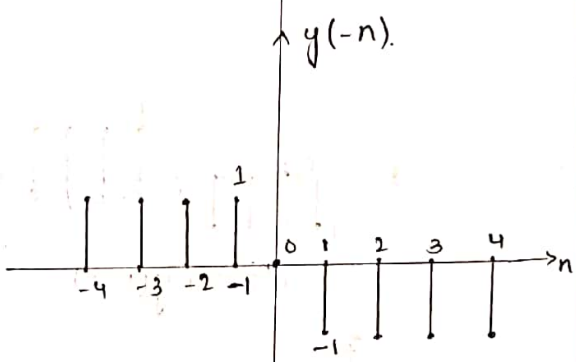


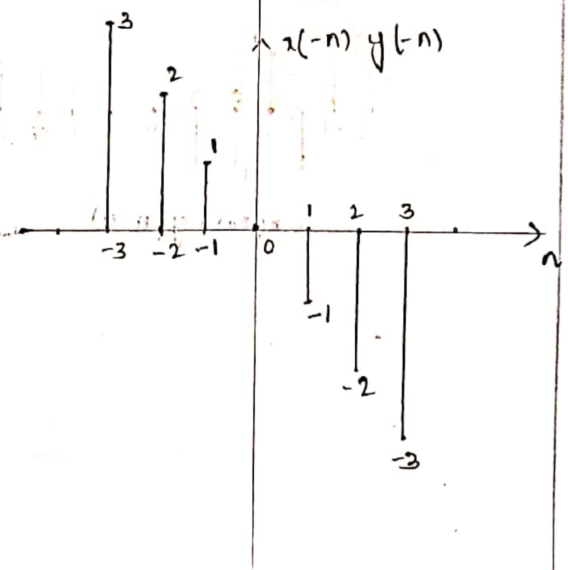




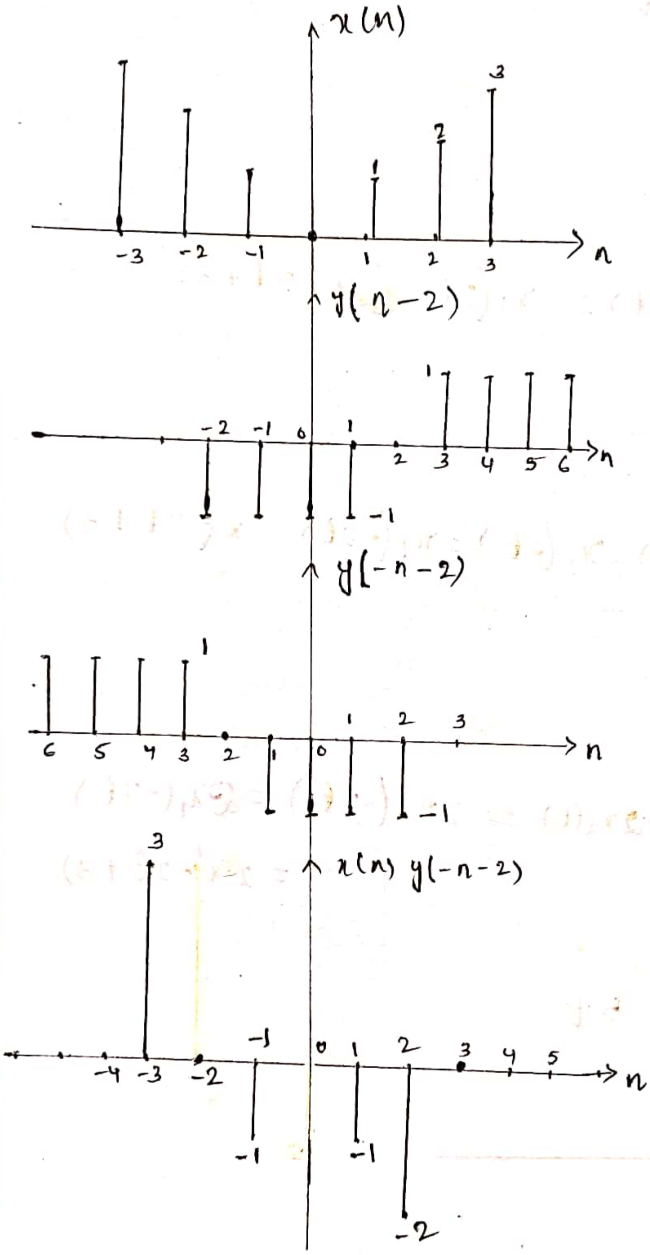
(ix) $x(-n)\,y(-n)$





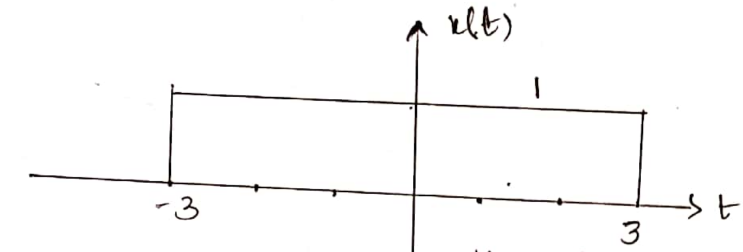


(x) $x(n) \cdot y(-2-n)$
$= x(n)\, y(-n-2)$

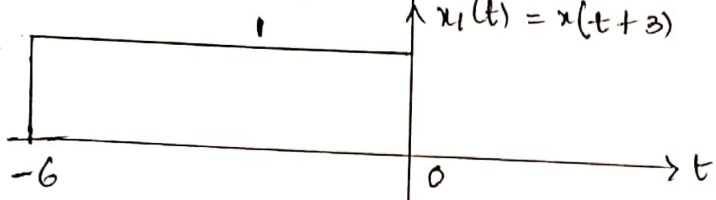


Given $x(t)$, Sketch

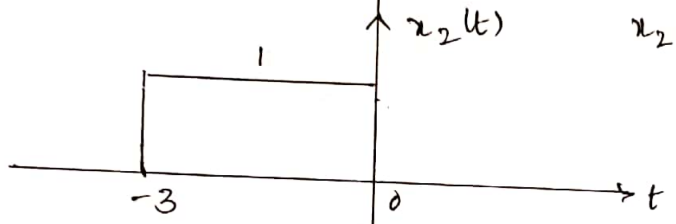
$$y(t) = 2x(-2t+3) + 3$$



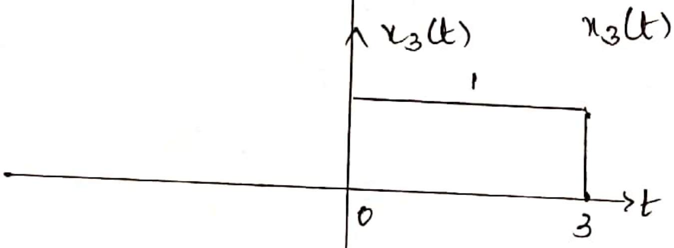
$x_1(t) = x(-t+3)$



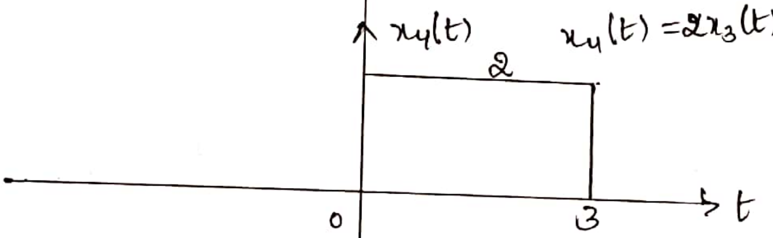
$x_2(t) = x_1(2t) = x(2t+3)$

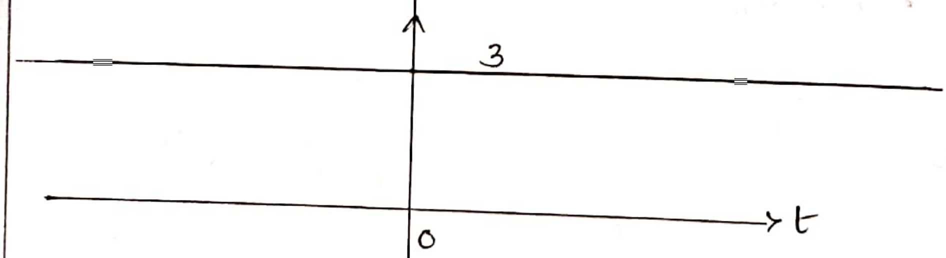


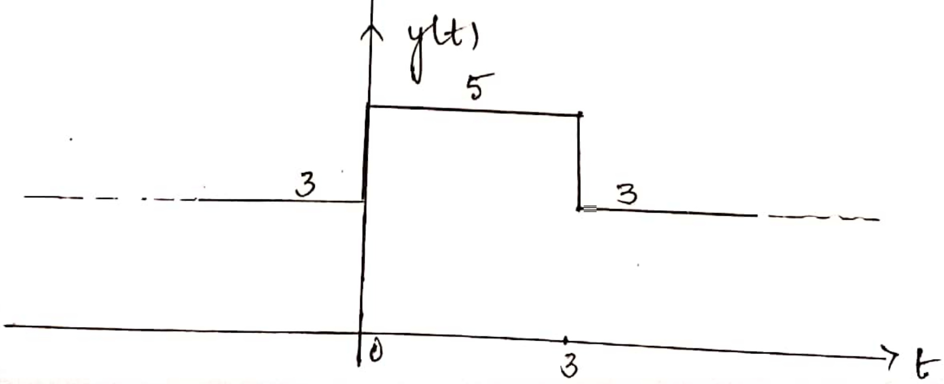
$x_3(t) = x_2(-t) = x_1(-2t) = x(-2t+3)$



$x_4(t) = 2x_3(t) = 2x_2(-t) = 2x_1(-2t)$
$= 2x(-2t+3)$







ELEMENTARY SIGNALS [SELF STUDY]

The basic Elementary signals component are:

1) Exponential signal.
2) Step signal
3) Ramp signal
4) Impulse signal
5) Sinusoidal signal.

1) Exponential Signal.

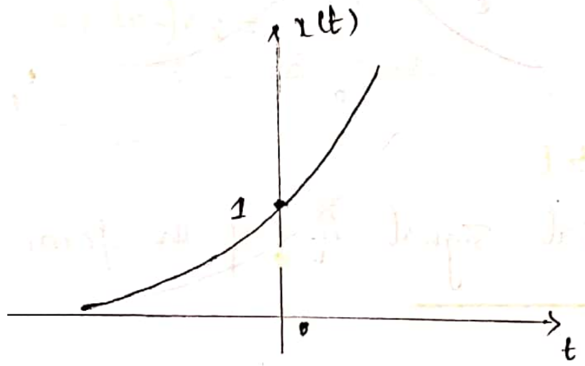
A continuous time exponential signal is of the form

$x(t) = B\,e^{\alpha t}$

If $\alpha > 0$; $x(t)$ is a exponentially growing signal.
If $\alpha < 0$; $x(t)$ is a exponentially decaying signal.

