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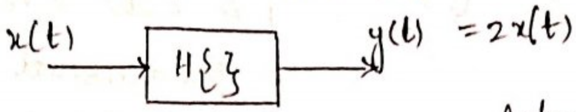
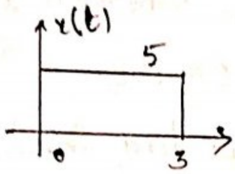


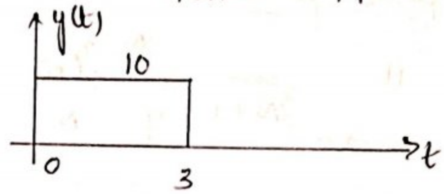
Actual i/p

fig ①



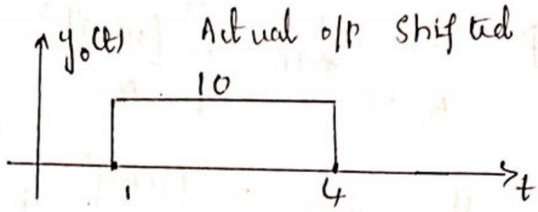
Actual o/p

fig ②



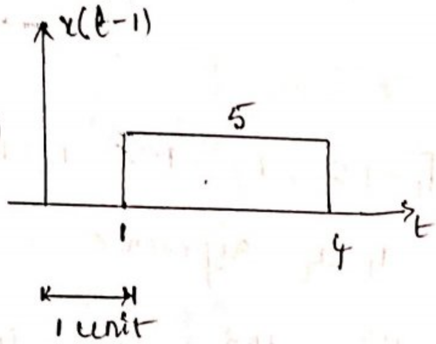
Actual o/p shifted

fig ③



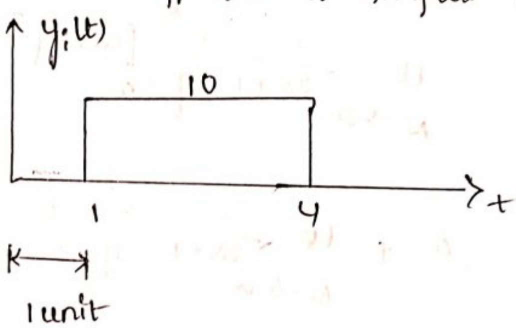
shifted i/p

fig ④



o/p due to shifted i/p

fig ⑤



The system is time invariant because:

shifted o/p = o/p due to shifted i/p

i.e., $y_0(t) = y_i(t)$; compare fig ③ & ⑤

OR

Shift w.r.t resulted in identical shift in the o/p.

compare fig ④ and 5 ; 1 unit shift in both.

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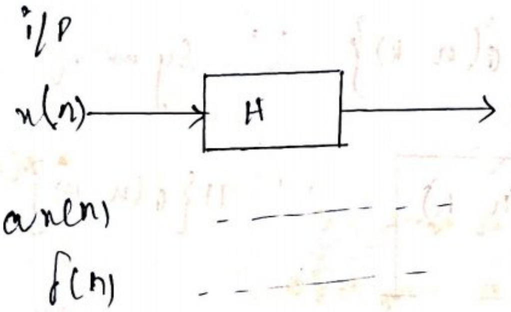
LINEAR TIME INVARIANT SYSTEMS (LTI Systems)

Time domain representation of LTI system.

A system which satisfies the linearity & time invariance property is called LTI system.

I. CONVOLUTION SUM

Consider an LTI system.



$y(n) = H\{x(n)\}$

$y(n) = aH\{x(n)\}$ ∵ Linear.

$h(n) = H\{\delta(n)\}$

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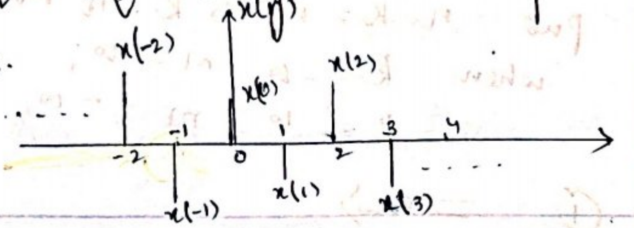
Impulse Response: $h(n)$

It is the response of the system when the i/p is unit impulse signal. i.e. $\delta(n)$

If the impulse response of a given system is known, we can determine the o/p of sym for ~~using~~ ~~convo~~ any i/p using convolution sum.

To derive expression for Convolution Sum:

Consider an i/p sequence $x(n)$ as shown in figure ①, also it can be expressed as weighted sum of delayed impulses as.



$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \longrightarrow ①$$

$$x(n) = \dots + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + \dots$$

But the system o/p is given as

$$y(n) = H\{x(n)\} \rightarrow (2)$$

put (1) in (2)

$$y(n) = H\left\{ \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) \right\}$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, H\{\delta(n-k)\} \quad \because \text{Sym is linear.}$$

$$\boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)} \quad \because H\{\delta(n-k)\} = h(n-k)$$

$$\boxed{y(n) = x(n) * h(n)}$$

In general the o/p of LTI Sym is the convolution of i/p $x(n)$ & impulse response $h(n)$.

Commutative property

$$x(n) * h(n) = h(n) * x(n)$$

consider

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \longrightarrow (1)$$

put $n-k = m \Rightarrow k = n-m$

when $k = -\infty \quad m = \infty;$

$k = \infty \quad m = -\infty;$

(1) $\Longrightarrow$

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$$x(n) * h(n) = \sum_{M=\infty}^{-\infty} x(n-m)\, h(m)$$

$$= \sum_{m=-\infty}^{\infty} h(m)\, x(n-m)$$

$$x(n) * h(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$x(n) * h(n) = h(n) * x(n) = RHS$$

Prove the following

1) $x(n) * \delta(n) = x(n)$
2) $x(n) * \delta(n-n_0) = x(n-n_0)$
3) $x(n) * u(n) = \sum_{k=-\infty}^{n} x(k)$
4) $x(n) * u(n-n_0) = \sum_{k=-\infty}^{n-n_0} x(k)$

(1) Consider LHS

$$x(n) * \delta(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(n)$$

$$= x(n)$$

$$= \underline{RHS}$$

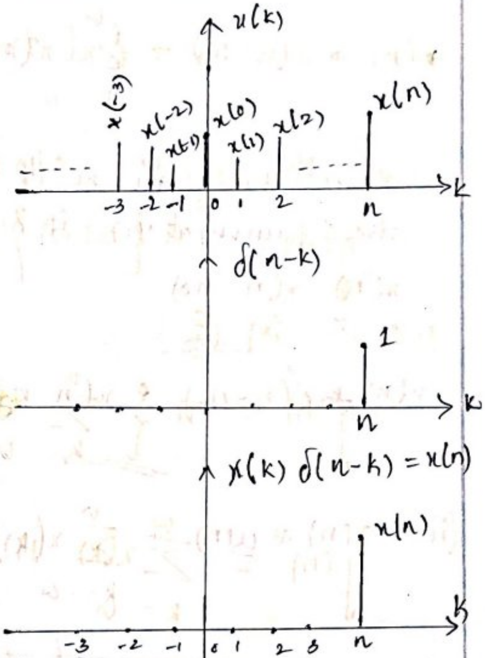


fig ①

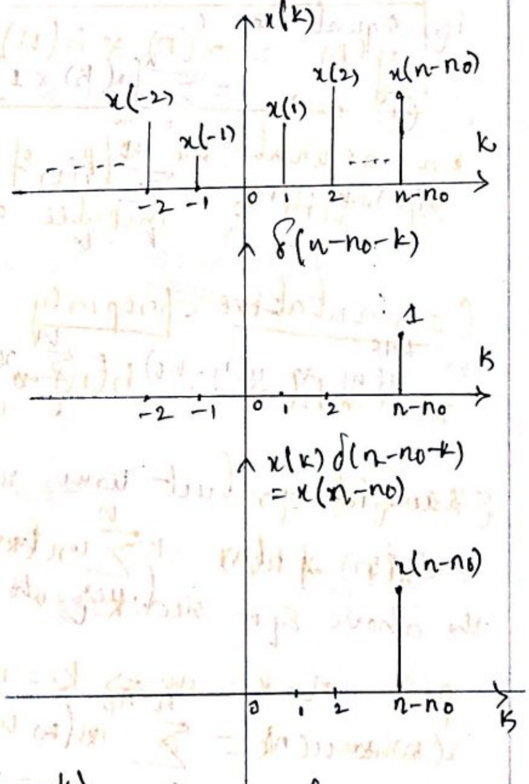


fig ②

(2) consider LHS

$$x(n) * \delta(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-n_0-k)$$

$$x(n) * \delta(n-n_0) = \sum_{k=-\infty}^{\infty} x(n-n_0)$$

Since there are no k terms in above summation it reduces to ~~x(n)~~ $x(n-n_0)$

$$x(n) * \delta(n-n_0) = x(n-n_0)$$

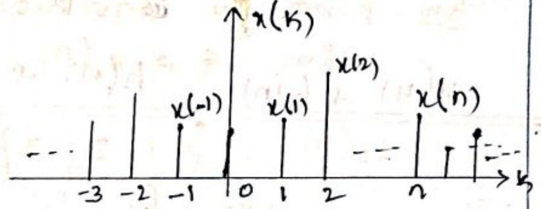
(iii) $$x(n) * u(n) = \sum_{k=-\infty}^{\infty} x(k)\, u(n-k) \rightarrow (1)$$

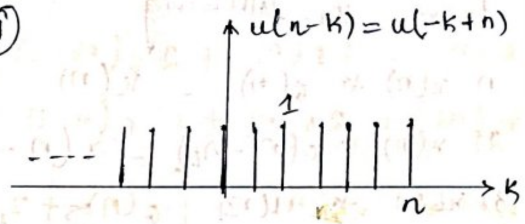
$$u(k) = \begin{cases} 1 & k \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

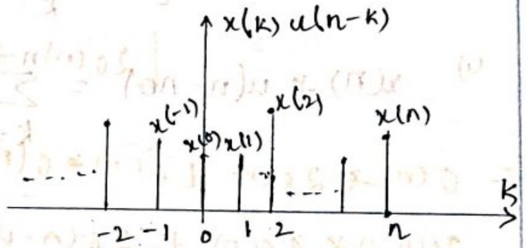
$$u(n-k) = \begin{cases} 1 & n-k \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(OR)

$$u(n-k) = \begin{cases} 1 & k \leq n \\ 0 & \text{otherwise} \end{cases}$$







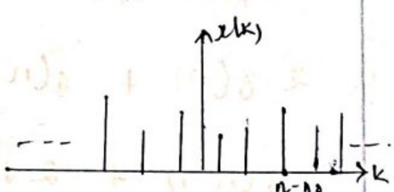
fig(3)

From figure(3) the product signal $x(k)\, u(n-k)$ exists from $k=-\infty$ to $k=n$ otherwise it is equal to 0.

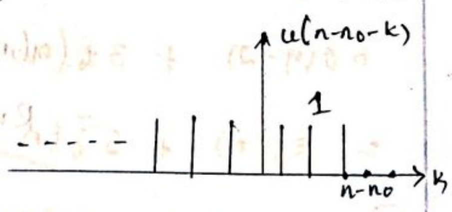
$$\therefore (1) \Rightarrow = \sum_{k=-\infty}^{n} [x(k) \times 1]$$

$$x(n) * u(n) = \sum_{k=-\infty}^{n} x(k)$$

iv) $$\text{LHS} = x(n) * u(n-n_0) = \sum_{k=-\infty}^{\infty} x(k)\, u(n-n_0-k)$$

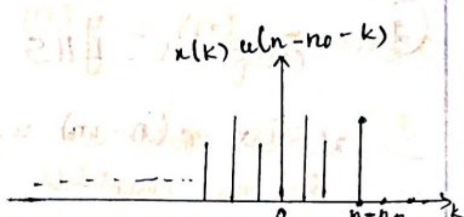


Since the product term $x(k)\, u(n-n_0-k) = x(k)$ exists from $k=-\infty$ to $n-n_0$ the above Eqn reduces to



$$x(n) * u(n) = \sum_{k=-\infty}^{n-n_0} x(k)$$

$$= \text{RHS}$$



(4)

Delayed impulse method or Delta method to compute convolution.

Note:

$$\delta(n-k) * \delta(n-m) = \delta(n-k-m)$$

Qn Find the convolution for the two sequences $x_1(n)$ & $x_2(n)$ given below.

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 3\} \quad x_2(n) = \{\underset{\uparrow}{2}, 1, 4\}.$$

$$x_1(n) = 1\delta(n) + 2\delta(n-1) + 3\delta(n-2)$$

$$x_2(n) = 2\delta(n) + 1\delta(n-1) + 4\delta(n-2)$$

$$x_1(n) * x_2(n) = [\delta(n) + 2\delta(n-1) + 3\delta(n-2)] * [2\delta(n) + \delta(n-1) + 4\delta(n-2)]$$

$$= \delta(n) * 2\delta(n) + \delta(n) * \delta(n-1) + \delta(n) * 4\delta(n-2)$$
$$2\delta(n-1) * 2\delta(n) + 2\delta(n-1) * \delta(n-1) + 2\delta(n-1) * 4\delta(n-2)$$
$$3\delta(n-2) * 2\delta(n) + 3\delta(n-2) * \delta(n-1) + 3\delta(n-2) * 4\delta(n-2)$$

$$= 2\delta(n) + \delta(n-1) + 4\delta(n-2)$$
$$4\delta(n-1) + 2\delta(n-2) + 8\delta(n-3)$$
$$6\delta(n-2) + 3\delta(n-3) + 12\delta(n-4)$$

$$= 2\delta(n) + 5\delta(n-1) + 12\delta(n-2) + 11\delta(n-3) + 12\delta(n-4)$$

$$x_1(n) * x_2(n) = y(n) = \{\underset{\uparrow}{2}, 5, 12, 11, 12\}$$

note: If there are 'p' samples in i/p & 'q' samples in the impulse response, then the convolution of samples with $x(n)$ & $h(n)$ contains $(p+q-1)$ samples

Qn ② compute Convolution

$x(n) = \{2, \underset{\uparrow}{3}, 2\}$ $h(n) = \{\underset{\uparrow}{3}, 2, 3\}$

$y(n) = x(n) * h(n)$

$= [2\delta(n+1) + 3\delta(n) + 2\delta(n-1)] *$
$[3\delta(n) + 2\delta(n-1) + 3\delta(n-2)]$

$= 2\delta(n+1) * 3\delta(n) + 2\delta(n+1) * 2\delta(n-1) + 2\delta(n+1) * 3\delta(n-2) +$
$3\delta(n) * 3\delta(n) + 3\delta(n) * 2\delta(n-1) + 3\delta(n) * 3\delta(n-2) +$
$2\delta(n-1) * 3\delta(n) + 2\delta(n-1) * 2\delta(n-1) + 2\delta(n-1) * 3\delta(n-2)$

$= 6\delta(n+1) + 4\delta(n) + 6\delta(n-1) +$
$9\delta(n) + 6\delta(n-1) + 9\delta(n-2) +$
$6\delta(n-1) + 4\delta(n-2) + 6\delta(n-3).$

$y(n) = 6\delta(n+1) + 13\delta(n) + 18\delta(n-1) + 13\delta(n-2) + 6\delta(n-3)$

$y(n) = \{6, \underset{\uparrow}{13}, 18, 13, 6\}$

Note: The sequence $x(n)$ starts at $n_1 = -1$
The sequence $h(n)$ starts at $n_2 = 0$
∴ the sequence $y(n)$ starts at
$n = n_1 + n_2 = -1 + 0 = -1$

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GRAPHICAL METHOD

The process of computing the convolution b/wn $x(n)$ & $h(n)$ involves the following four steps.