Ex 01: $x(t) = e^{2t}$

$B = 1 \quad \alpha = 2 > 0 \qquad$ for $t = 0 \quad x(0) = e^{0} = 1$

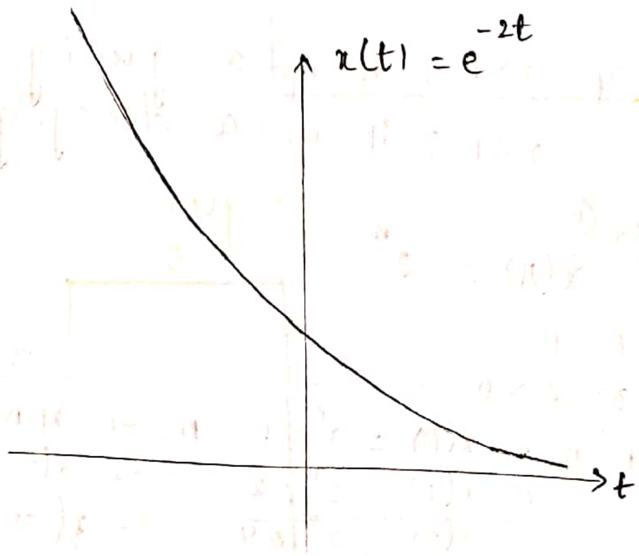


Ex 2: $x(t) = e^{-2t}$

$t = -\infty \quad x(-\infty) = e^{\infty} = \infty$

$t = 0 \quad x(0) = e^{0} = 1$

$t = \infty \quad x(\infty) = e^{-\infty} = 0$



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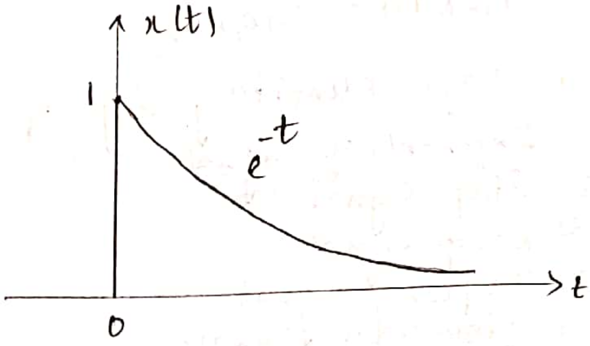
3) $x(t) = e^{-t} \quad t \geq 0$

$\quad\quad 0 \quad t < 0$

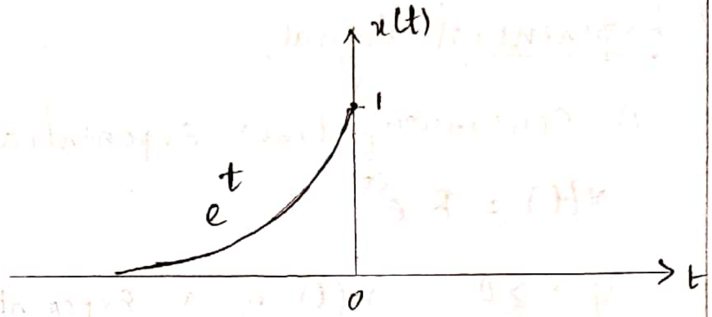


<u>soln</u> $t = 0 \quad x(0) = e^{0} = 1$

Here $B = 1 \quad \alpha = -1 < 0$

4) $x(t) = e^{t} \quad t \leq 0$

$\quad\quad 0 \quad t > 0$



<u>soln</u>

Here $B = 1 \quad \alpha = 1 > 0$

$t = 0 \quad x(0) = e^{0} = 1$

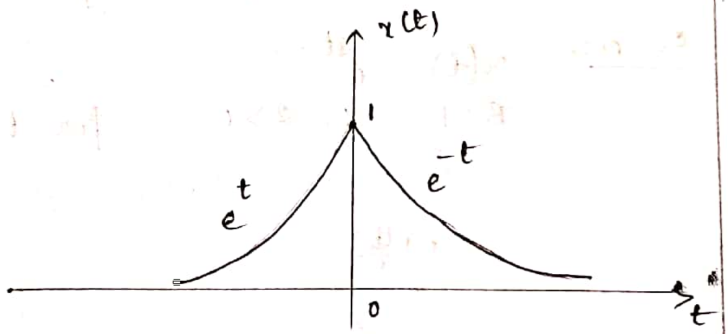
5) $x(t) = e^{-|t|}$

<u>soln</u> given $x(t) = e^{-|t|} = e^{-\pm t}$

for $t > 0 \quad x(t) = e^{-t}$

for $t < 0 \quad x(t) = e^{t}$

$t = 0 \quad x(0) = e^{0} = 1$



- The discrete time exponential signal is of the form

$x(n) = B\gamma^{n}$

If $\gamma > 1$; it is a growing exponential signal

$\gamma < 1$; it is a decaying exponential signal.

<u>Ex ①</u>

$x(n) = 2^{n}$

$B = 1$

$\gamma = 2 > 0$

At $n=0 \quad x(0) = 2^{0} = 1$

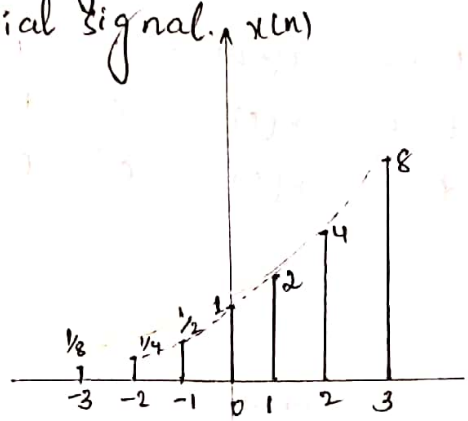
$\quad 1 \quad x(1) = 2^{1} = 2$

$\quad 2 \quad x(2) = 2^{2} = 4$

$n = -1 \quad x(-1) = 2^{-1} = 1/2$

$\quad -2 \quad x(-2) = 2^{-2} = 1/4$

$\quad 3 \quad x(-3) = 2^{-3} = \frac{1}{8}$

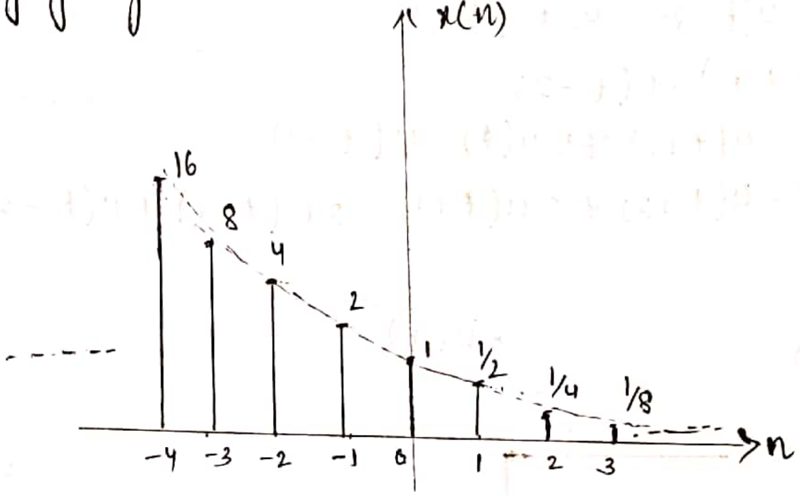


⇒ ②

$x(n) = \left(\frac{1}{2}\right)^n$

$B=1 \quad r = \frac{1}{2} < 0$; so decaying signal

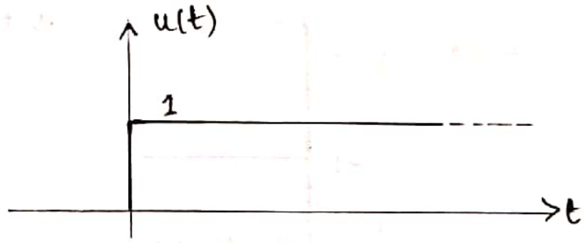
n	$x(n) = (1/2)^n$
0	$x(0) = (1/2)^0 = 1$
1	$x(1) = (1/2)^1 = 1/2$
2	$x(2) = (1/2)^2 = 1/4$
-1	$x(-1) = (1/2)^{-1} = 2$
-2	$x(-2) = (1/2)^{-2} = 4$
-3	$x(-3) = (1/2)^{-3} = 8$



2) <u>Step signal</u>

The continuous time unit step signal is defined as

$u(t) = 1 \quad t \geq 0$
$ 0 \quad t < 0$



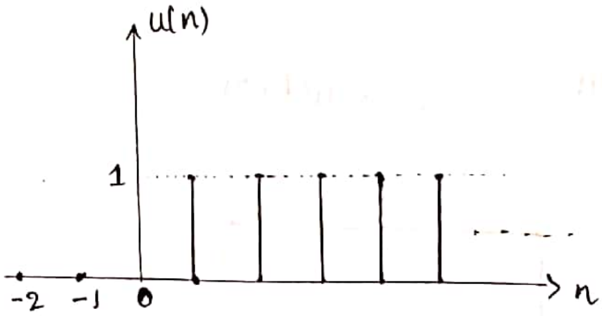
<u>Eg</u>: $x(t) = e^{-2t} u(t)$

it is written as

$x(t) = e^{-2t} \quad t \geq 0 \quad \because u(t) = 1$
$ = 0 \quad t < 0 \quad \because u(t) = 0.$

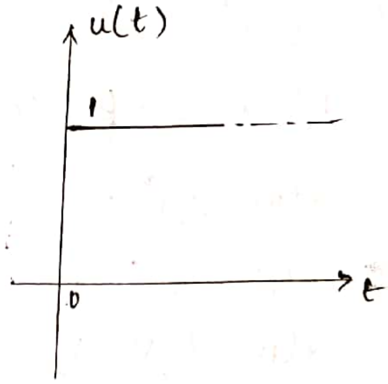
A discrete time unit step signal is defined as

$u(n) = 1 \quad n \geq 0$
$ 0 \quad n < 0$



Sketch the following signals; Given u(t)

1) $u(-t)$ 2) $-u(-t)$
3) $u(t-2)$ 4) $u(t+3)$
5) $u(t) - u(t-2)$
6) $-u(t+1) + 2u(t) - u(t-1)$
7) $-u(t+2) + 2u(t+1) - 2u(t-1) + u(t-2)$



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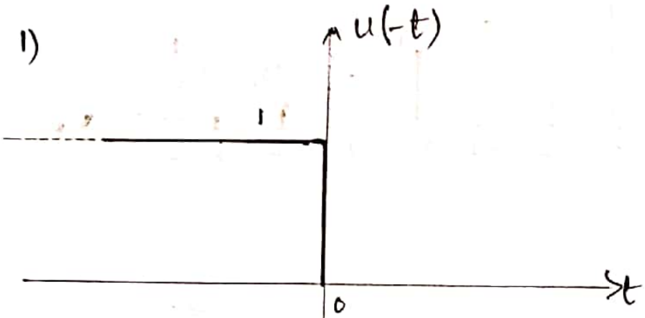
1)



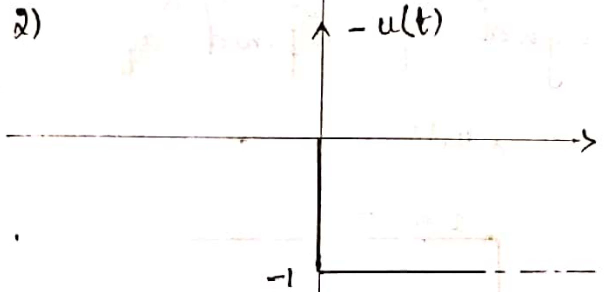
$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

we need $-t$, so

$-t \geq 0$

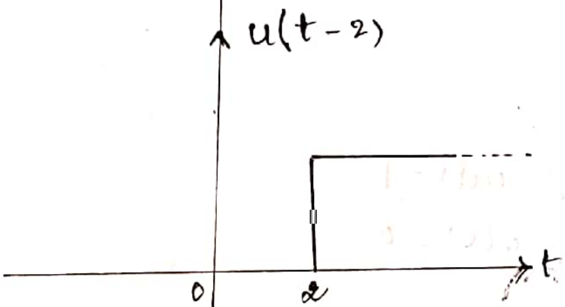
OR $t \leq 0$

2)



⇐ All the amplitude value inverted.

3)



shift u(t) towards right side by 2 units.