1) FOLDING:

Fold $h(k)$ about $k=0$ to obtain $h(-k)$

2) SHIFTING:

Shift $h(-k)$ by 'n' units to the right if n is +ve OR to the left if n is −ve. to obtain $h(n-k)$

3) MULTIPLICATION:

Multiply $x(k)$ by $h(n-k)$ to obtain the product sequence.

4) SUMMATION:

Sum all the values of product sequence to obtain the output $y(n)$

obt.

Qn

1) Obtain the convolution of $x_1(n)$ & $x_2(n)$ by making use of graphical method.

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 3\}, \quad x_2(n) = \{\underset{\uparrow}{2}, 1, 4\}$$

soln

Let $x_1(n) = x(n)$, $x_2(n) = h(n)$

By definition

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

For $n=0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$$

$y(0) = 0+0+2+0+0$

$y(0) = \underline{2}$

$n=1$

$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$

$= 0+1+4+0$

$y(1) = \underline{5}$

$n=2$

$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$

$= 4+2+6$

$= \underline{12}$

$n=3$

$y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$

$= 0+8+3$

$y(3) = 11$

$n=4;\ y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k)$

$= 0+12+0$

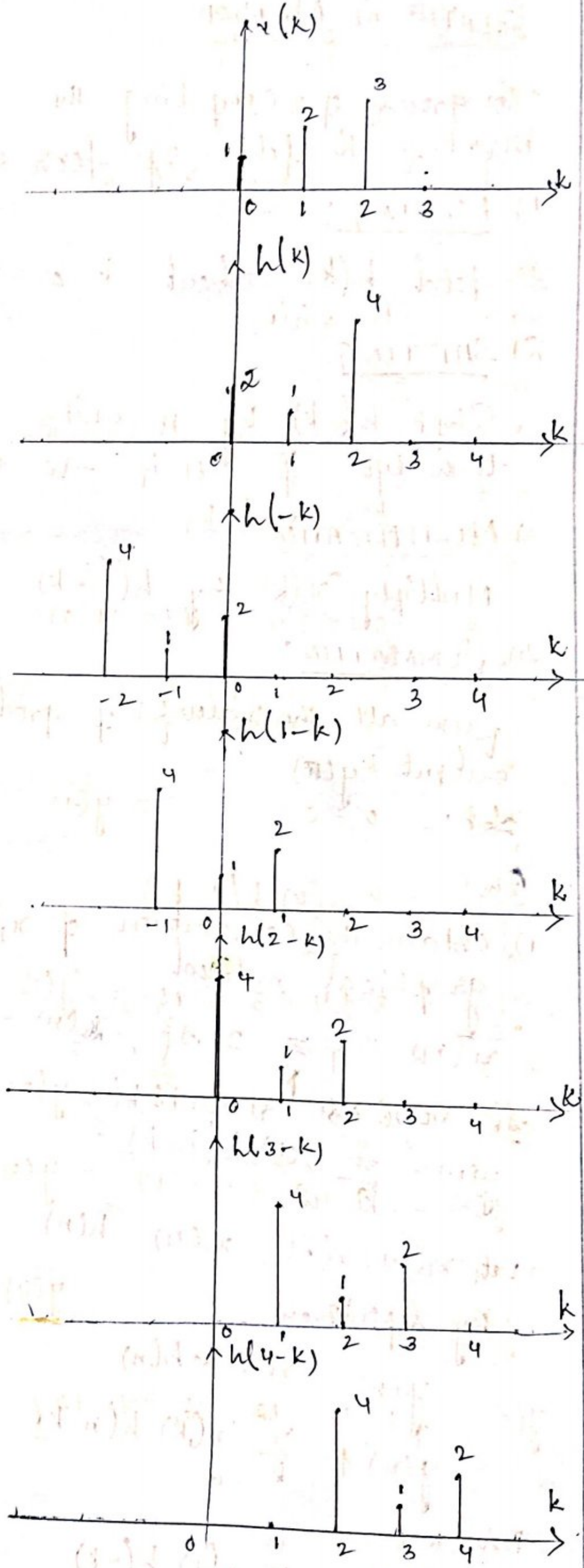
$\underline{y(4) = 12}$

$n=5$

$y(5) = \sum_{k=-\infty}^{\infty} x(k)\, h(5-k)$

$y(5) = \underline{0}$

$y(n) = \{\underset{\uparrow}{2}, 5, 12, 11, 12\}$



Qn (2) Given

$x(n) = \{1, \underset{\uparrow}{2}, 3, 4\}$

$h(n) = \{\underset{\uparrow}{1}, 2, 3\}$

get the convolution $x(n)$ & $h(n)$

Soln

$y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$n=0$

$$y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$$

$y(0) = 0+2+2 = 4 = y(0)$

$n=-1$

$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$$

$y(-1) = 0+0+1 = y(-1)$

$n=1$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$

$y(1) = 3+4+3 = 10 = y(1)$

$n=2$

$y(2) = 6+6+4 = 16 = y(2)$

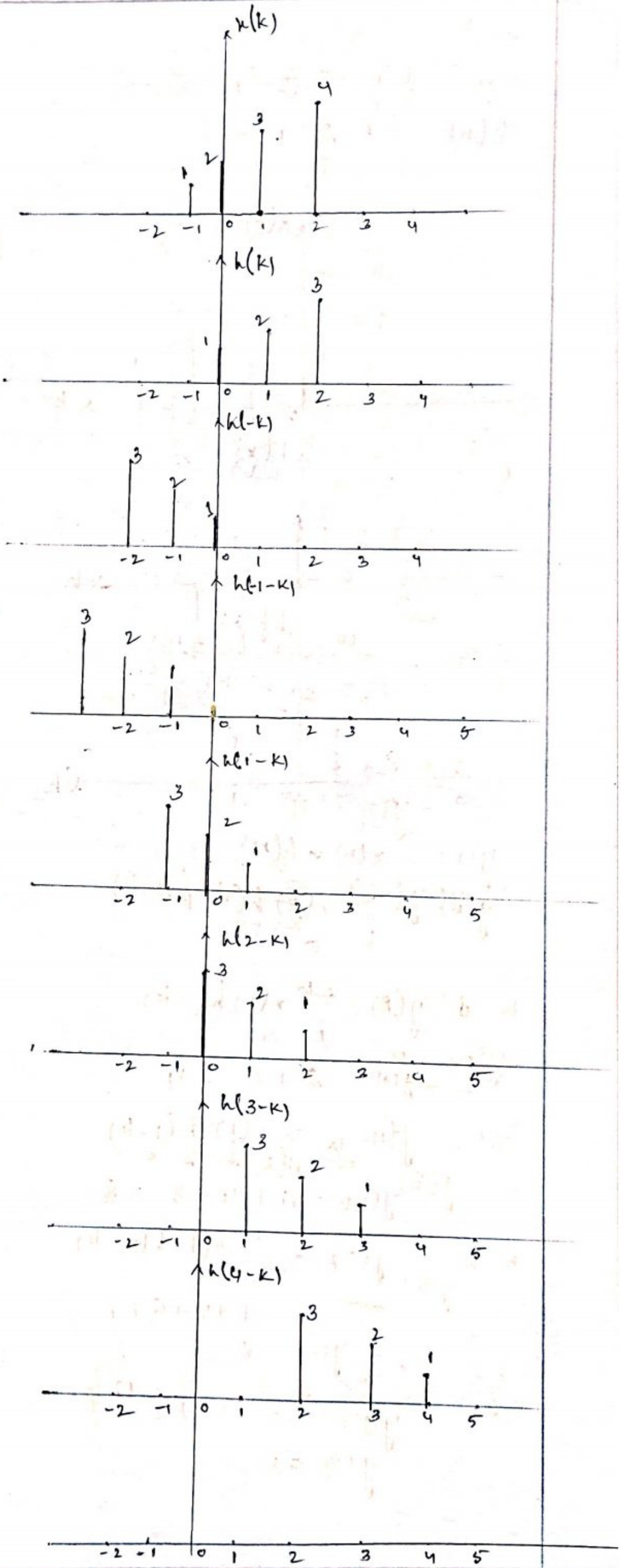
$n=3$

$y(3) = 9+8+0 = 17 = y(3)$

$n=4$

$y(4) = 12 = y(4)$

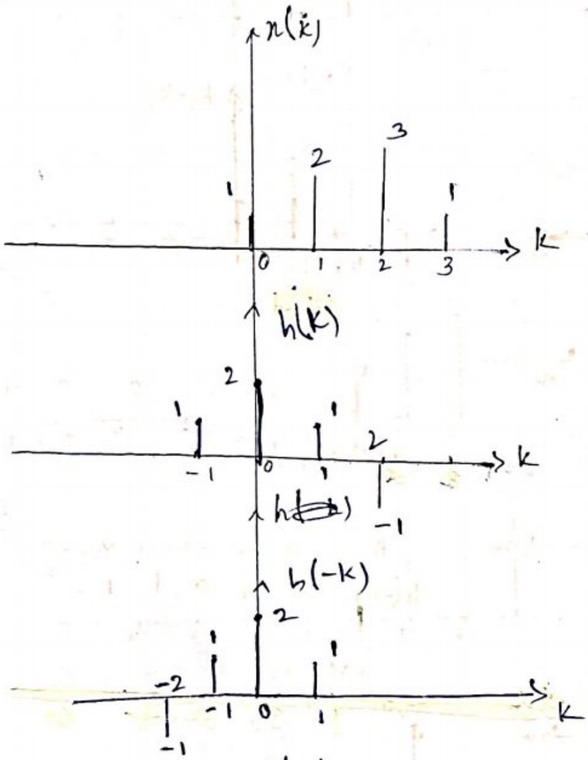
$y(n) = \{1, \underset{\uparrow}{4}, 10, 16, 17, 12\}$



Qn (3)

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$

$h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$



k	0	1	2	3
$x(k)$	1	2	3	1

k	-2	-1	0	1
$h(-k)$	-1	1	2	1

k	-2	-1	0	1	2
$h(1-k)$		-1	1	2	1

k	-1	0	1	2	3
$h(2-k)$		-1	1	2	1

k	1	2	3	4
$h(3-k)$	-1	1	2	1

k	2	3	4	5	6
$h(4-k)$	-1	1	2	1	

k	3	4	5	6
$h(5-k)$	-1	1	2	1

k	-3	-2	-1	0
$h(-1-k)$	-1	1	2	1

k	-4	-3	-2	-1
$h(-2-k)$	-1	1	2	1

$y(n) = x(n) * h(n)$

$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$n=0$; $y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$

$y(0) = 2 + 2 = 4$

$n=1$ $\quad y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$

$y(1) = 1 + 4 + 3 = 8$

$n=2$ $\quad y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$

$= -1 + 2 + 6 + 1$

$y(2) = 8$

$n=3$ $\quad y(3) = -2 + 3 + 2$

$y(3) = 3$

$n=4$ $\quad y(4) = -3 + 1$

$y(4) = -2$

$n=5$ $\quad y(5) = -1$

$n=-1$ $\quad y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$

$y(-1) = 1$

$n=-2$ $\quad y(-2) = \sum_{k=-\infty}^{\infty} x(k)\, h(-2-k)$

$= 0$

$y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$

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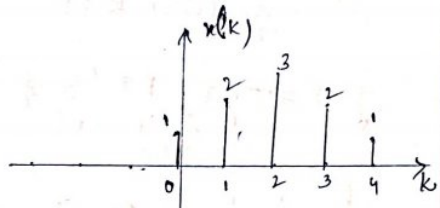
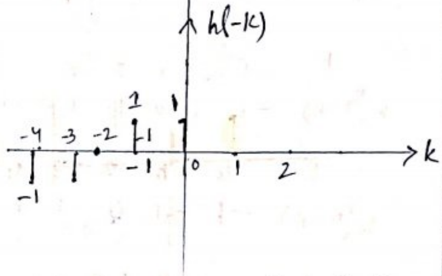
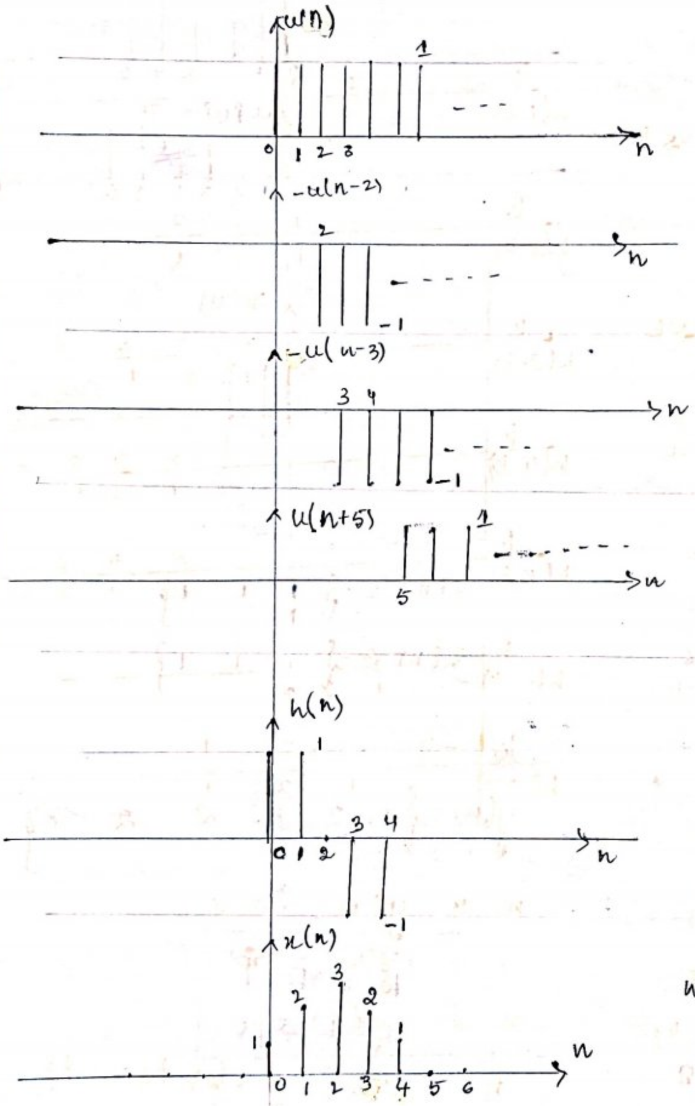
(4)