OR

$u(t) = 1 \quad t \geq 0$

$\quad\quad 0 \quad$ otherwise

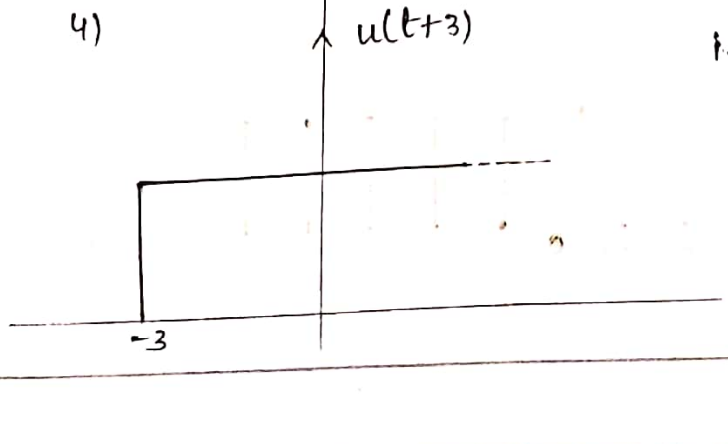
But we need $t-2$

$t-2 \geq 0$

$t \geq 2$

i.e., $u(t) = 1 \quad t \geq 2$

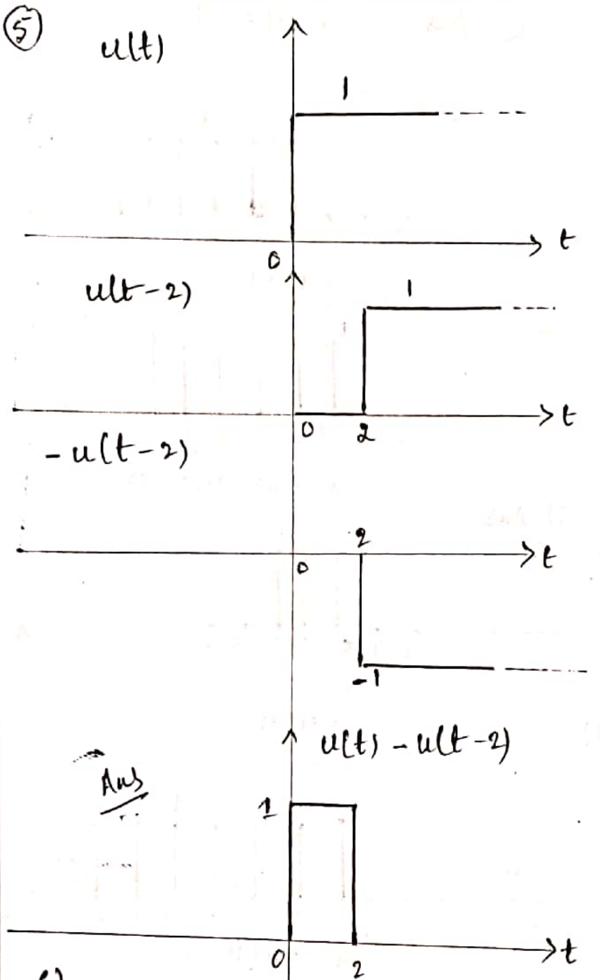
4)



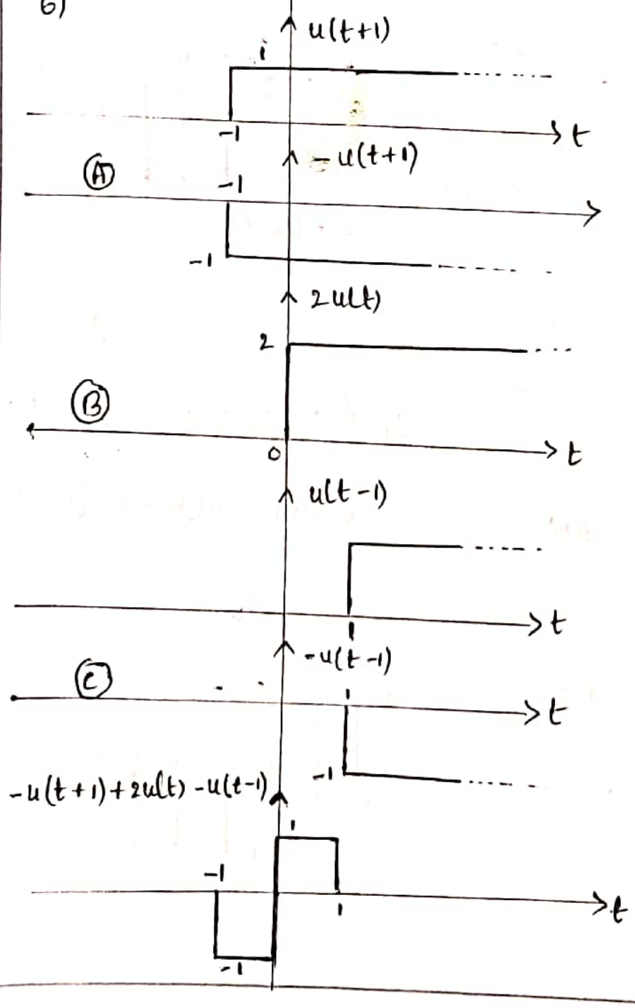
⇐ u(t) shifted to left by 3 units.

Scanned by CamScanner

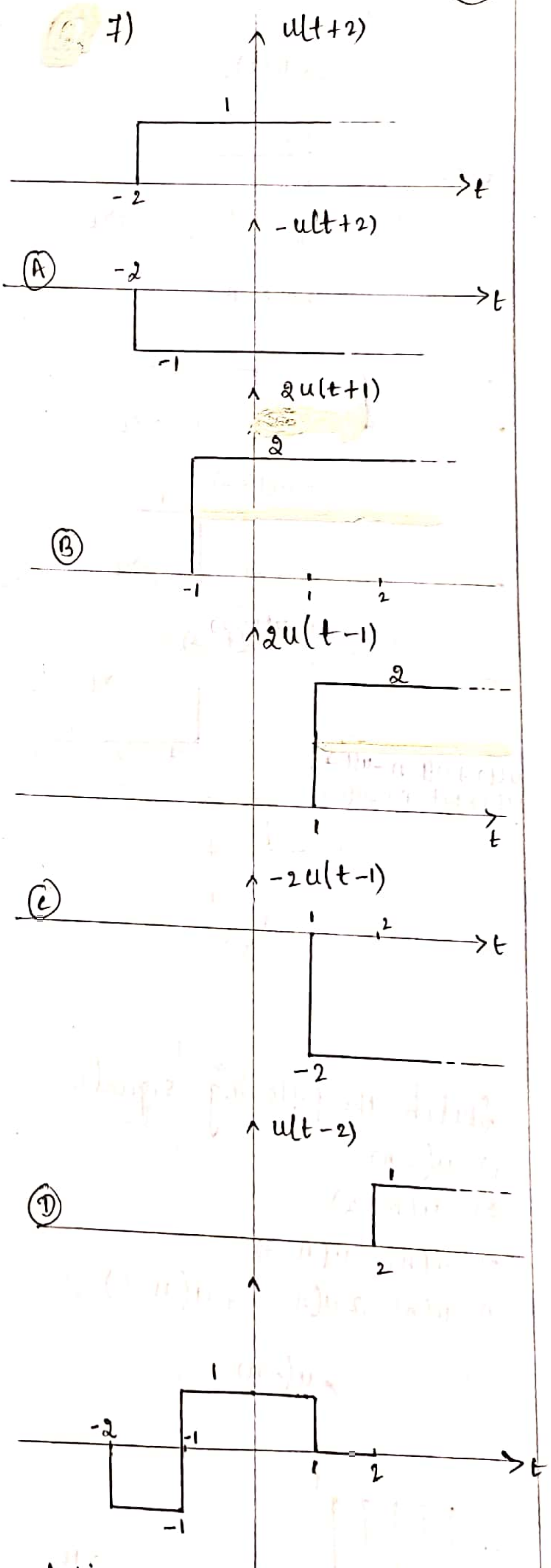
5)



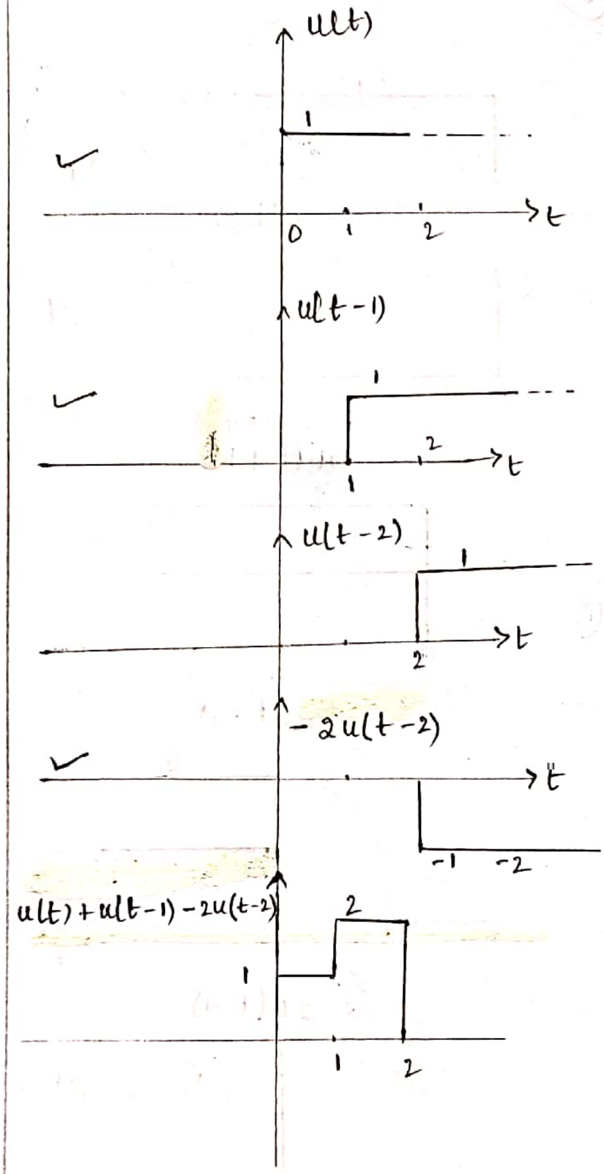
6)



7)

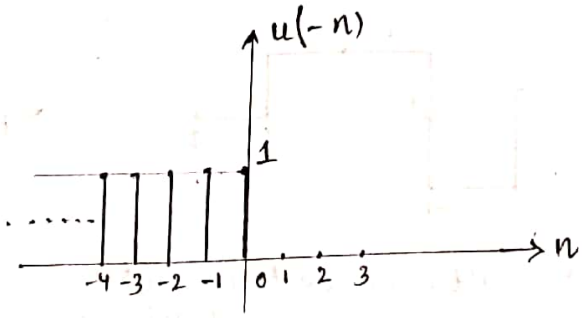


Ans: $-u(t+2)+2u(t+1)-2u(t-1)+u(t-2)$

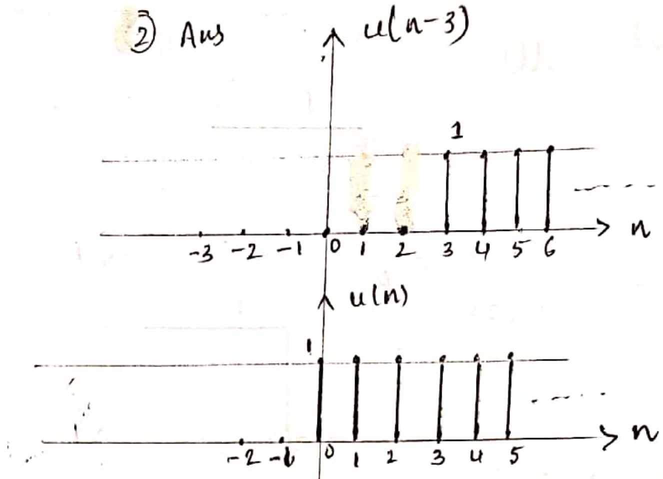


Sketch the following signals.