$$x(n) = \begin{cases} n+1 & 0 \le n \le 2 \\ 5-n & 3 \le n \le 5 \\ 0 & \text{Else where.} \end{cases}$$

$$h(n) = u(n) - u(n-2) - u(n-3) + u(n-5)$$







	0	1	2	3	4
$x(k)$	1	2	3	2	1

k	-4	-3	-2	-1	0
$h(-k)$	-1	-1	0	1	1

$y(0) = 1$

$n=1$

$$y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$$

k	-3	-2	-1	0	1
$h(1-k)$	-1	-1	0	1	1

$y(1) = 1 + 2 = \underline{3}$

$n=2$

$$y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$$

k	-2	-1	0	1	2
$h(2-k)$	-1	-1	0	1	1

$y(2) = 0 + 2 + 3 = 5$

$y(2) = \underline{5}$

$y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$n=0 \quad y(0) = \sum x(k)\, h(-k)$

$$y(3) = \sum_{k=-\infty}^{\infty} x(k)\,h(3-k)$$

$$y(3) = -1 + 3 + 2 = 4$$

k	0	1	2	3	4
x(k)	1	2	3	2	1

k	-1	0	1	2	3
h(3-k)	-1	-1	0	1	1

k	0	1	2	3	4
h(4-k)	-1	-1	0	1	1

$$y(4) = -1 - 2 + 2 + 1$$

$$\underline{y(4) = 0}$$

k	1	2	3	4	5
h(5-k)	-1	-1	0	1	1

$$y(5) = -2 - 3 + 1$$

$$\underline{y(5) = -4}$$

k	2	3	4	5	6
h(6-k)	-1	-1	0	1	1

$$y(6) = -3 - 2 = \underline{-5}$$

k	3	4	5	6	7
h(7-k)	-1	-1	0	1	1

$$y(7) = -2 - 1$$

$$\underline{y(7) = -3}$$

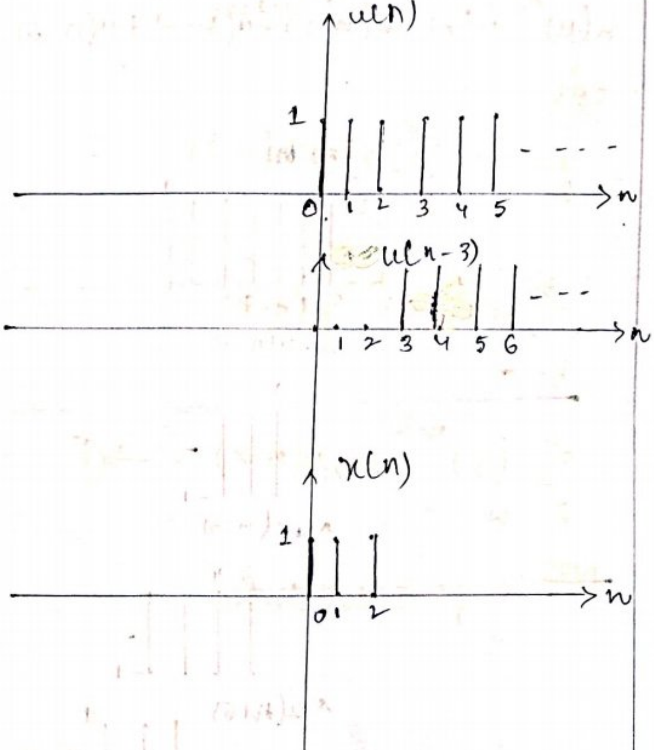
k	4	5	6	7	8
h(8-k)	-1	-1	0	1	1

$$\underline{y(8) = -1}$$

$$y(n) = \{\underset{\uparrow}{1}, 3, 5, 4, 0, -4, -5, -3, -1\}$$

Qn 5 $\quad x(n) = u(n) - u(n-3)$

$h(n) = u(n) - u(n-3)$



$$x(n) = \{\underset{\uparrow}{1}\ 1\ 1\}$$

$$h(n) = \{\underset{\uparrow}{1}\ 1\ 1\}$$

$$y(n) = \{\underset{\uparrow}{1}, 2, 3, 2\ 2\ 0\}$$

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Formula Method

(A) $x(n) = u(n)$

(1) $h(n) = (1/2)^n u(n)$

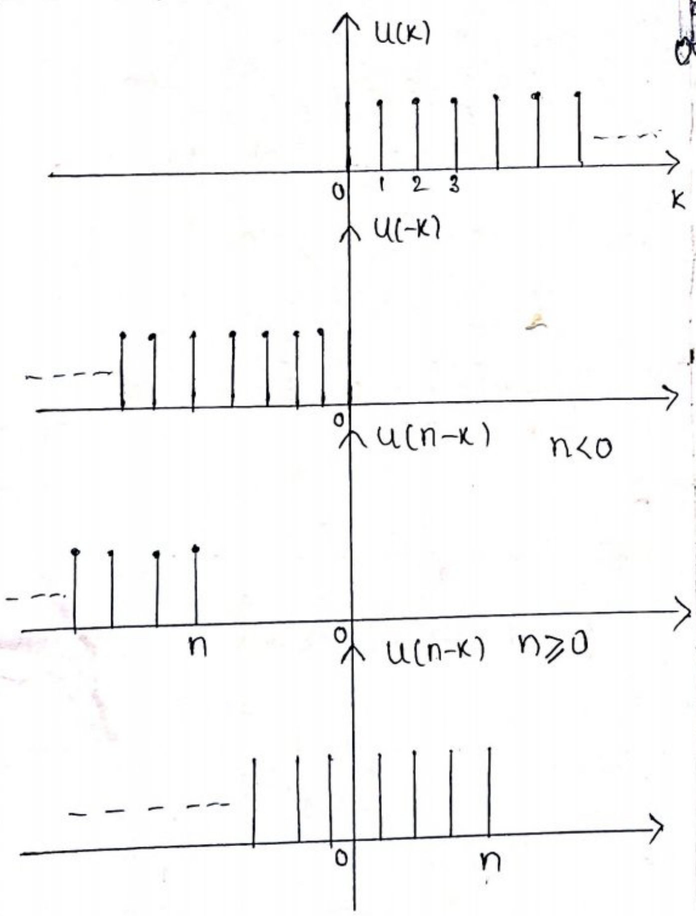
soln

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} u(k) (1/2)^{n-k} u(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (1/2)^{n-k} u(k)\, u(n-k) \longrightarrow (1)$$

Note: Now to find the limits.



Keeping $u(k)$ fixed & moving $u(n-k)$ left & right for different values of n to find the overlap. (refer fig (A) & (B))

case i ; $n<0$;

There is no overlap ∴ $u(k)\, u(n-k) = 0$

Eqn (1) ⟹ $y(n) = 0 \longrightarrow (2)$

case ii : $n \geq 0$.

Overlap is from $0 \leq k \leq n$

Eqn (1) ⟹

$$y(n) = \sum_{k=0}^{n} (1/2)^{n-k} \cdot 1$$

$$= (1/2)^n \sum_{k=0}^{n} (1/2)^{-k}$$

$$= (1/2)^n \sum_{k=0}^{n} 2^k$$

$$= (1/2)^n \frac{2^{n+1} - 1}{2 - 1}$$

$$= 2^{-n} \frac{(2^n \cdot 2^1 - 1)}{1}$$

$$y(n) = (2 - 2^{-n}) \longrightarrow (3)$$

for $n > 0$

combining ② & ③

$$y(n) = \begin{cases} 0 & n<0 \\ (2-2^{-n}) & n \geq 0 \end{cases}$$

OR

$$y(n) = (2-2^{-n})\, u(n)$$

Qn 2) $x(n) = \left(\frac{1}{4}\right)^n u(n)$

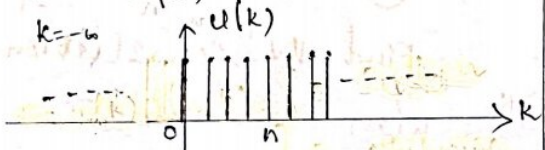
$h(n) = (1/2)^n u(n)$

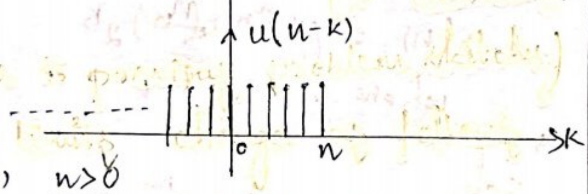
$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u(k) (1/2)^{n-k} u(n-k)$$

$$y(n) = \left(\frac{1}{2}\right)^n \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k \left(\frac{1}{2}\right)^{-k} u(k)\, u(n-k) \longrightarrow ①$$





case i) $n \geq 0$

$$y(n) = (1/2)^n \sum_{k=0}^{n} \left(\frac{2}{4}\right)^k \cdot 1$$

$$= \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} (1/2)^k$$

$$= \left(\frac{1}{2}\right)^n \left[\frac{1-\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}}\right]$$

$$= \left(\frac{1}{2}\right)^n \frac{1-\left(\frac{1}{2}\right)^n \frac{1}{2}}{\frac{1}{2}}$$

$$= 2^{-n}\, 2^1 \left[1 - 2^{-n}\, 2^{-1}\right]$$

$$y(n) = 2^{-n+1} - 2^{-2n}$$

$$y(n) = \left[2^{-n+1} - 2^{-2n}\right] \text{ for } n \geq 0 \longrightarrow ②$$

case ii) for $n<0$

there is no overlap.

$u(k)\, u(n-k) = 0$

using this in Eqn ①

$$y(n) = 0 \quad \text{for } n<0 \longrightarrow ③$$

combining ② & ③

$$y(n) = \begin{cases} 2^{-n+1} - 2^{-2n} & n \geq 0 \\ 0 & n<0 \end{cases}$$

OR

$$y(n) = \left(2^{-n+1} - 2^{-2n}\right) u(n)$$

Qn (3) 3

An LTI system is characterized by $h(n) = \left(\frac{3}{4}\right)^n u(n)$. Determine the o/p of the system at $n = 5, -5, 10$. where $x(n) = u(n)$

soln $y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} u(k) \left(\frac{3}{4}\right)^{n-k} u(n-k)$$

$$= \left(\frac{3}{4}\right)^n \sum_{k=-\infty}^{\infty} \left(\frac{4}{3}\right)^k u(k)\, u(n-k) \longrightarrow (1)$$

Refer to previous sketches.

Case i) $n < 0$

there is no overlap

$u(k)\, u(n-k) = 0$

$(1) \Longrightarrow$

$y(n) = 0. \longrightarrow (2)$

Case ii) $n \geq 0$

$u(k)\, u(n-k) = 1$

$(1) \Longrightarrow$

$$y(n) = \left(\frac{3}{4}\right)^n \sum_{k=0}^{n} (4/3)^k$$

$$y(n) = \left(\frac{3}{4}\right)^n \frac{\left(\frac{4}{3}\right)^{n+1} - 1}{\frac{4}{3} - 1}$$

$$y(n) = \left(\frac{3}{4}\right)^n \cdot \frac{\left(\left(\frac{4}{3}\right)^n \left(\frac{4}{3}\right) - 1\right)}{\frac{1}{3}}$$

$$y(n) = \frac{\left[\left(\frac{3}{4}\right)^n \left(\frac{4}{3}\right)^n \left(\frac{4}{3}\right) - \left(\frac{3}{4}\right)^n\right]}{\frac{1}{3}}$$

$$y(n) = 3\left[(1)^n \left(\frac{4}{3}\right) - \left(\frac{3}{4}\right)^n\right]$$

Put $n = 5$

$$y(n) = 3\left[(1)^5 \left(\frac{4}{3}\right) - \left(\frac{3}{4}\right)^5\right]$$

$$y(n) = 3.2881$$

Qn (4) 4

Find the convolution of

$h(n) = \alpha^{-n} u(-n)$

$x(n) = \alpha^n u(n)$

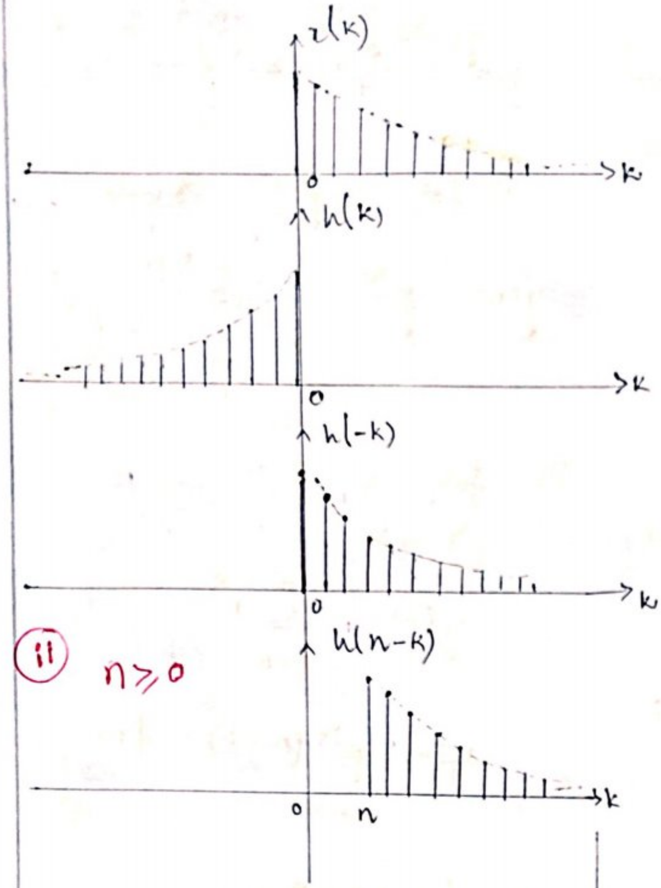
where $0 < \alpha < 1$

soln

$y(n) = x(n) * h(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \longrightarrow (1)$$

$$\cancel{y(n) = \sum_{k=-\infty}^{\infty} \alpha^{+k} u(+k)\, \alpha^{n+k} u(-n+k)}$$



(ii) $n \geq 0$

case i) $n < 0$

overlap b/wn $x(k)$ & $h(n-k)$ is from 0 to ∞.

$\therefore$ Eqn ① $\Longrightarrow$

$$y(n) = \sum_{k=0}^{\infty} \alpha^{k} \alpha^{-(n-k)} \cdot 1$$

$$= \sum_{k=0}^{\infty} \alpha^{k} \alpha^{-n} \alpha^{k}$$

$$= \alpha^{-n} \sum_{k=0}^{\infty} \alpha^{2k}$$

$$= \alpha^{-n} \sum_{k=0}^{\infty} (\alpha^{2})^{k}$$

Note

1) $\sum_{k=n}^{\infty} a^{k} = \frac{a^{n}}{1-a}$

2) $\sum_{k=0}^{\infty} a^{k} = \frac{1}{1-a}$

$$y(n) = \alpha^{-n} \frac{1}{1-\alpha^{2}}$$

$$y(n) = \frac{\alpha^{-n}}{1-\alpha^{2}} \quad \text{for } n<0 \longrightarrow ②$$

case ii) $n \geq 0$.

overlap is from n to ∞

$\therefore$ Eqn ① $\Rightarrow$

$$y(n) = \alpha^{-n} \sum_{k=n}^{\infty} (\alpha^{2})^{k}$$

$$= \alpha^{-n} \frac{(\alpha^{2})^{n}}{1-\alpha^{2}}$$

$$= \frac{\alpha^{-n} \alpha^{2n}}{1-\alpha^{2}}$$

$$y(n) = \frac{\alpha^{n}}{1-\alpha^{2}} \longrightarrow ③$$

combining ② & ③

$$y(n) = \begin{cases} \frac{\alpha^{-n}}{1-\alpha^{2}} & n<0 \\ \frac{\alpha^{n}}{1-\alpha^{2}} & n \geq 0. \end{cases}$$

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$$y(n) = \frac{\alpha^{|n|}}{1-\alpha^2}$$

Qn 5

5) $h(n) = \beta^n u(n) \quad |\beta| < 1$

$x(n) = \alpha^n u(n) \quad |\alpha| < 1$

Find $y(n)$ for

(i) $\alpha \neq \beta$ (ii) $\alpha = \beta$ (iii) $\alpha = \beta = 1$

Soln

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k) h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \alpha^k u(k) \cdot \beta^{n-k} u(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \beta^n \left(\frac{\alpha}{\beta}\right)^k u(k) u(n-k) \longrightarrow 1$$

Case I for $n < 0$ (refer figure below)

No overlap, b/wn $u(k)$ & $u(n-k)$

Eqn 1 $\Rightarrow$

$$y(n) = \sum_{k=-\infty}^{\infty} \beta^n \left(\frac{\alpha}{\beta}\right) 0$$

$$y(n) = 0 \longrightarrow 2$$

Case II $n \geq 0$

overlap is from 0 to n

Eqn 1 $\Longrightarrow$

$$y(n) = \beta^n \sum_{k=0}^{n} \left(\frac{\alpha}{\beta}\right)^k \longrightarrow 3$$

i) for $\alpha \neq \beta$ 3 $\Longrightarrow$

$$y(n) = \beta^n \frac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}} \text{ for } n \geq 0 \longrightarrow 4$$

ii) for $\alpha = \beta$ in eqn 3

3 $\Rightarrow$

$$y(n) = \beta^n \sum_{k=0}^{n} 1^k$$

$$y(n) = \beta^n (n+1) \text{ for } n \geq 0 \longrightarrow 5$$

iii) for $\alpha = \beta = 1$

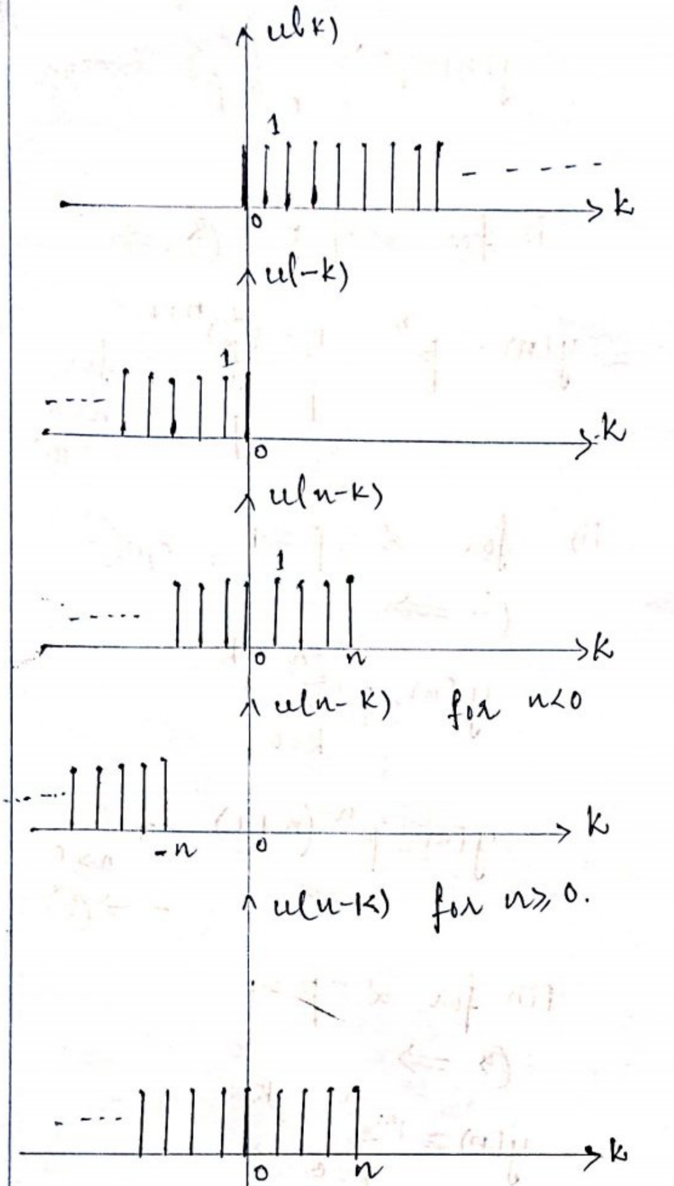
3 $\Rightarrow$

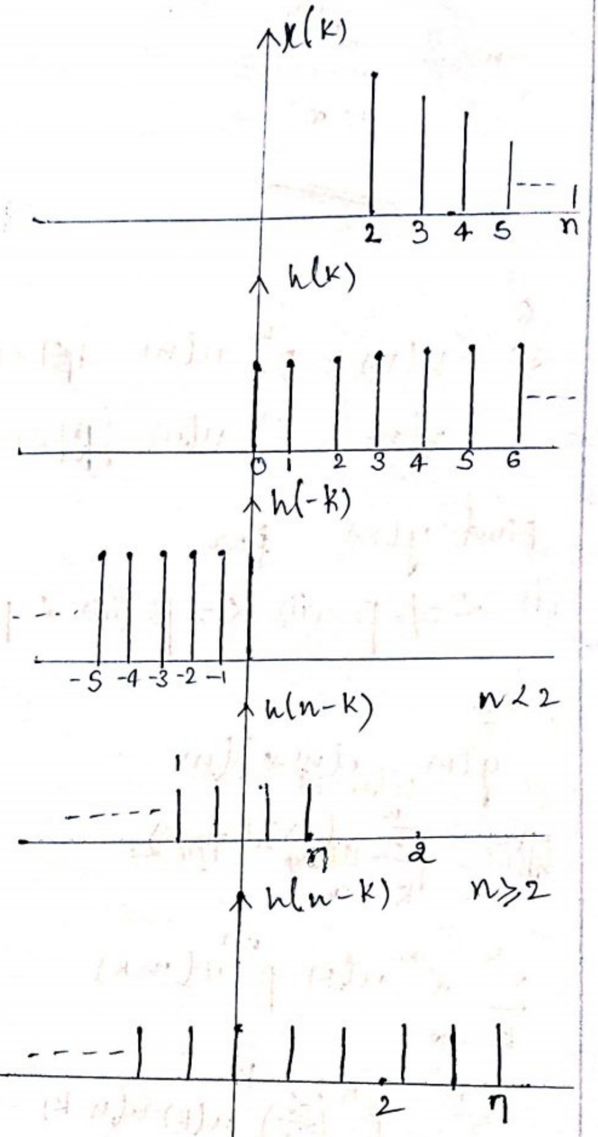
$$y(n) = \sum_{k=0}^{n} 1^k$$

$$y(n) = (n+1) \text{ for } n \geq 0 \longrightarrow 6$$

Combining 4 5 6

$$y(n) = \begin{cases} \beta^n \dfrac{1 - \left(\frac{\alpha}{\beta}\right)^{n+1}}{1 - \frac{\alpha}{\beta}} u(n); & \alpha \neq \beta \\ \beta^n (n+1) u(n); & \alpha = \beta \\ (n+1) u(n); & \alpha = \beta = 1 \end{cases}$$





Qn 6

$x(n) = (1/2)^n \, u(n-2)$

$h(n) = u(n)$

Sol:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \longrightarrow (1)$$

Case i) when $n<2$

No overlap $\therefore$ Eqn (1) $\Rightarrow$

$$y(n) = 0 \longrightarrow (2)$$

Case ii) for $n \geq 2$; over lap is from 2 to n; Eqn (1) $\Rightarrow$

$$y(n) = \sum_{k=2}^{n} \left[(1/2)^k\right] \cdot 1$$

Qn 7 7

$x(n) = u(n-3)$

$h(n) = \beta^n u(n)$. $|\beta| < 1$

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Qu 8 8

$x(n) = \alpha^n u(-n)$

$h(n) = u(-n)$

$0 < \alpha < 1$

Qn (9) (9)

$$x(n) = \begin{cases} 1 & 0 \le n \le 4 \\ 0 & \text{elsewhere.} \end{cases}$$

$$h(n) = \begin{cases} \alpha^n & 0 \le n \le 6 \\ 0 & \text{otherwise.} \end{cases}$$

$\alpha > 1$

Soln

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) \longrightarrow (1).$$

i) when $n < 0$

$$(1) \Longrightarrow y(n) = 0.$$

ii) when $n \geq 0$ & $n \leq 4$

i.e., $0 \leq n \leq 4$

$h(k)$:	$0 \le n \le 6$
$h(n-k)$:	$0 \le n-k \le 6$
	$-n \le -k \le 6-n$
	$n \geq k \geq n-6$
$h(n-k)$	$n-6 \le k \le n$

$$y(n) = \sum_{k=0}^{4} 1 \cdot \alpha^{n-k}$$

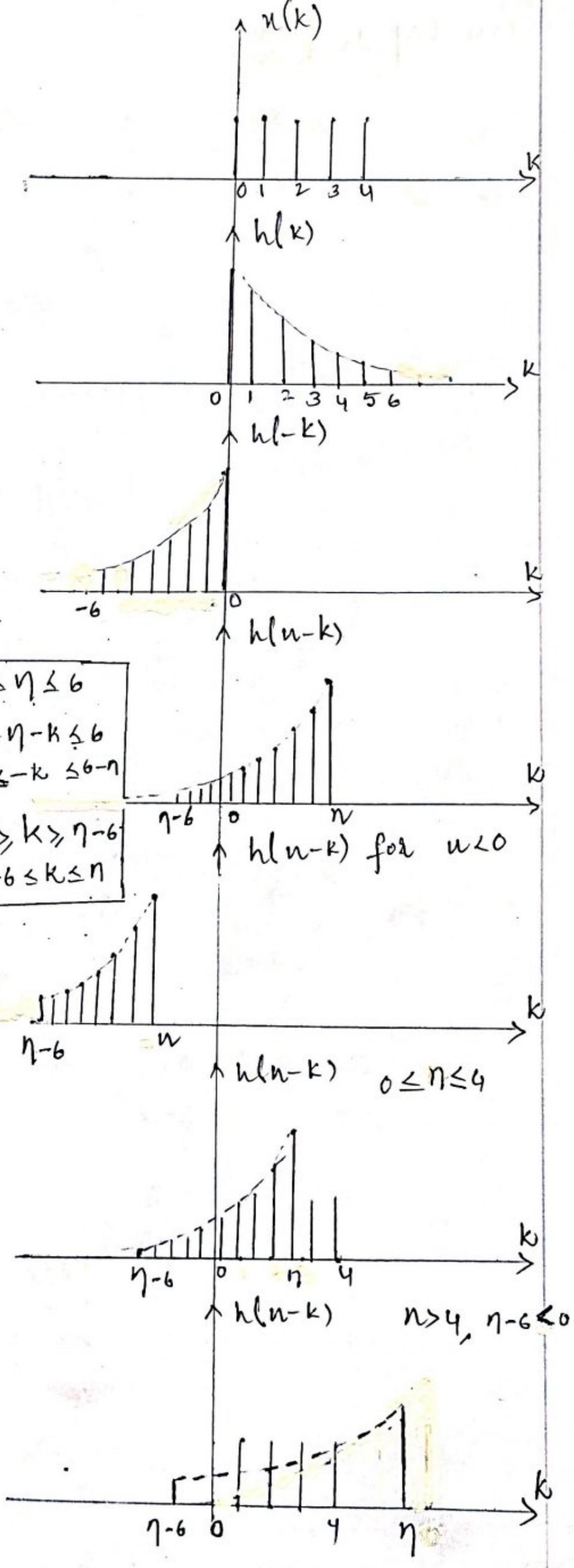
$$= \alpha^n \sum_{k=0}^{4} \left(\frac{1}{\alpha}\right)^k$$

$$= \alpha^n \frac{1 - \left(\frac{1}{\alpha}\right)^{n+1}}{1 - \frac{1}{\alpha}}$$

$$= \alpha^n \frac{1 - \left(\frac{1}{\alpha}\right)^{n+1}}{\frac{\alpha - 1}{\alpha}}$$

$$= \frac{\alpha^{n+1}\left(1 - \alpha^{-n-1}\right)}{\alpha - 1}$$

$$y(n) = \frac{\alpha^{n+1} - 1}{\alpha - 1} \quad \text{for } 0 \le n \le 4$$

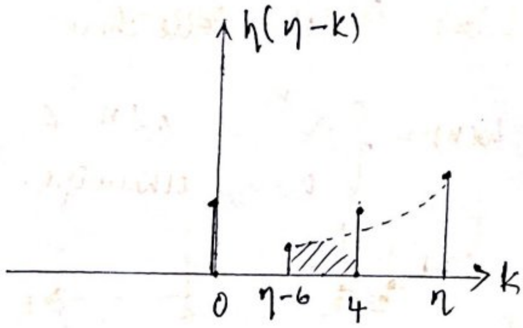


iii) $n > 4$ $\quad n-6 < 0$; $4 \le n \le 6$

over lap is from

iv) $0 \le n-6 \le 4$

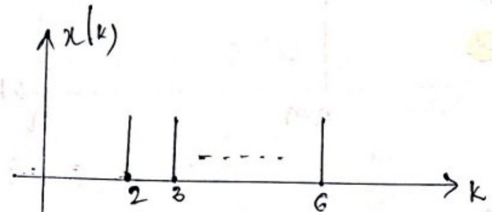
$6 \le n \le 10$.