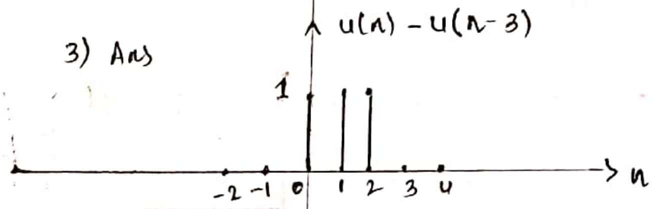
1) $u(-n)$
2) $u(n-3)$
3) $u(n) - u(n-3)$
4) $u(n) - 2u(n-3) + u(n-6)$



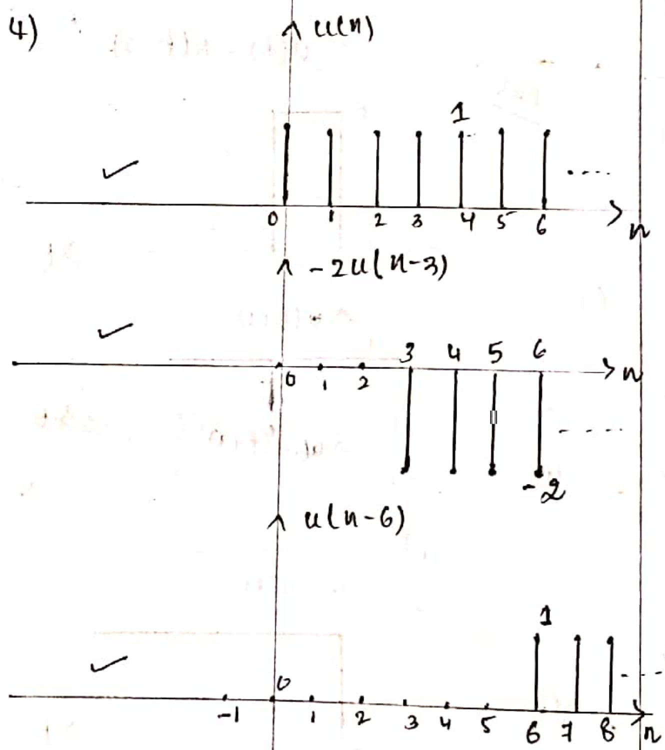
2) Ans



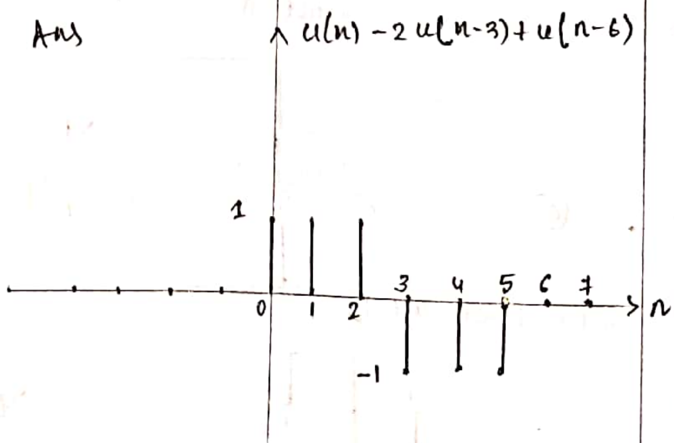
3) Ans



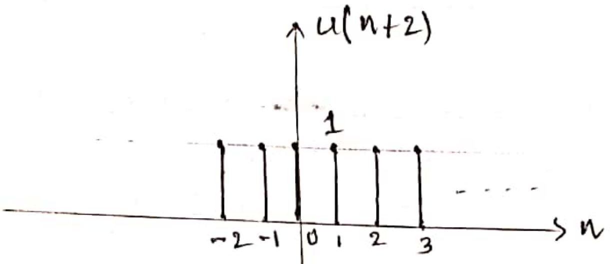
4)

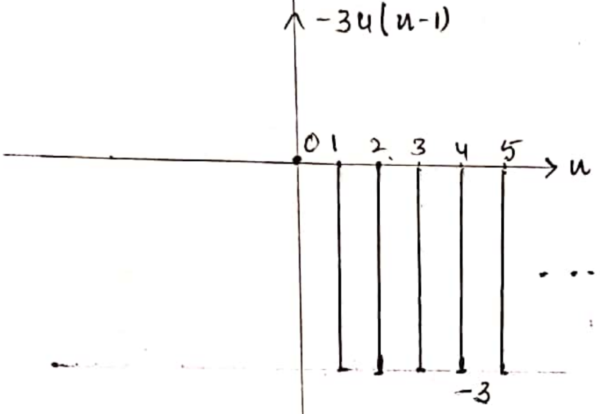


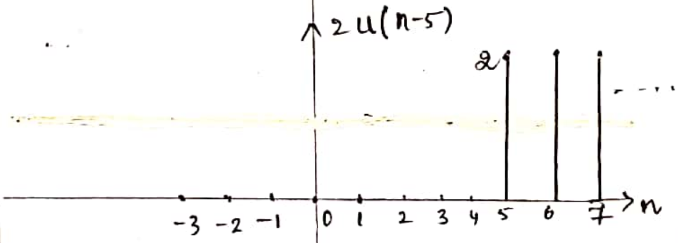
Ans



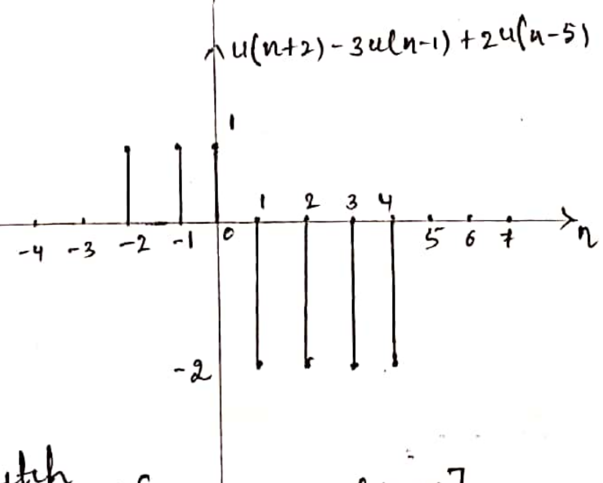
5) $u(n+2) - 3u(n-1) + 2u(n-5)$







Ans

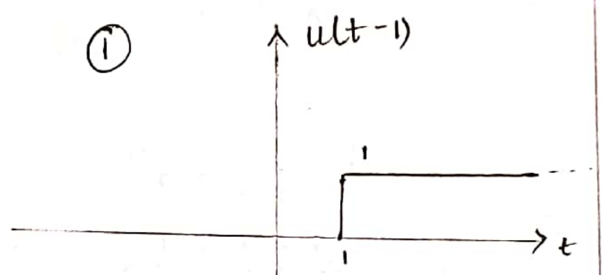


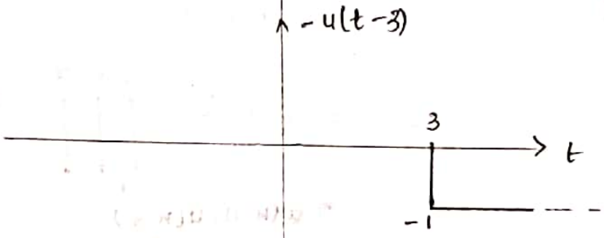
Sketch

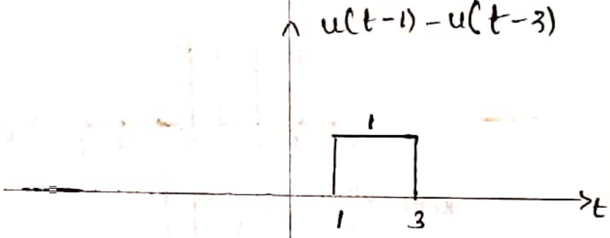
1) $x(t) = e^{-t}[u(t-1) - u(t-3)]$

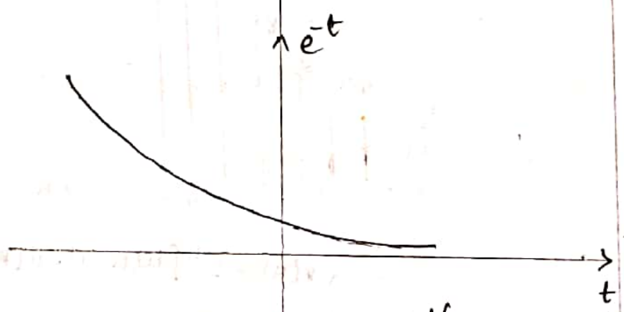
2) $x(n) = 2^n[u(n-1) - u(n-5)]$

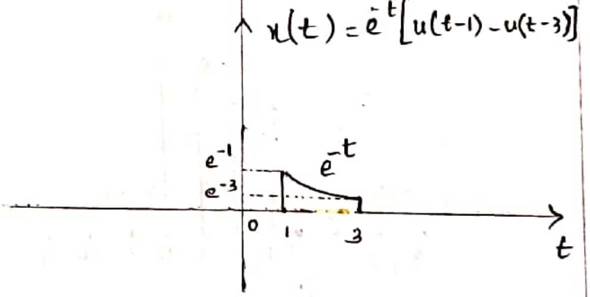
(1)

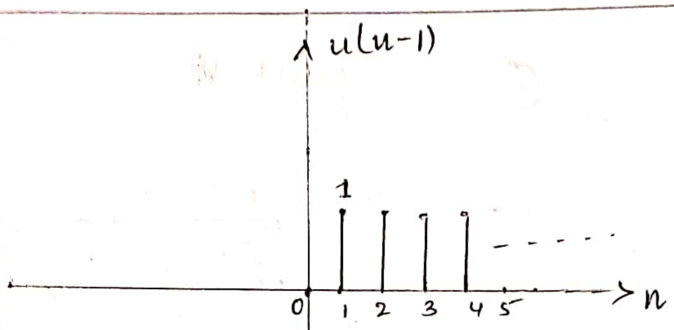


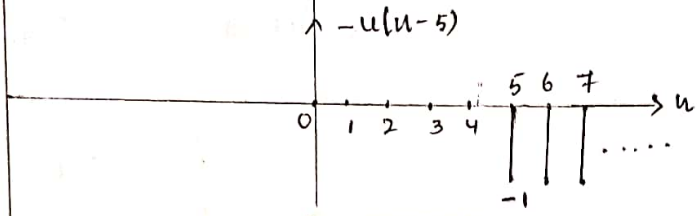


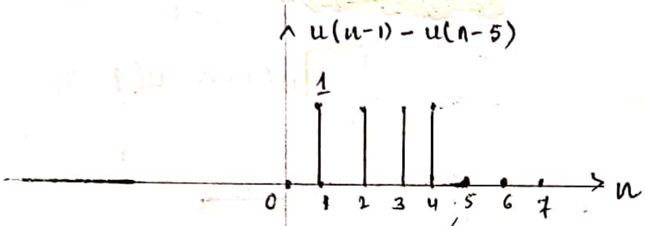


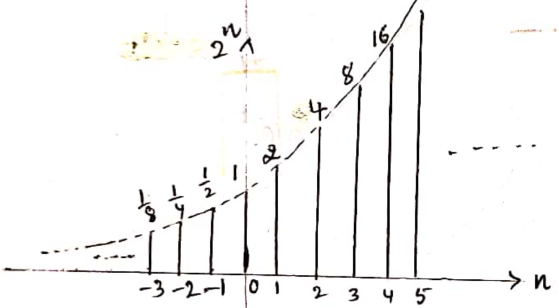





u(n-1)
1
0 1 2 3 4 5
n


-u(n-5)
5 6 7
0 1 2 3 4
-1
n


u(n-1) - u(n-5)
1
0 1 2 3 4 5 6 7
n


2^n
1/8 1/4 1/2 1 2 4 8 16
-3 -2 -1 0 1 2 3 4 5
n

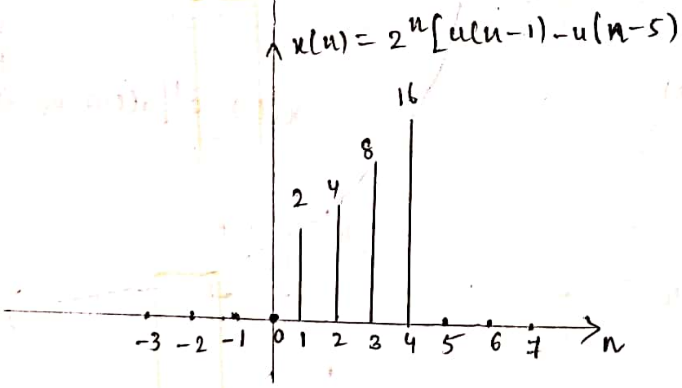
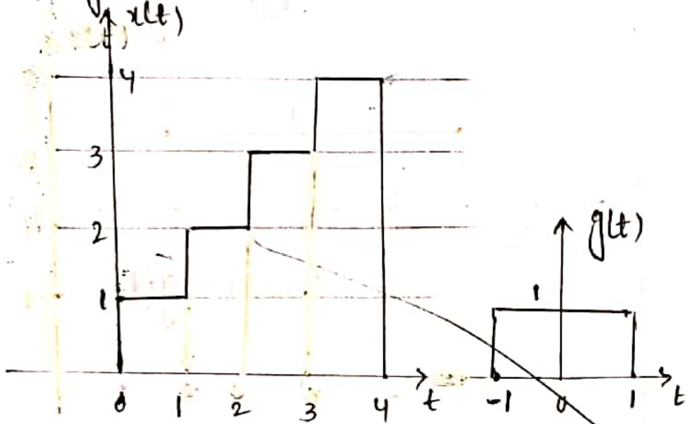
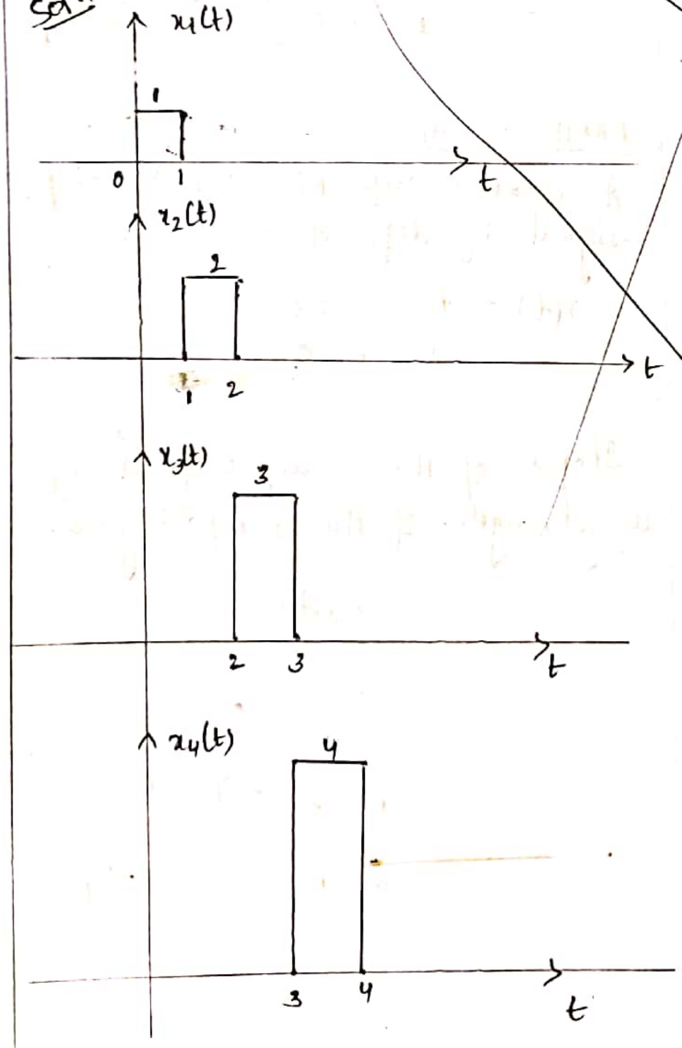