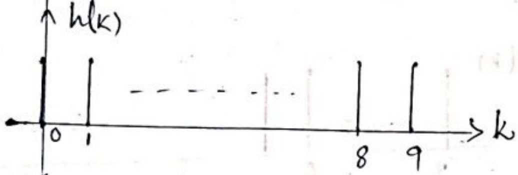


Qn 10 (10)

$x(n) = u(n-2) - u(n-7)$

$h(n) = u(n) - u(n-10)$





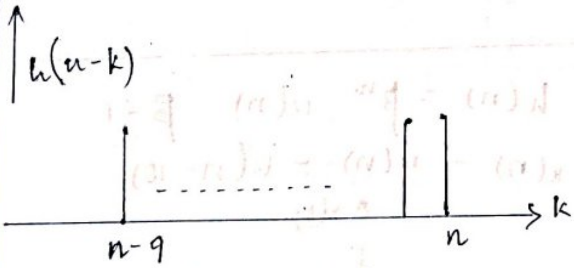
$h(k): 0 \le k \le 9$

$h(n-k): \quad 0 \le n-k \le 9$

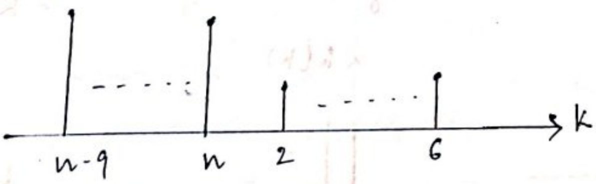
$\quad -n \le -k \le 9-n$

$n \geq k \geq n-9$

$n-9 \le k \le n$



i) for $n < 2$

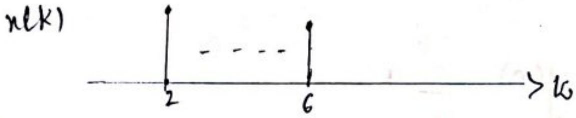


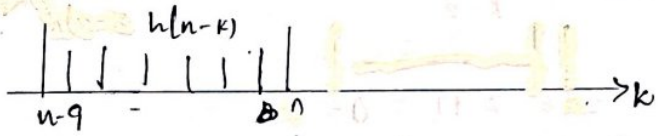
no overlap

$\therefore \; y(n) = 0.$

ii) For $n \geq 2$ & $n \le 6$

i.e., $2 \le n \le 6$





overlapping is from 2 to n.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=2}^{n} 1 \cdot 1$$

$$= n - 2 + 1$$

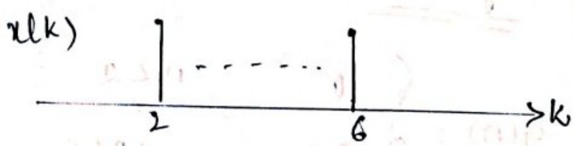
$$\underline{\underline{y(n) = n-1}}$$

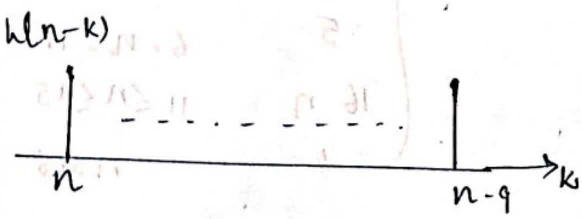
Recall: $\displaystyle\sum_{k=N_1}^{N_2} 1 = N_2 - N_1 + 1$

iii) $n > 6$ but $n-9 < 2$

$n > 6$, $n < 11$

$6 < n < 11$





$\therefore$ of overlapping is from 2 to 6

$$y(n) = \sum_{k=2}^{6} 1$$

$= 6 - 2 + 1 = 05$

$y(n) = 5$

iv) $n-9 \geq 2$ & $n-9 \leq 6$

$n \geq 11$, $n \leq 15$

$11 \leq n \leq 15$

overlap is from $n-9$ to 6.

$$y(n) = \sum_{k=n-9}^{6} 1$$

$y(n) = 6-(n-9)+1$

$= 6-n+9+1$

$y(n) = 16-n$

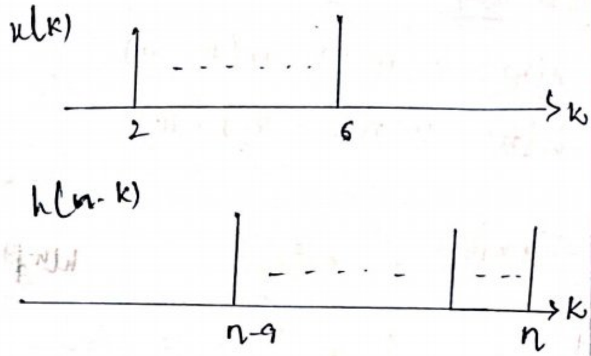
v) $n-9 > 6$; $n > 15$

No overlap

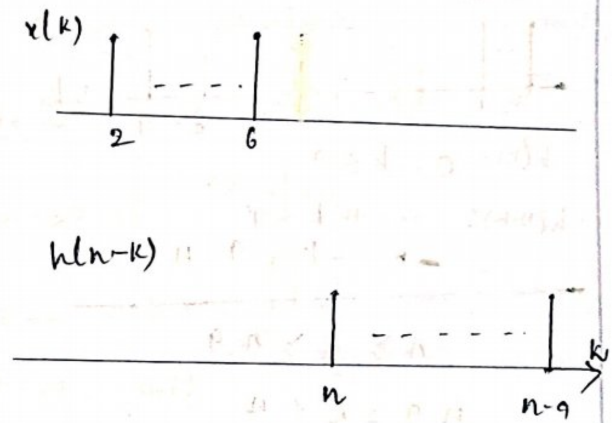
$y(n) = 0.$

$$y(n) = \begin{cases} 0 & n < 2 \\ n-1 & 2 \leq n \leq 6 \\ 5 & 6 < n < 11 \\ 16-n & 11 \leq n \leq 15 \\ 0 & n > 15 \end{cases}$$

Case (iv)

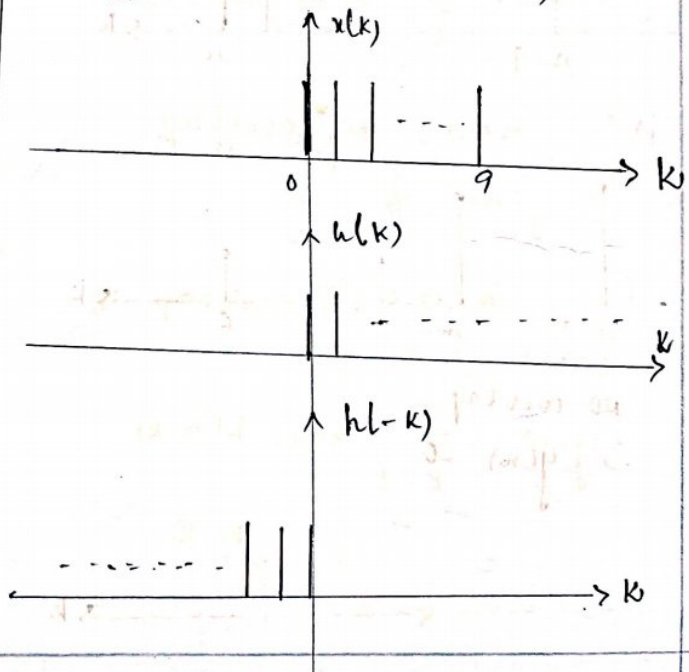


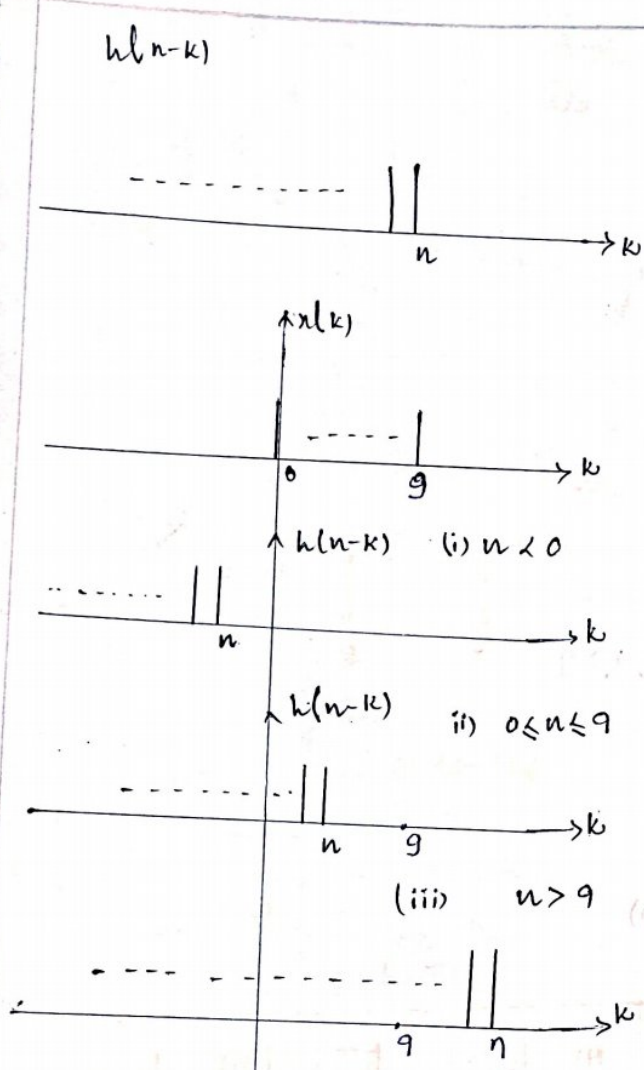
Case (v)



II) $h(n) = \beta^n u(n)$ $\beta < 1$

$x(n) = u(n) - u(n-10)$





$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

i) $n<0$; no overlap

$$y(n) = 0$$

ii) $0 \le n \le 9$; overlap is from 0 to n

$$y(n) = \sum_{k=0}^{n} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} 1 \quad \beta^{n-k}$$

$$= \beta^{n} \sum_{k=0}^{n} \beta^{-k}$$

$$= \beta^{n} \sum_{k=0}^{n} \left(\frac{1}{\beta}\right)^{k}$$

$$= \beta^{n} \frac{\left(\frac{1}{\beta}\right)^{n+1} - 1}{\frac{1}{\beta} - 1}$$

$$= \beta^{n} \frac{(\beta)^{-n-1} - 1}{\frac{1-\beta}{\beta}}$$

$$= \frac{\beta^{n+1}\left[\beta^{-n-1} - 1\right]}{1-\beta}$$

$$y(n) = \frac{1 - \beta^{n+1}}{1 - \beta}$$

(iii) for $n>9$,

The overlap is from 0 to 9

$$y(n) = \sum_{k=0}^{9} 1 \cdot \beta^{n-k}$$

$$= \beta^{n} \sum_{k=0}^{9} \left(\frac{1}{\beta}\right)^{k}$$

$$= \beta^{n} \frac{(1/\beta)^{9+1} - 1}{1/\beta - 1}$$

$$y(n) = \beta^{n} \frac{\left(\frac{1}{\beta}\right)^{10} - 1}{\frac{1}{\beta} - 1}$$

$$= \beta^{n} \frac{1-\beta^{10}}{\beta^{10}} \times \frac{\beta}{1-\beta}$$

$$y(n) = \beta^{n-9}\left[\frac{1-\beta^{10}}{1-\beta}\right]$$

$$y(n) = \begin{cases} 0 & ; n<0 \\ \frac{1-\beta^{n+1}}{1-\beta} & 0 \le n \le 9 \\ \beta^{n-9}\left[\frac{1-\beta^{10}}{1-\beta}\right] & ; n>9 \end{cases}$$

(12) $x(n) = (1/2)^{n} u(n)$

$h(n) = \delta(n) - 1/2\, u(n-1)$

Find the convolution of $x(n)$ & $h(n)$

Sol:

$y(n) = x(n) * h(n)$

$= (1/2)^{n} u(n) * \left[\delta(n) - 1/2\, u(n-1)\right]$

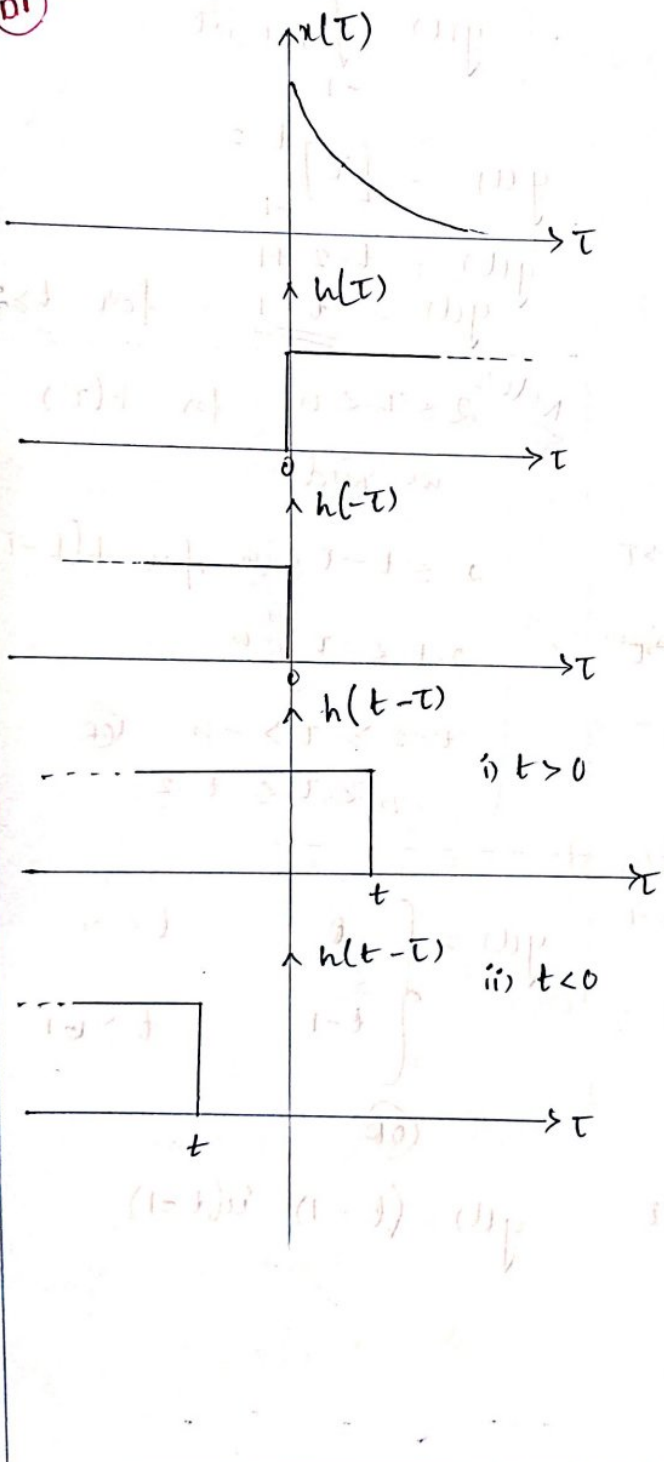
$= (1/2)^{n} u(n) * \delta(n) - (1/2)^{n} u(n) * 1/2\, u(n-1)$

$= (1/2)^{n} u(n) - (1/2)$

1) Consider a continuous time LTI system with unit impulse response $h(t) = u(t)$ & i/p $x(t) = e^{-at}u(t)$ $a > 0$; find the o/p of the system.

Soln

D1



$$x(t) = e^{-at}u(t)$$

$$h(t) = u(t)$$

$$y(t) = x(t) * h(t)$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

i) for $t < 0$; No overlap

$y(t) = 0$

ii) for $t \geq 0$; overlap is from 0 to t.

$$y(t) = \int_{\tau=0}^{t} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_0^t e^{-a\tau} u(\tau)\, u(t-\tau)$$

$$= \int_0^t e^{-a\tau}$$

$$= \left[\frac{e^{-a\tau}}{-a}\right]_0^t$$

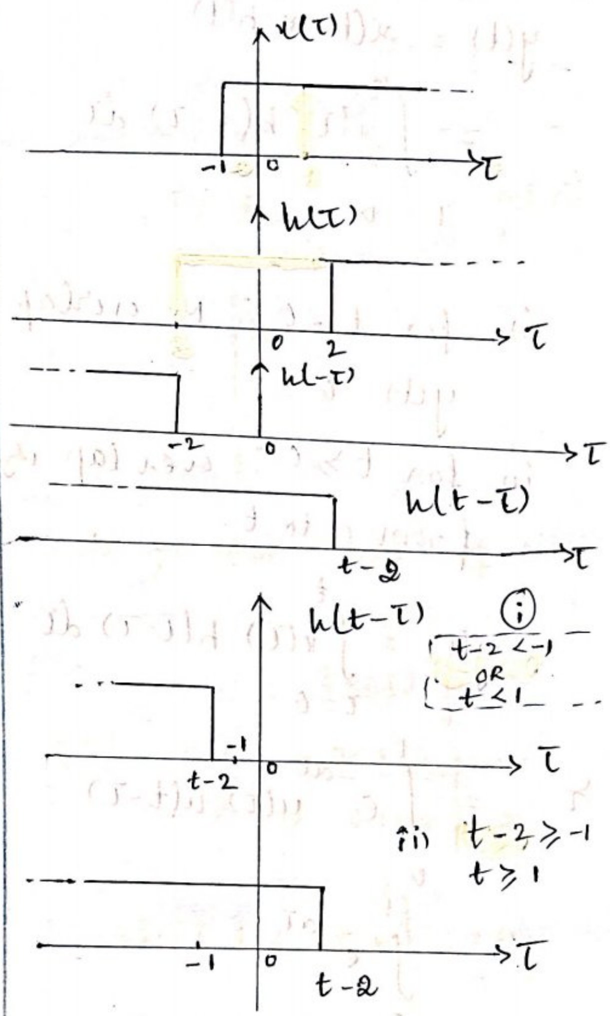
$$= -\frac{1}{a}\left[e^{-at} - 1\right]$$

$$= \frac{1}{a}\left[1 - e^{-at}\right]$$

$$\therefore\ y(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{a}(1-e^{-at}) & t \geq 0 \end{cases}$$

(2) $x(t) = u(t+1)$

(02) $h(t) = u(t-2)$



$$y(t) = x(t) * h(t)$$

$$= \int_{\tau=-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\, d\tau$$

i) for $t-2 < -1$; $t < 1$

No overlap.

$\therefore\ y(t) = 0$

ii) for $t-2 \geq -1$, $t \geq 1$ overlap is from -1 to $t-2$

$$\therefore\ y(t) = \int_{-1}^{t-2} 1.1\, d\tau$$

$$y(t) = [\tau]_{-1}^{t-2}$$

$$y(t) = t-2+1$$

$$y(t) = \underline{t-1} \quad \text{for } \underline{t \geq 1}$$

Note: $2 \leq \tau < \infty$ for $h(\tau)$

we need

$2 \leq t-\tau < \infty$ for $h(t-\tau)$

$2-t \leq -\tau < \infty$

$t-2 \geq \tau > -\infty$ (OR)

$-\infty < \tau \leq t-2$

$$y(t) = \begin{cases} 0 & t < 1 \\ t-1 & t \geq 1 \end{cases}$$

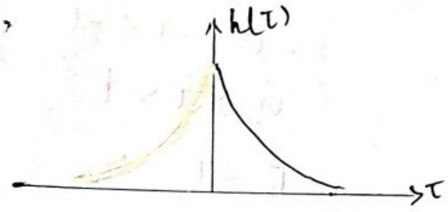
OR

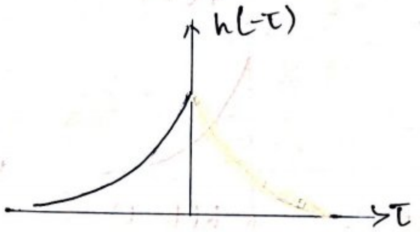
$$y(t) = (t-1)\, u(t-1)$$

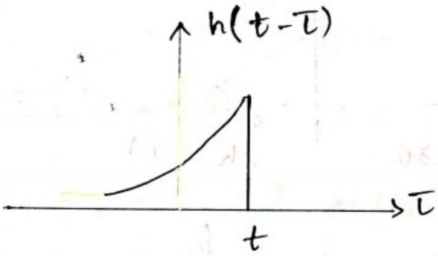
(3) $x(t) = e^{-3t} u(t)$

03

$h(t) = e^{-2t} u(t)$







for $h(\tau)$; $0 < \tau < \infty$

for $h(t-\tau)$: $0 < t-\tau < \infty$

$-t < -\tau < \infty$

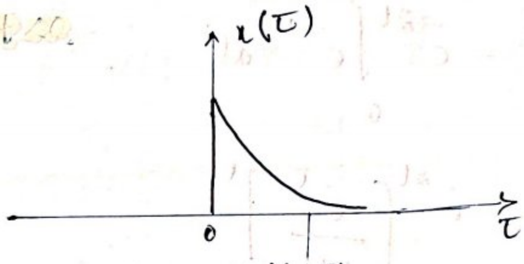
$t \quad \tau \quad -\infty$

OR $-\infty < \tau \leq t$

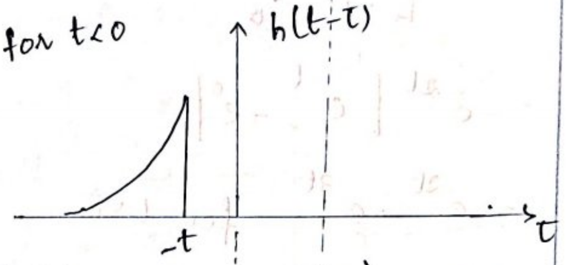
$y(t) = x(t) * h(t)$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

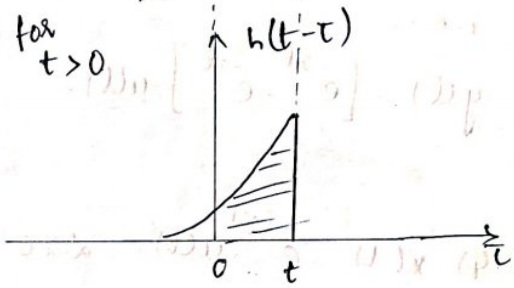
i)



i) for $t<0$



ii) for $t>0$



i) for $t<0$: there is no overlap ∴

$y(t) = 0$.

ii) for $t>0$; the overlap is from 0 to t.

$$y(t) = \int_0^t x(\tau) \cdot h(t-\tau)\, d\tau$$

$$= \int_0^t e^{-3\tau} \cdot e^{-2(t-\tau)}\, d\tau$$

$$= e^{-2t} \int_0^t e^{-3\tau + 2\tau}\, d\tau.$$

$$= e^{-2t}\int_0^t e^{-\tau}\, d\tau$$

$$= e^{-2t}\left[\frac{e^{-\tau}}{-1}\right]_0^t$$

$$= -e^{-2t}\left[e^{-t} - e^{0}\right]$$

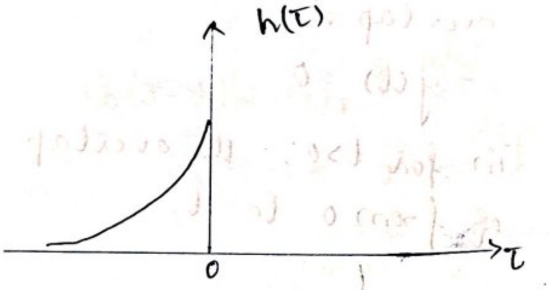
$$= e^{-2t} - e^{-3t} \quad \text{for } t>0.$$

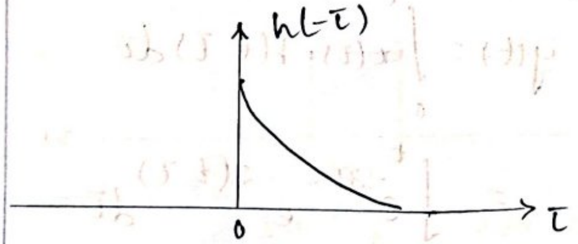
i.e.,

$$y(t) = \left[e^{-2t} - e^{-3t}\right] u(t).$$

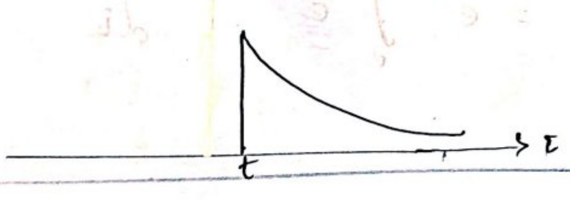
4) $x(t) = e^{-\alpha t} u(t) \quad \alpha > 0$

04) $h(t) = e^{\alpha t} u(-t)$





$h(t-\tau)$



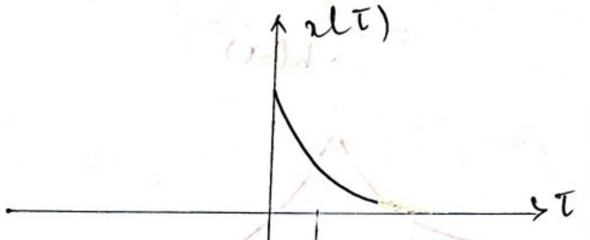
for $h(\tau)$ $\quad -\infty < \tau < 0$

$$-\infty < t-\tau < 0$$

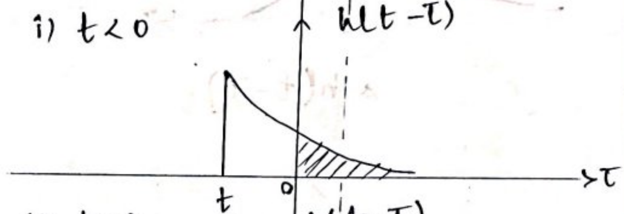
$$-\infty < -\tau < -t$$

$$\infty > \tau > t$$

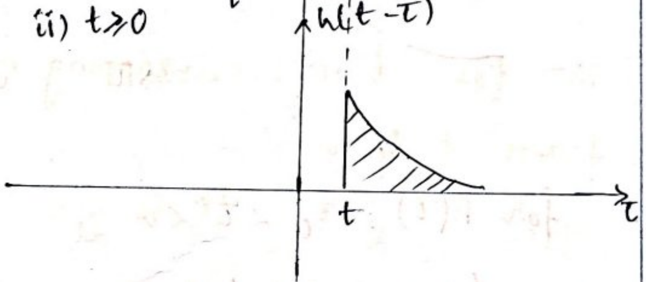
$$t < \tau < \infty$$



i) $t<0$



ii) $t \geq 0$



i) for $t<0$ overlapping is from 0 to ∞ ∴

$$y(t) = \int_0^\infty x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_0^\infty e^{-\alpha\tau} \cdot e^{\alpha(t-\tau)}\, d\tau$$

$$= \int_0^\infty e^{-\alpha\tau} \cdot e^{\alpha t}\, e^{-\alpha\tau}\, d\tau$$

$$= e^{\alpha t}\int_0^\infty e^{-2\alpha\tau}\, d\tau$$

$$= e^{\alpha t}\left[\frac{e^{-2\alpha}}{-2\alpha}\right]_0^\infty$$

$$= -e^{\alpha t}\left[\frac{e^{-\infty}-e^{0}}{2\alpha}\right]$$

$$= -e^{\alpha t}\frac{(0-1)}{2\alpha}$$

$$= \frac{e^{\alpha t}}{2\alpha} \quad \text{for } t<0.$$

ii) for $t \geq 0$; overlapping is from t to ∞

$$y(t) = \int_t^\infty e^{-\alpha\tau}\, e^{\alpha(t-\tau)}\, d\tau$$

$$= e^{\alpha t}\int_t^\infty e^{-2\alpha\tau}\, d\tau$$

$$= e^{t}\int_t^\infty e^{-2\alpha\tau}\, d\tau$$

$$= e^{\alpha t}\left[\frac{e^{-2\alpha\tau}}{-2\alpha}\right]_t^\infty$$

$$= \frac{-e^{\alpha t}}{2\alpha}\left[e^{-\infty} - e^{-2\alpha t}\right]$$

$$= \frac{-e^{\alpha t}}{2\alpha}\left[0 - e^{-2\alpha t}\right]$$

$$= \frac{e^{-\alpha t}}{2\alpha} \quad \text{for } t \geq 0$$

$$y(t) = \begin{cases} \dfrac{e^{\alpha t}}{2\alpha} & t<0 \\[2mm] \dfrac{e^{-\alpha t}}{2\alpha} & t \geq 0 \end{cases}$$