x(n) = 2^n[u(n-1) - u(n-5)]
2 4 8 16
-3 -2 -1 0 1 2 3 4 5 6 7
n

Figure shows a staircase like signal $x(t)$ that can be thought of as superposition of 4 rectangular pulses. Starting with the rectangular pulse $g(t)$ shown in fig, construct this waveform. Express $x(t)$ in terms of $g(t)$.



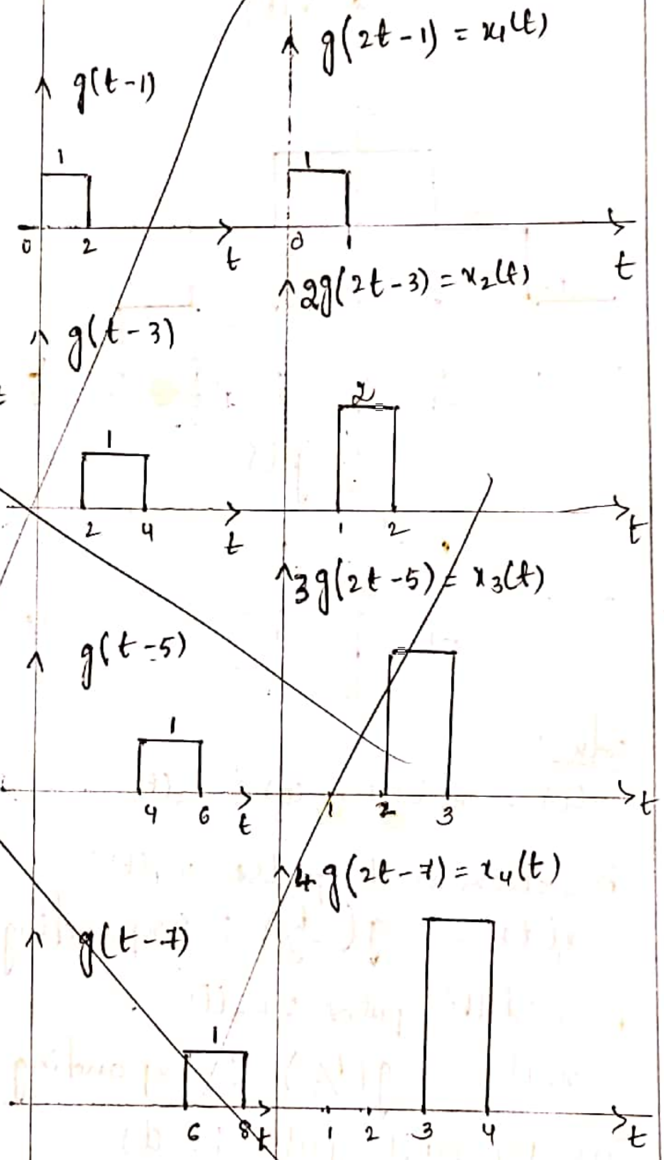
Soln



It is ~~found~~ seen that

$x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$

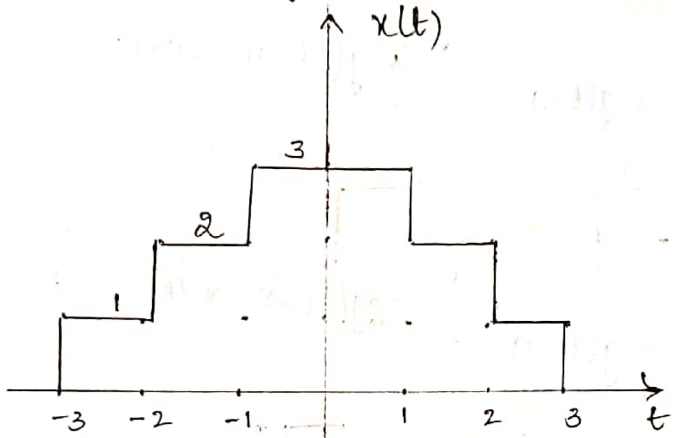
Now to express x_1 x_2 x_3 & x_4 in terms of $g(t)$

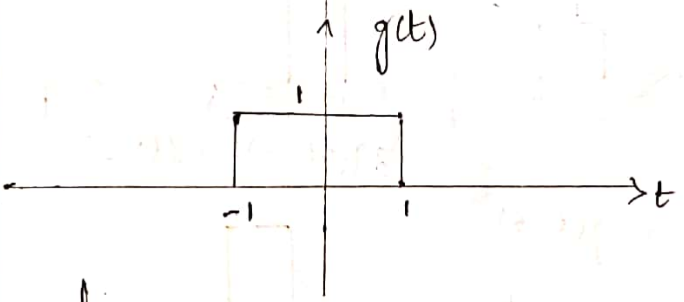


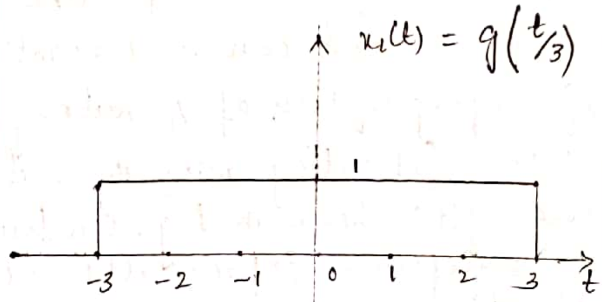
For $x_1(t)$: shift $g(t)$ right 1 unit; Now signal reside in $0 \le t \le 2$; So compress time by taking at to its half

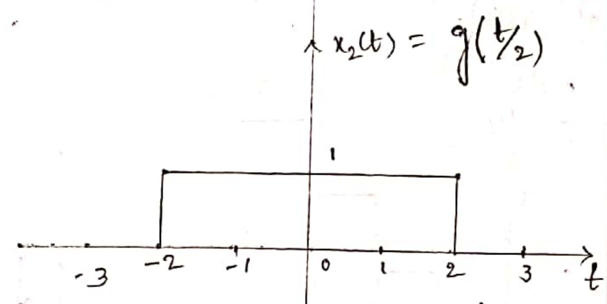
Q2

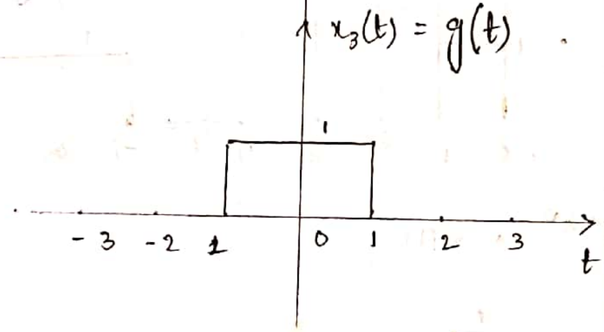
Figure shows a pulse $x(t)$ that may be thought of as superposition of 3 rectangular pulses. Starting from rectangular pulse $g(t)$, construct $x(t)$. Express $x(t)$ in terms of $g(t)$.











Soln:

$x(t) = x_1(t) + x_2(t) + x_3(t)$

i) bottom most pulse $x_1(t)$

$x_1(t) = g\left(\frac{t}{3}\right)$: Expanding

ii) middle pulse : $x_2(t)$

$x_2(t) = g(t/2)$: Expanding

iii) Top most pulse : $x_3(t)$

$x_3(t) = g(t)$: No change

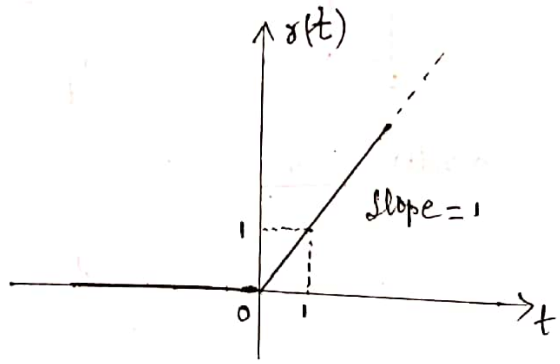
$$x(t) = g(t/3) + g(t/2) + g(t)$$

RAMP SIGNAL

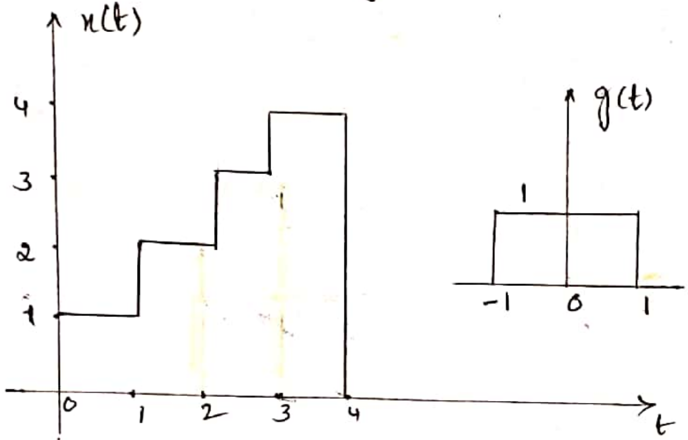
A continuous time unit ramp signal is defined as

$$r(t) = t \quad t \geq 0$$
$$= 0 \quad t < 0$$

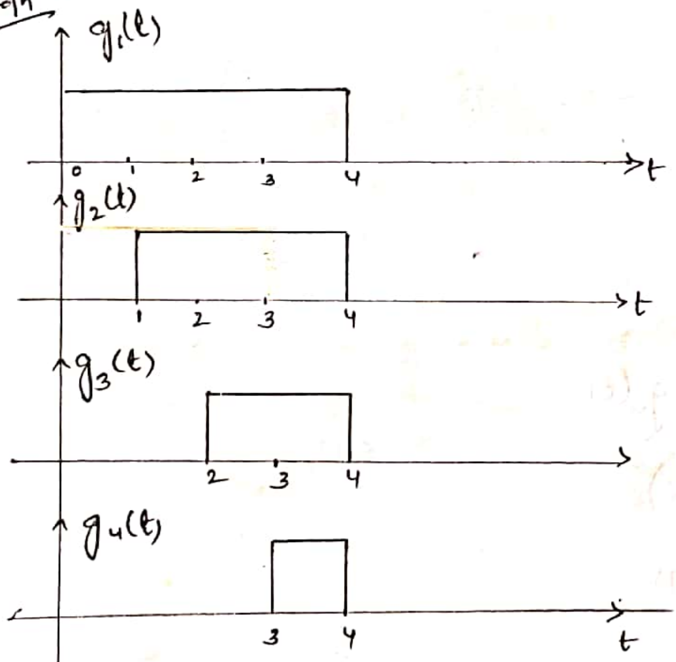
Slope of the ramp signal is the strength of the ramp signal.



Qn/ Figure shows a staircase like signal $x(t)$ that can be thought of as superposition of 4 rectangular pulses. Starting ~~from~~ with rectangular pulse $g(t)$ shown in figure. construct this waveform. Express $x(t)$ in terms of $g(t)$.



Soln



From figure

$$x(t) = g_1(t) + g_2(t) + g_3(t) + g_4(t)$$

To express $g_i(t)$ in terms of $g(t)$

$g_i(t) = g(at-b)$, where

$$\boxed{a = \frac{\text{width of } g(t)}{\text{width of } g_i(t)}}$$

To find b

$$\boxed{at - b = 0}$$

$t \rightarrow$ mid point of $g_i(t)$

1) $g_1(t) = g(at-b)$

$a = \frac{2}{4} = \frac{1}{2}$

$t = 2$

$at - b = 0$

$\frac{1}{2} 2 - b = 0$ $\quad \boxed{b = 1}$

$$\boxed{g_1(t) = g\left(\tfrac{1}{2}t - 1\right)}$$

2) $g_2(t) = g(at-b)$

$a = \frac{2}{3}$

$t = \frac{5}{2}$

$at - b = 0$

$\frac{2}{3}\frac{5}{2} - b = 0$ $\quad b = \frac{5}{3}$

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$$\boxed{g_2(t) = g\left(\frac{2}{3}t - \frac{5}{3}\right)}$$

$g_3(t) = g(at - b)$

$a = \frac{2}{2}$ $\underline{a=1}$

$t = 3$

$at - b = 0$

$1(3) - b = 0$ $\underline{b = 3}$

$$\boxed{g_3(t) = g(t-3)}$$

$g_4(t) = g(at - b)$

$a = \frac{2}{1}$ $\underline{a = 2}$

$at - b = 0$

$2\left(\frac{7}{2}\right) - b = 0$ $\underline{b = 7}$

$$\boxed{g_4(t) = g(2t - 7)}$$

Thus

$= g_1(t) + g_2(t) + g_3(t) + g_4(t)$

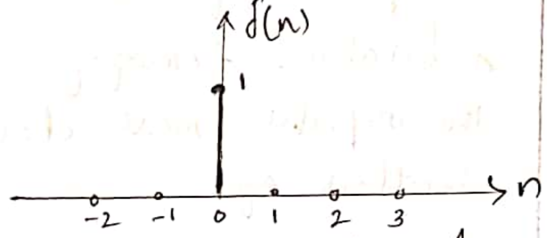
$x(t) = g(\frac{1}{2}t - 1) + g\left(\frac{2}{3}t - \frac{5}{3}\right)$

$g(t-3) + g(2t - 7)$

IMPULSE FUNCTION.

The discrete time version of the UNIT IMPULSE is defined as

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



The continuous time version of unit impulse is defined by the following pair of relations.

$$\delta(t) = 0 \quad \text{for } t \neq 0. \longrightarrow ①$$

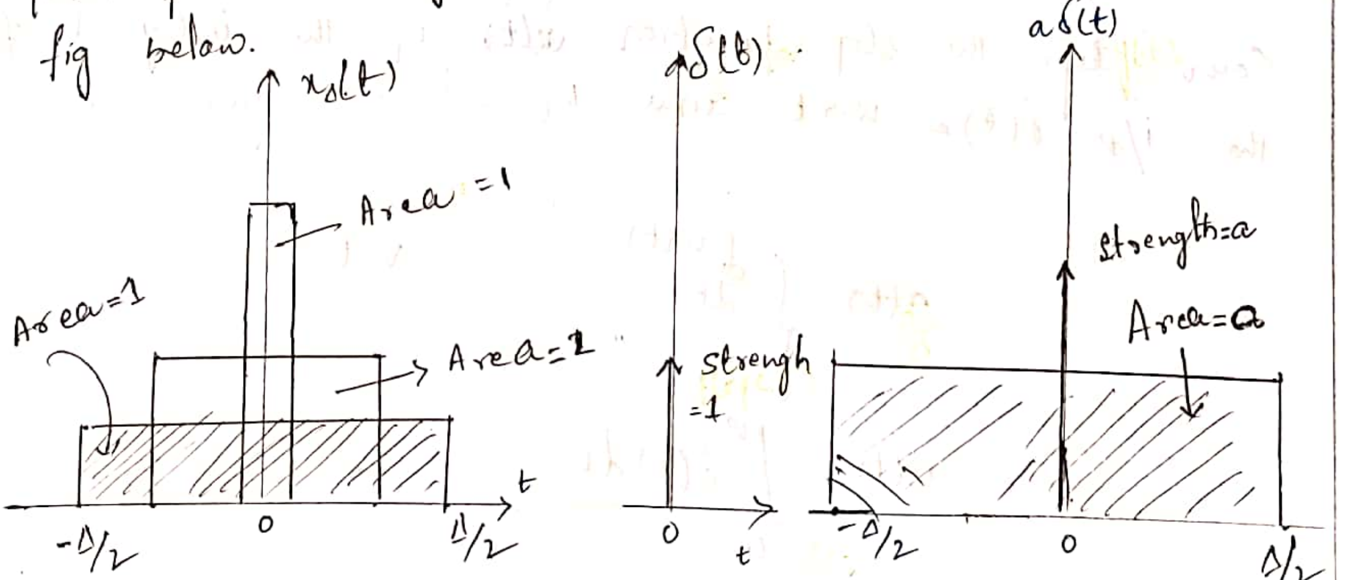
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1 \longrightarrow ②$$

Eqn ① says that the impulse $\delta(t)$ is zero everywhere except at the origin.

Eqn ② says that the total area under the unit impulse is UNITY.

→ The impulse function $\delta(t)$ is also called as Dirac delta function.

The unit impulse $\delta(t)$ can be viewed as the limiting form of a rectangular pulse of unit area, as shown in fig below.



Specifically, the duration of the pulse is decreased, and its amplitude is increased, such that the area under the pulse is maintained ~~at~~ constant at unity. As the A duration decreases, the rectangular pulse approximates the impulse more closely. This in generally can be written as

$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$

where $x_\Delta(t)$ is any pulse that is an even function of time t with duration Δ & unit area.

The AREA under the pulse defines the STENGTH ~~under~~ of the impulse.

An impulse strength a is written as

$a\delta(t)$

Also we can write.

$$\delta(t) = \frac{d\,u(t)}{dt}$$

Conversely, the step function $u(t)$ is the integral of the i/p $\delta(t)$ w.r.t time is and it is given as.

$$\cancel{\delta(t) = \int \frac{d\,u(t)}{dt}} \qquad \cancel{\forall t}$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau .$$

Unit impulse function is even function of time t.

$\therefore \quad \delta(-t) = \delta(t)$

Properties of Unit impulse function

1) $\delta(-t) = \delta(t)$

2) $x(t) \cdot \delta(t) = x(t)\big|_{t=0} \delta(t) = x(0)\,\delta(t)$

Here $x(0)$ is the strength of impulse signal at $t=0$

3) $x(t)\,\delta(t-t_0) = x(t)\big|_{t=t_0} \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$.

4) $\delta(at) = \frac{1}{a}\,\delta(t)$ for $a>0$.

5) $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = \underbrace{\int_{-\infty}^{0^-} x(t)\,\delta(t)\,dt}_{0} + \int_{0^-}^{0^+} x(t)\,\delta(t)\,dt + \underbrace{\int_{0^+}^{\infty} x(t)\,\delta(t)\,dt}_{0}$

$= x(t)\big|_{t=0} = x(0)$; if $x(t)$ is continuous at $t=0$.

6) $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t)\big|_{t=t_0}$ {provided $x(t)$ is continuous at $t=t_0$.}

$= x(t_0)$

7) $\delta(at-b) = \delta\left[a\left(t-\frac{b}{a}\right)\right] = \frac{1}{a}\delta\left(t-\frac{b}{a}\right)$

8) $\frac{d[u(t)]}{dt} = \delta(t)$

9) $u(n) - u(n-1) = \delta(n)$

10) $\sum_{k=0}^{\infty} \delta(n-k) = u(n)$

11) $\int_{-\infty}^{\infty} \delta(t)\,dt = 1$

Evaluate the following

① $\int_{-2}^{1} (3t^2+1)\,\delta(t)\,dt$ recall $\int_{-2}^{1} x(t)\,\delta(t)\,dt = x(0)$

Here $x(t) = 3t^2+1$

$$\int_{-2}^{1} (3t^2+1)\,\delta(t)\,dt = (3t^2+1)\Big|_{t=0} = \underline{\underline{1}}$$

② $\int_{1}^{2} (3t^2+1)\,\delta(t)\,dt = 0$

$\because$ the given limit 1 to 2 doesn't include $t=0$ at which $\delta(t)$ exists. $\therefore$ value of integral is equal to 0.

3) $\int_{-\infty}^{\infty} (t^2+\cos\pi t)\,\delta(t-1)\,dt$

$= (t^2+\cos\pi t)\Big|_{t-1=0;\ t=1}$

$= 1 + \cos\pi \cdot 1$

$= \underline{\underline{0}}$

4) $\int_{-\infty}^{\infty} e^{-t}\,\delta(2t-2)\,dt$

$= \int_{-\infty}^{\infty} e^{-t}\,\delta[2(t-1)]\,dt$

$= \int_{-\infty}^{\infty} e^{-t}\,\frac{1}{2}\,\delta(t-1)\,dt$ $\quad \because \delta(at) = \frac{1}{a}\delta(t)$

$= \frac{1}{2}\,e^{-t}\Big|_{t=1}$

$= \underline{\underline{\frac{1}{2e}}}$

5) $x(t) = \int_{-\infty}^{\infty} 4t^2\,\delta(2t-4)\,dt$

$x(t) = \int_{-\infty}^{\infty} 4t^2\,\delta[2(t-2)]\,dt$

$= \int_{-\infty}^{\infty} 4t^2\,\frac{1}{2}\,\delta(t-2)\,dt$

$= 2\,t^2\Big|_{t=2}$

$= 2 \cdot 2^2 = \underline{\underline{8}}$

6)

$$x(t) = \int_0^\infty 4t^2\, \delta(t-1)\, dt$$

$$= 4t^2 \Big|_{t=1}$$

$$= 4 \cdot 1^2 = \underline{\underline{4}}.$$

7)

$$\int_{-4}^{2} \cos 2\pi t\ \delta(2t+1)\, dt$$

$$= \int_{-4}^{2} \cos 2\pi t\ \delta\left[2\left(t+\tfrac{1}{2}\right)\right] dt$$