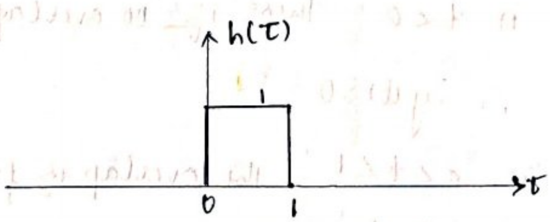
05

5) $x(t) = e^{-t}u(t)$

$$h(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

soln

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\cdot h(t-\tau)\cdot d\tau$$



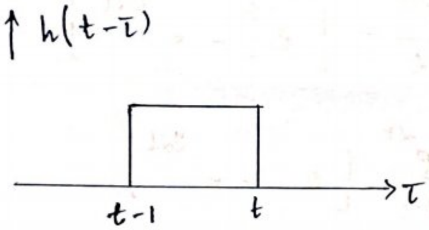
For $h(\tau)$: $0 \leq \tau \leq 1$

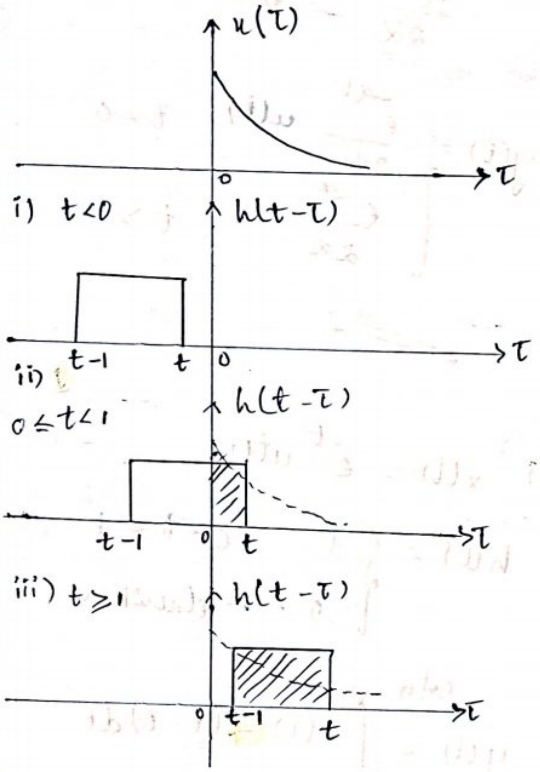
for $h(t-\tau)$: $0 \leq t-\tau \leq 1$

$-t \leq -\tau \leq 1-t$

$t \geq \tau \geq t-1$

$t-1 \leq \tau \leq t$





i) $t<0$; there is no overlap

$\therefore\ y(t)=0$

ii) $0<t<1$; the overlap is from 0 to t

$$y(t)=\int_0^t x(\tau)\,h(t-\tau)\,d\tau$$

$$=\int_0^t e^{-\tau}\ 1\ d\tau$$

$$=\left[\frac{e^{-\tau}}{-1}\right]_0^t$$

$$=-1\left(e^{-t}-e^{0}\right)$$

$$y(t)=\underline{1-e^{-t}}$$

iii) $t-1\geq 0$; $t\geq 1$;

overlap is from $t-1$ to t

$$y(t)=\int_0^1 e^{-\tau}\cdot 1\ d\tau$$

$$=\left[\frac{e^{-\tau}}{-1}\right]_{t-1}^{t}$$

$$=-1\left[e^{-t}-e^{-(t-1)}\right]$$

$$y(t)=\underline{e^{1-t}-e^{-t}}$$

$$y(t)=\begin{cases}0 & t<0\\ 1-e^{-t} & 0\leq t<1\\ e^{1-t}-e^{-t} & t\geq 1\end{cases}$$

Note: ii) $t\geq 0$ but $t-1<0$
$t\geq 0$ $\quad$ $t<1$
i.e., $\boxed{0\leq t<1}$

For above problem

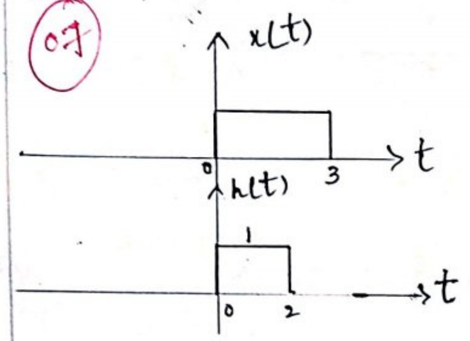
6) $x(t) = e^{-3t}\, u(t)$

06

$h(t) = u(t-1)$

7

07



Find the convolution of $x(t)$ & $h(t)$.

<u>Soln</u>

For $h(\tau)$: $\quad 0 \le \tau \le 2$

for $h(t-\tau)$: $\quad 0 \le t-\tau \le 2$

$$-t \le -\tau \le 2-t$$

$$t \ge \tau \ge t-2$$

$$t-2 \le \tau \le t$$

i) $t<0$; No overlap

$y(t) = 0$

ii) $t \ge 0 \quad t-2<0$

$t \ge 0 \quad t<2$

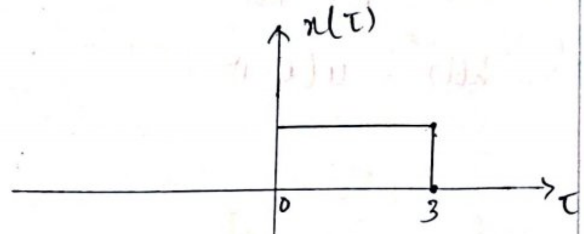
$0 \le t < 2$; overlap is from 0 to t

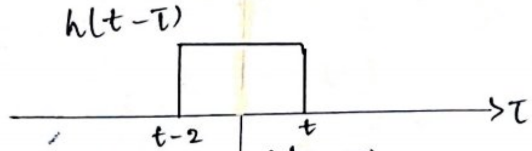
$$y(t) = \int_0^t 1 \cdot 1 \, d\tau$$

$$= [\tau]_0^t$$

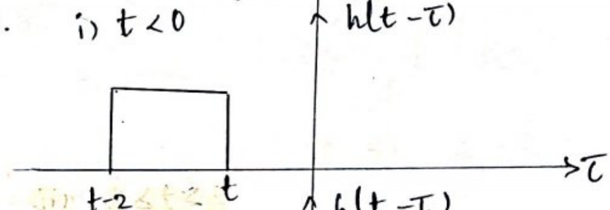
$$= t - 0$$

$$= \underline{t}$$

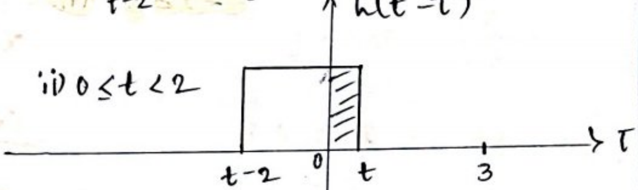




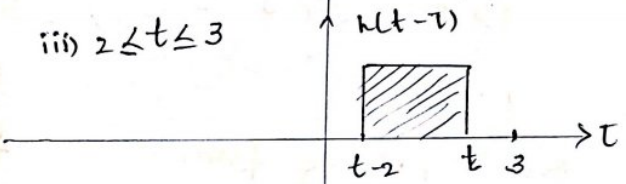
i) $t<0$



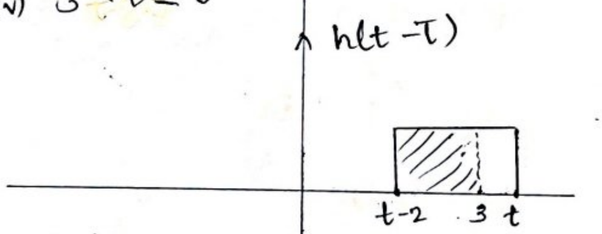
ii) $0 \le t < 2$



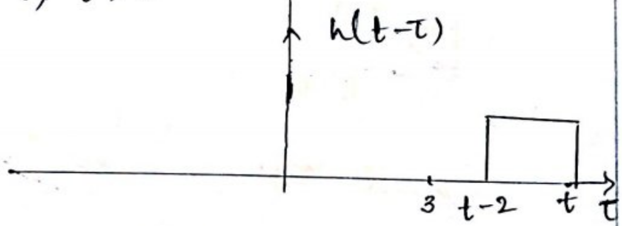
iii) $2 \le t \le 3$



iv) $3 \le t \le 5$



v) $t>5$



iii) $t<3$ & $t-2 \;\; 0$

$t \le 3 \qquad t \ge 2$

$2 \le t \le 3$

overlapping is from
$t-2$ to t

$$y(t) = \int_{t-2}^{t} 1 \cdot 1 \, d\tau$$

$$= [\tau]_{t-2}^{t}$$

$$= t-(t-2)$$

$$= 2 \quad 2 \leq t \leq 3$$

iv) $t > 3$ $\quad t-2 \leq 3$

$t > 3$ $\quad t \leq 5$

$3 < t \leq 5$

overlapping is from
$t-2$ to 3

$$y(t) = \int_{t-2}^{3} 1 \cdot 1 \, d\tau$$

$$= [\tau]_{t-2}^{3}$$

$$= 3-(t-2)$$

$$= 5-t \quad 3 < t \leq 5$$

v) $t-2 > 3$; $t > 5$

No overlapping

$y(t) = 0$

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$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 2 \\ 2 & 2 \leq t \leq 3 \\ 5-t & 3 < t \leq 5 \\ 0 & t > 5 \end{cases}$$

08

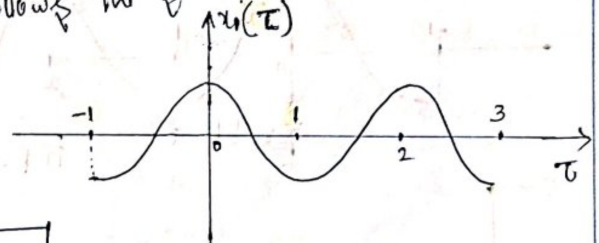
8) Convolute the 2 continuous time signals

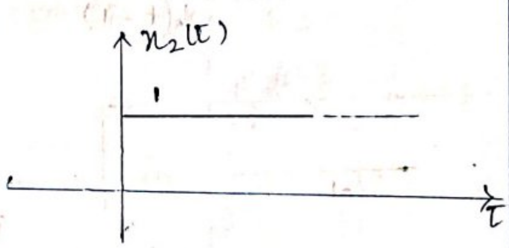
$x_1(t) = \cos \pi t \, [u(t+1) - u(t-3)]$

$x_2(t) = u(t)$

Soln Sketching $x_1(t)$ & $x_2(t)$.

Since $u(t+1) - u(t-3)$ exists from $t=-1$ to $t=3$, $x_1(t)$ follows the same limit





$y(t) = x_1(t) * x_2(t)$

$$y(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

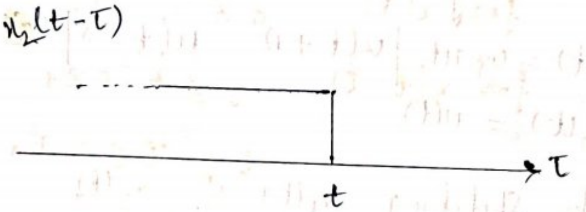
for $x_2(\tau)$: $0 < \tau < \infty$

for $x_2(t-\tau)$: $0 \le t-\tau \le \infty$

$-t \le -\tau \le \infty$

$t \ge \tau \ge -\infty$

$-\infty \le \tau \le t$



i) $t < -1$; No overlap

$y(t) = 0$

ii) $t \ge -1$ & $t \le 3$

$-1 \le t \le 3$

overlap is from -1 to 3

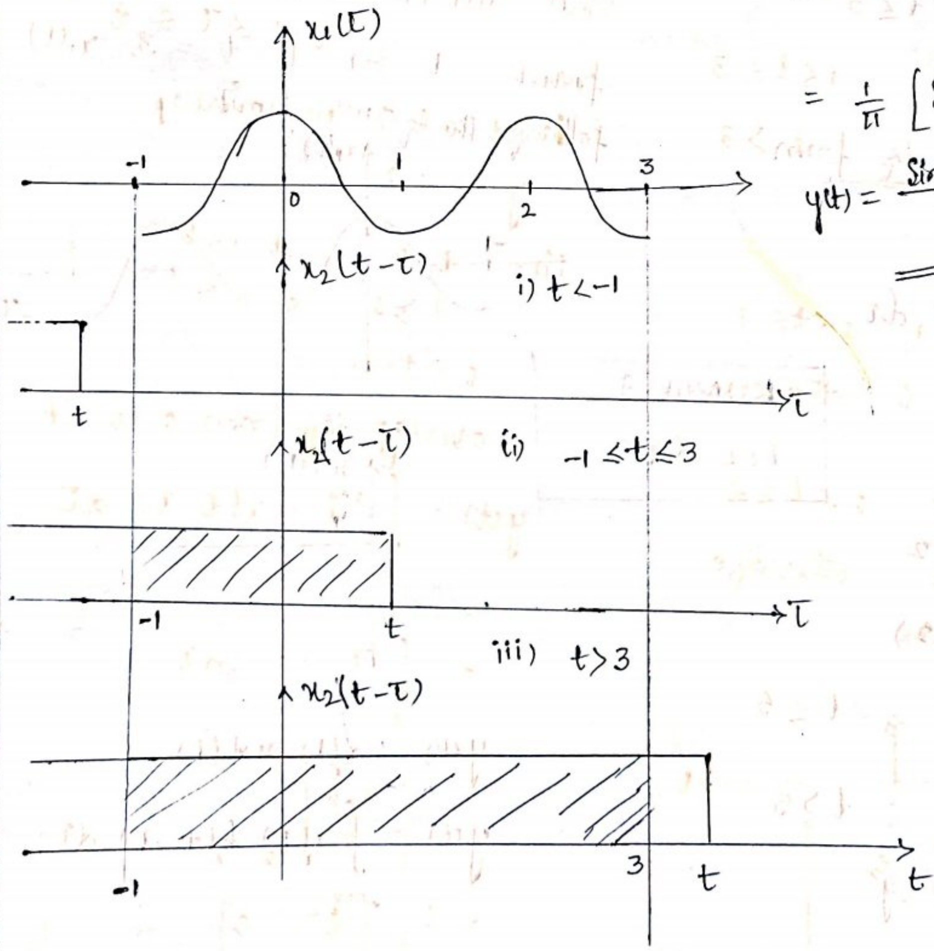
$$y(t) = \int_{-1}^{t} x_1(\tau)\, x_2(t-\tau)\, d\tau$$