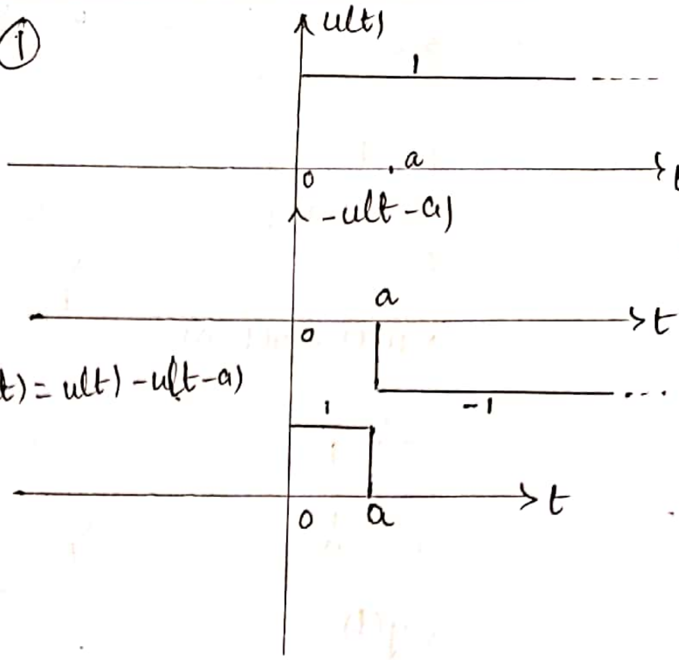
$$= \int_{-4}^{2} \cos 2\pi t\ \frac{1}{2}\ \delta\left(t+\tfrac{1}{2}\right) dt$$

$$= \frac{1}{2} \cos 2\pi t \Big|_{t=-1/2}$$

$$= \frac{1}{2} \cos\left(-\cancel{2}\pi \frac{1}{\cancel{2}}\right)$$

$$= \underline{\underline{-\frac{1}{2}}}.$$

1

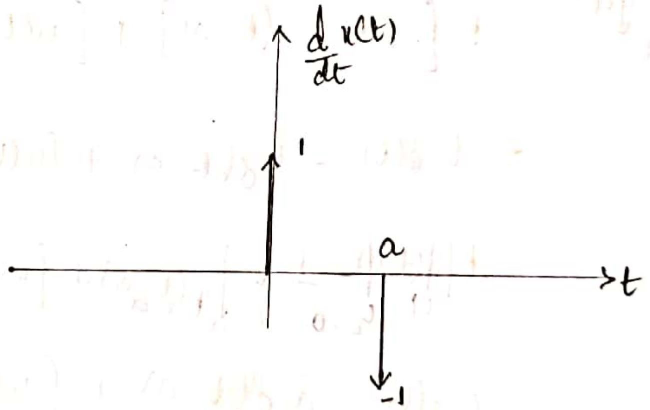


$x(t) = u(t) - u(t-a)$

Also

$$x(t) = u(t) - u(t-a)$$

$$\frac{d}{dt} x(t) = \delta(t) - \delta(t-a)$$



8 Find & sketch the first derivative of the following signal.

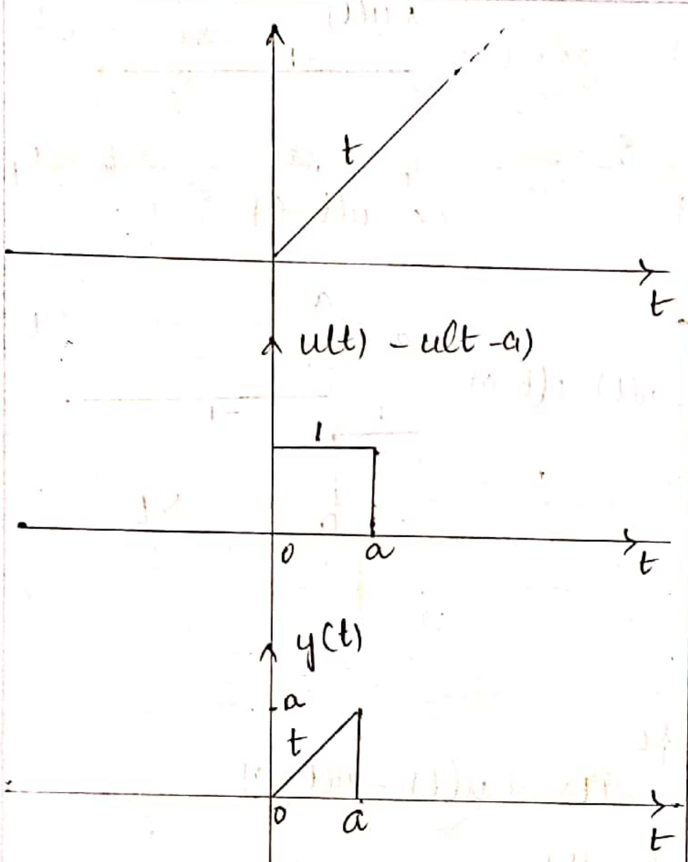
1) $x(t) = u(t) - u(t-a)$ $a>0$.

2) $y(t) = t\,[u(t) - u(t-a)]$

3) ~~$x(t) = \mathrm{sgn}(t)$~~

2

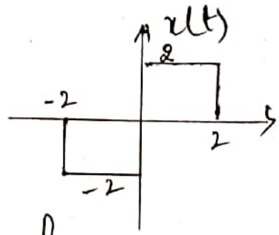
PTO.



Qn

$\Rightarrow$ Sketch $x(t)\left[\delta(t+1) - \delta(t-1)\right]$

given $x(t)$

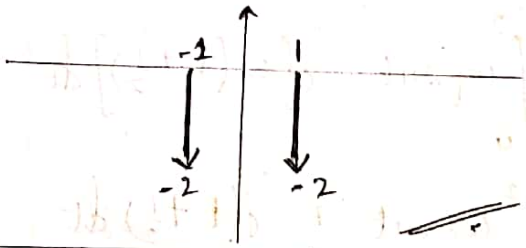


<u>soln</u>

$x(t)\left[\delta(t+1) - \delta(t-1)\right]$

$= x(t)\Big|_{t=-1}\delta(t+1) - x(t)\Big|_{t=1}\delta(t-1)$

$= x(-1)\,\delta(t+1) - x(1)\,\delta(t-1)$



Now

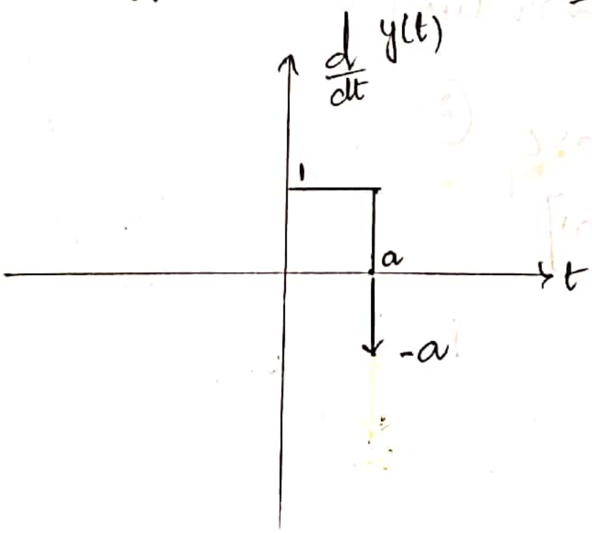
$$\frac{d\,y(t)}{dt} = t\left[\delta(t) - \delta(t-a)\right] + \left[u(t) - u(t-a)\right]$$; using product rule.

$$= t\,\delta(t) - t\,\delta(t-a) + \left[u(t) - u(t-a)\right]$$

$$= t\Big|_{t=0}\delta(t) - t\Big|_{t=a}\delta(t-a) + \left[u(t) - u(t-a)\right]$$

$$= 0\,\delta(t) - a\,\delta(t-a) + \left[u(t) - u(t-a)\right]$$

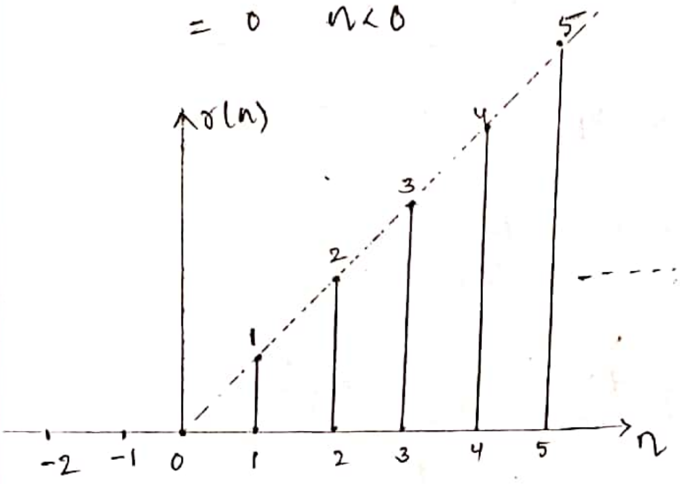
$$= -a\,\delta(t-a) + \left[u(t) - u(t-a)\right]$$



Q) A discrete time unit ramp signal is defined as

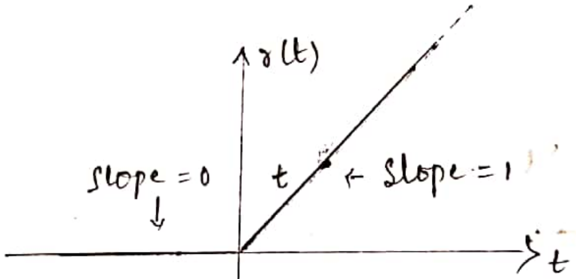
$r(n) = n \quad n \geq 0$

$\quad\;\; = 0 \quad n < 0$

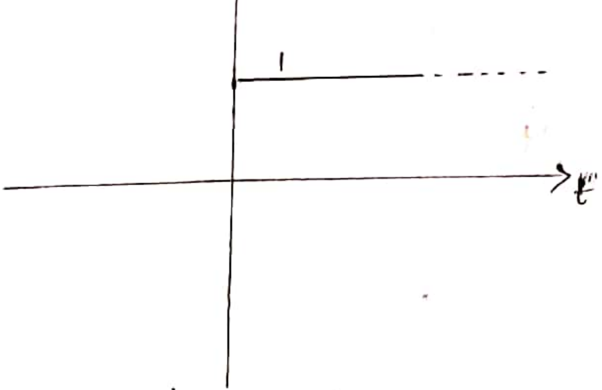


Note:

$\begin{pmatrix}\text{UNIT STEP}\\ \text{function}\end{pmatrix} \underset{\text{Differentiate}}{\overset{\text{Integrate}}{\rightleftarrows}} \begin{pmatrix}\text{UNIT RAMP}\\ \text{function}\end{pmatrix}$



$u(t) = \frac{d\, r(t)}{dt}$



$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau \quad = 0 \quad t < 0$

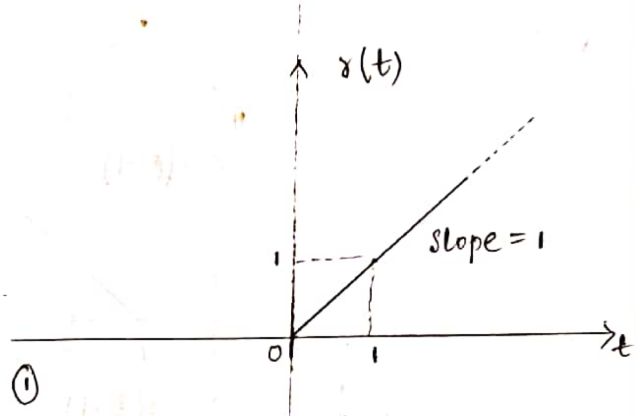
$\qquad\qquad\qquad\qquad\quad = t \quad t \geq 0$

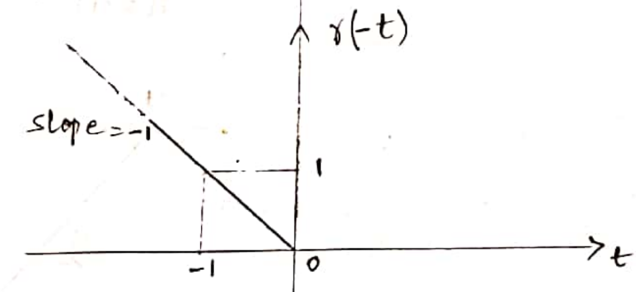
Sketch the following signals

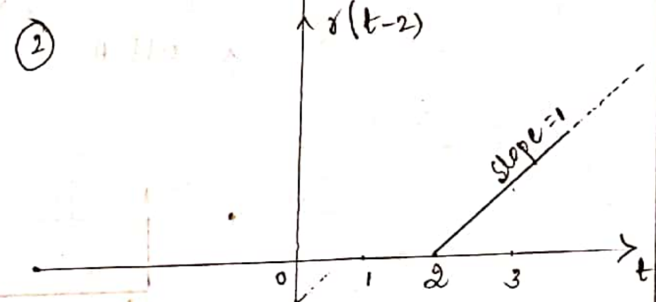
1) $r(-t)$ 2) $r(t-2)$ 3) $r(t+3)$

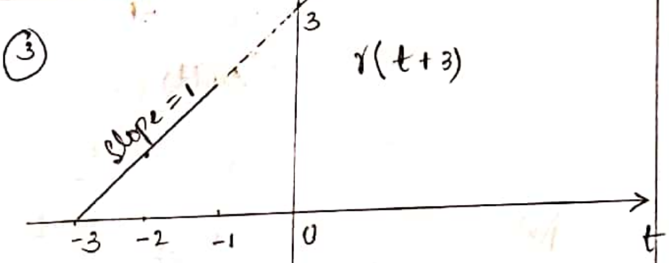
4) $-r(t)$ 5) $r(t) - r(t-1) - u(t-1)$

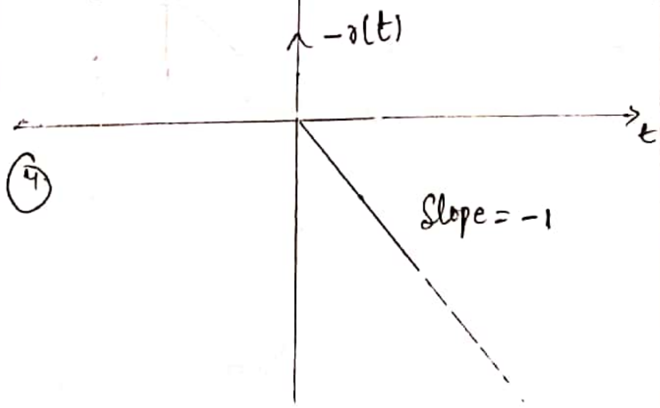
6) $r(t+2) - r(t+1) - r(t-1) + r(t-2)$



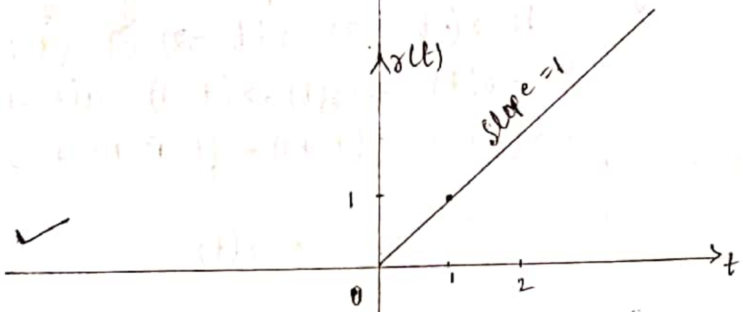


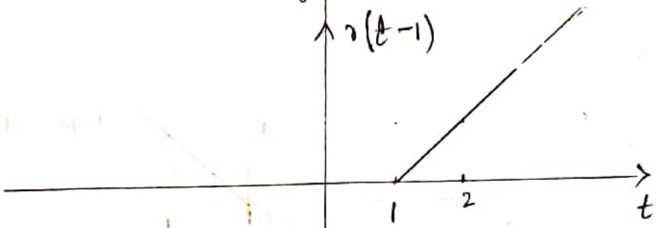


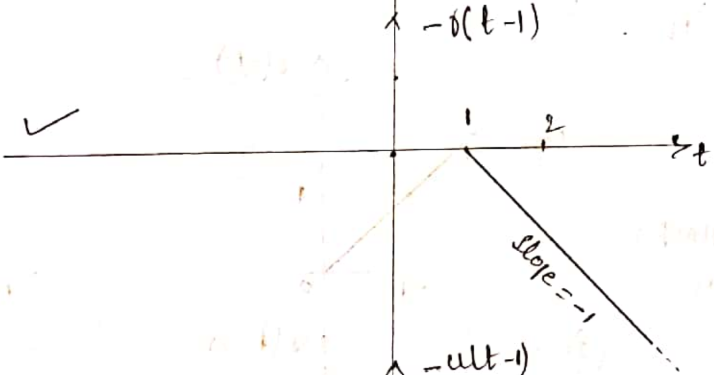


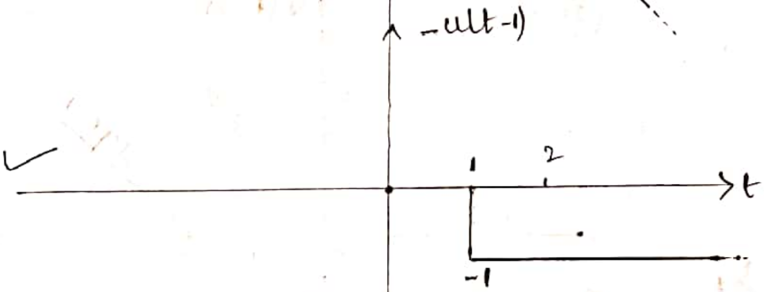


5) $x(t) = r(t) - r(t-1) - u(t-1)$

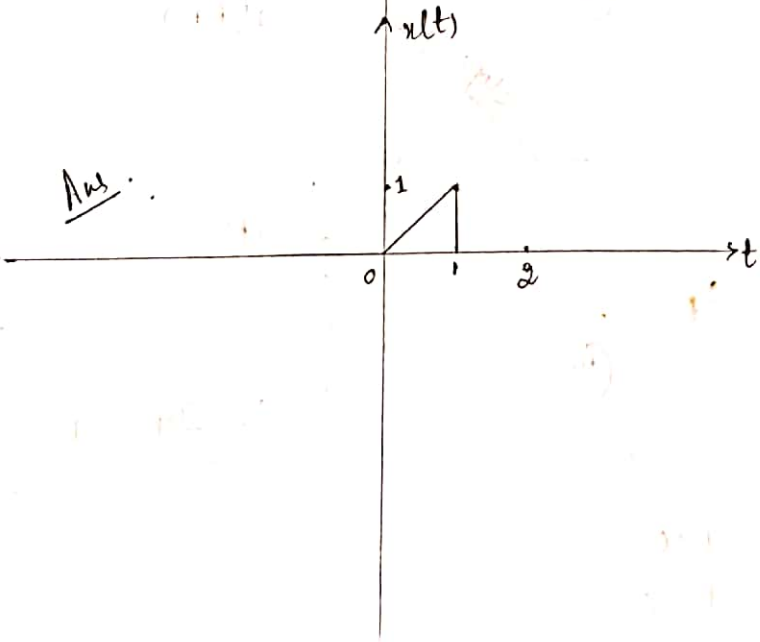




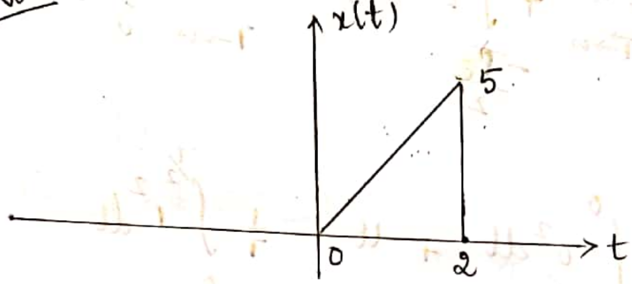




Ans.



Note:



To sketch $x(t-3)$

Shift $x(t)$ RIGHT side by 3 unit.

To sketch $x(t+3)$

Shift $x(t)$ LEFT side by 3 unit

Problem : 1

To Sketch $x\left(-\frac{1}{2}t-3\right)$

Step 1: Shift $x(t)$ right by 3 unit; we get $x(t-3)$

Step 2: Scale $x(t-3)$ by $1/2$; we get $x\left(\frac{1}{2}t-3\right)$

Step 3: Take reflection of $x\left(\frac{1}{2}t-3\right)$; we get $x\left(-\frac{1}{2}t-3\right)$

Problem 2

To Sketch $x\left(-\frac{1}{2}t+3\right)$

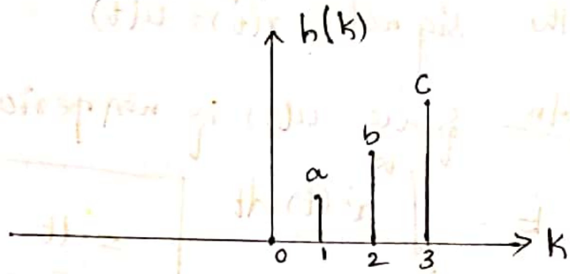
Step 1: Shift $x(t)$ Left by 3 unit; we get $x(t+3)$

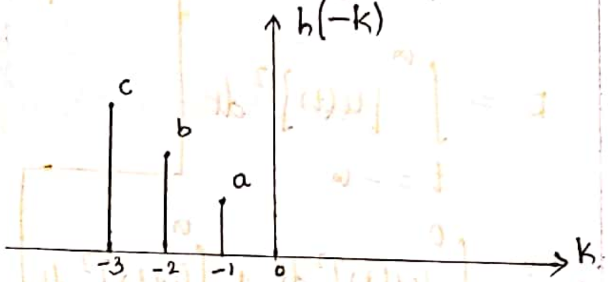
Step 2: Scale $x(t+3)$ by $1/2$; we get $x\left(\frac{1}{2}t+3\right)$

Step 3: take reflection of $x\left(\frac{1}{2}t+3\right)$ we get $x\left(-\frac{1}{2}t+3\right)$

Precedence:
First: SHIFT
Second: SCALE
Third: REFLECTION.

It is same for Discrete Signal $x(n)$ also.





To sketch $h(n-k)$

i) for $n=2$; $h(2-k)$

Shift $h(-k)$ right by 2 unit; we get $h(2-k)$

ii) for $n=-2$; $h(-2-k)$

Shift $h(-k)$ left by 2 unit we get $h(-2-k)$

Note

$$\sum_{k=N_1}^{N_2} 1 = (\text{upper limit} - \text{Lower limit} + 1)$$

$$\sum_{k=N_1}^{N_2} = (N_2 - N_1 + 1)$$

Find the energy or power of the signal. $x(t) = u(t)$

Soln since $u(t)$ is non periodic

$$E = \int_{t=-\infty}^{\infty} x^2(t)\, dt$$

$$E = \int_{t=-\infty}^{\infty} [u(t)]^2 dt$$

$$= \int_{t=-\infty}^{0} [u(t)]^2 dt + \int_{t=0}^{\infty} [u(t)]^2 dt$$

$$= 0 + \int_{t=0}^{\infty} [u(t)]^2 dt$$

$$= \int_0^{\infty} 1^2 dt$$

$$= [t]_0^{\infty}$$

$$E = \infty$$

$\therefore x(t) = u(t)$ is not an energy signal.

To find power:

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} [u(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{0} [u(t)]^2 dt + \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} [u(t)]^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{0} 0^2 dt + \lim_{T\to\infty} \frac{1}{T}\int_{0}^{T/2} 1^2 dt$$

$$= 0 + \lim_{T\to\infty} \frac{1}{T}[t]_0^{T/2}$$

$$= 0 + \lim_{T\to\infty} \frac{1}{T}\left[\frac{T}{2} - 0\right]$$

$$= \lim_{T\to\infty} \frac{1}{T}\frac{T}{2}$$

$$\boxed{P = \frac{1}{2}\ \text{Watt}} < \infty$$

$\therefore x(t) = u(t)$ is a power signal.

Qn Find energy / power.

$x(n) = u(n)$

Soln: $u(n)$ is non periodic

$$E = \sum_{n=-\infty}^{\infty} x^2(n)$$

$$= \sum_{n=-\infty}^{\infty} [u(n)]^2$$

$$= \sum_{n=-\infty}^{-1} [u(n)]^2 + \sum_{n=0}^{\infty} [u(n)]^2$$

$$= 0 + \sum_{n=0}^{\infty} 1^2$$

$$E = \infty$$

$E = \infty$

$\therefore$ $x(n) = u(n)$ is not an Energy signal.

To find power:

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2(n)$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} [u(n)]^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{-1} [u(n)]^2 +$$

$$\lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} [u(n)]^2$$

$$= 0 + \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1^2$$

$$= \lim_{N \to \infty} \frac{N+1}{2N+1}$$

$$= \lim_{N \to \infty} \frac{\cancel{N}(1 + \frac{1}{N})}{\cancel{N}(2 + \frac{1}{N})}$$

$$= \frac{1 + \frac{1}{\infty}}{2 + \frac{1}{\infty}}$$

$$P = \frac{1}{2} \text{ watt}$$

$< \infty$

$\therefore$ unit step signal is a Power signal.

Q. $x(t)$ is sum of $x_1(t)$, $x_2(t)$ & $x_3(t)$ ~~which periodic~~ with fundamental period 1.2, 0.8 & 1.04. Determine whether $x(t)$ is periodic or not.

If it is periodic find the fundamental period.

Soln:

Given: $T_1 = 1.2$, $T_2 = 0.8$, $T_3 = 1.04$ sec

Taking T_1 as reference.

$$\frac{T_1}{T_2} = \frac{1.2}{0.8} = \frac{3}{2} = \frac{P}{q} \text{; rational}$$

$$\frac{T_1}{T_3} = \frac{1.2}{1.04} = \frac{15}{13} = \frac{P'}{q'} \text{; rational}$$

$\therefore$ the sum signal is periodic

The fundamental period

$T = T_1$ (LCM of all denominators)

$= T_1 (2 \cdot 13)$

$= T_1 (26)$

$= 1.2 \times 26$

$$\boxed{T = 31.2 \text{ sec}}$$

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