$$= \int_{-1}^{t} \cos \pi\tau \cdot 1 \, d\tau$$

$$= \left[\frac{\sin \pi\tau}{\pi}\right]_{-1}^{t}$$

$$= \frac{1}{\pi}\left[\sin \pi t - 0\right]$$

$$y(t) = \frac{\sin \pi t}{\pi}$$



iii) when $t>3$

overlap is from -1 to 3

$$y(t) = \int_{-1}^{3} \cos \pi\tau \cdot 1 \, d\tau$$

$$= \left[\frac{\sin \pi\tau}{\pi}\right]_{-1}^{3}$$

$$= \frac{1}{\pi}\left[\sin 3\pi - \sin(-1\pi)\right]$$

$$= \frac{1}{\pi}[0] = 0$$

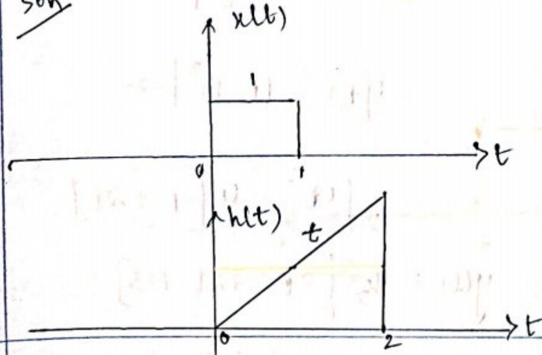
$$y(t) = \begin{cases} 0 & t<-1 \\ \dfrac{\sin \pi t}{\pi} & -1 \le t \le 3 \\ 0 & t>3 \end{cases}$$

(9) 09

$$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$h(t) = \begin{cases} t & 0 \le t \le 2 \\ 0 & \text{otherwise.} \end{cases}$$

Soln



Here we keep $h(t)$ fixed & vary $x(t)$

$$y(t) = x(t) * h(t) \quad \text{(OR)}$$

$$y(t) = h(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

for $x(\tau)$ $\quad 0 \le \tau \le 1$

for $x(t-\tau)$ $\quad 0 \le t-\tau \le 1$

$$-t \le -\tau \le 1-t$$

$$t \ge \tau \ge t-1$$

$$t-1 \le \tau \le t$$

i) $t<0$; No overlap

$y(t) = 0$

ii) $t \ge 0$ $\quad t-1<0$

$t \ge 0$ $\quad t<+1$

$0 \le t < 1$

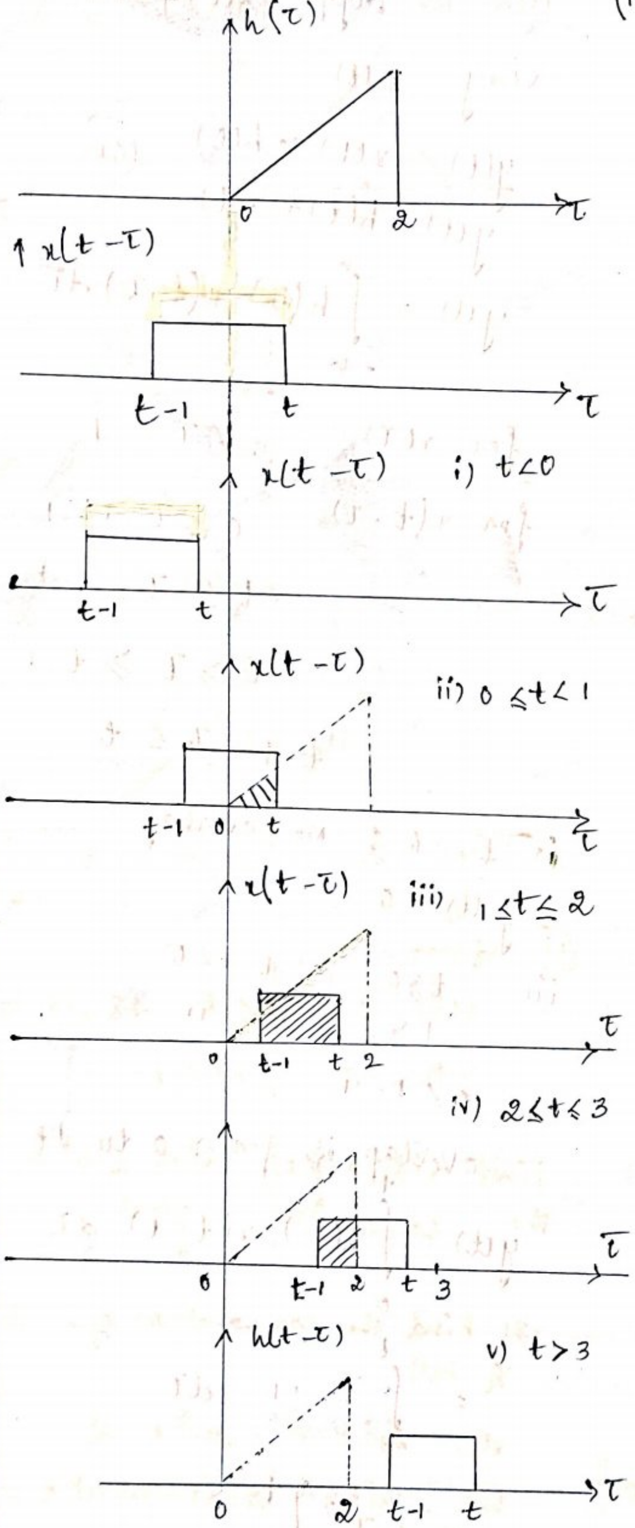
overlap is from 0 to t

$$y(t) = \int_{0}^{t} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{0}^{t} \tau \cdot 1 \, d\tau$$

$$= \left[\frac{\tau^2}{2}\right]_{0}^{t}$$

$$= \frac{1}{2}\left[t^2 - 0\right] = \frac{t^2}{2}$$



(iii) $t \le 2 \quad t-1 \ge 0$

$t \le 2 \quad t \ge 1$

$1 \le t \le 2$

overlap is from $t-1$ to t

$$y(t) = \int_{t-1}^{t} \tau \cdot 1 \, d\tau$$

$$= \left[\frac{\tau^2}{2}\right]_{t-1}^{t}$$

$$= \frac{t^2 - (t-1)^2}{2}$$

$$= \frac{1}{2}\left[t^2 - t^2 - 1 + 2t\right]$$

$$y(t) = \frac{1}{2}\left[2t - 1\right]$$

(iv) $t > 2$ & $t-1 \le 2$

$t > 2 \quad t \le 3$

$2 < t \le 3$

$$y(t) = \int_{t-1}^{2} \tau \cdot 1 \, d\tau$$

$$= \left[\frac{\tau^2}{2}\right]_{t-1}^{2}$$

$$= \frac{1}{2}\left[2^2 - (t-1)^2\right]$$

$$= \frac{1}{2}\left[4 - t^2 - 1 + 2t\right]$$

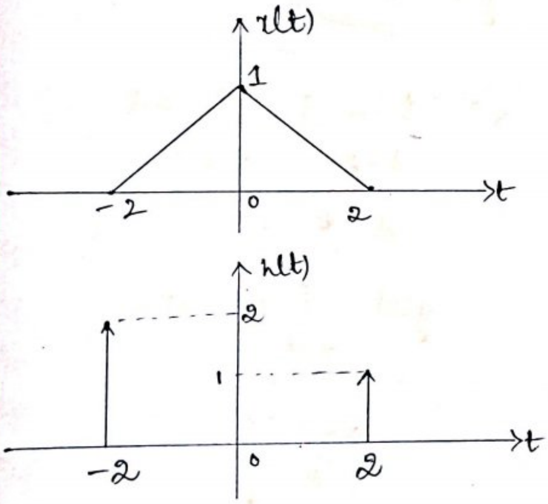
$$y(t) = \frac{1}{2}\left[-t^2 + 2t + 3\right]$$

V) when $t>3$ there is no overlap $\therefore$ $y(t)=0$

$$y(t)=\begin{cases} 0 & t<0 \\ \frac{t^2}{2} & 0\le t\le 1 \\ \frac{1}{2}(2t-1) & 1\le t\le 2 \\ \frac{1}{2}(-t^2+2t+3) & 2\le t\le 3 \\ 0 & t>3 \end{cases}$$

(10)

(10) Obtain the convolution of $x(t)$ & $h(t)$. Also express the o/p in terms of $x(t)$



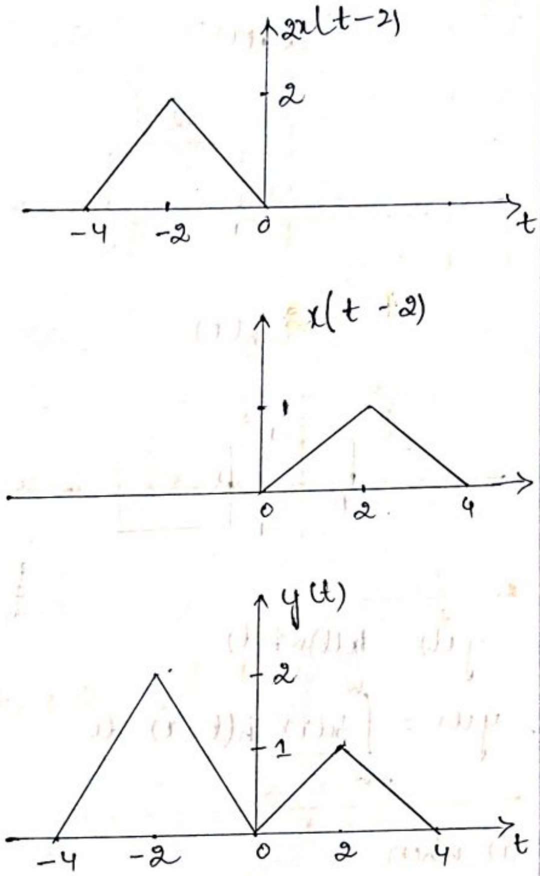
Soln

$$y(t) = x(t) * h(t)$$

$$y(t) = x(t) * [2\delta(t+2) + \delta(t-2)]$$

$$= x(t) * 2\delta(t+2) + x(t) * \delta(t-2)$$

$$y(t) = 2x(t+2) + x(t-2)$$



(11) Given

(11) $x(t) = \delta(t) - 2\delta(t-1) + 8\delta(t-2)$

$y(t) = 2 \quad -1\le t\le 1$

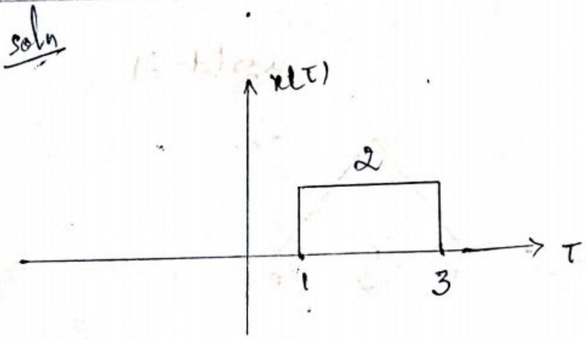
Find $x(t) * y(t)$, also sketch the convoluted signal.

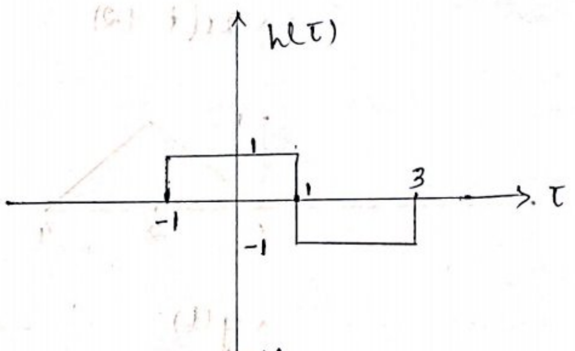
12) Find the convolution of $x(t)$ & $h(t)$.

$x(t) = 2u(t-1) - 2u(t-3)$

$h(t) = u(t+1) - 2u(t-1) + u(t-3)$

soln





$$y(t) = h(t) * x(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

i) when

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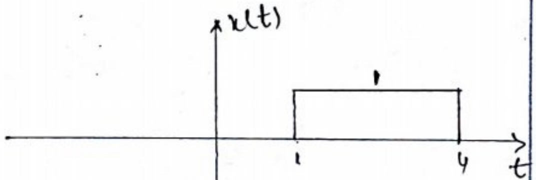
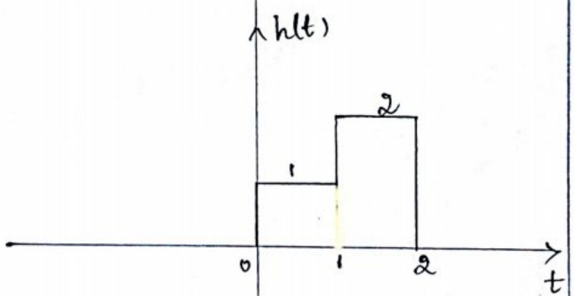
12) $x(t) = u(t-1) - u(t-4)$

$h(t) = u(t) + u(t-1) - 2u(t-2)$





13

13)
$x(t) = (t + 2t^2)[u(t+1) - u(t-1)]$
$h(t) = 2u(t+2)$.

14) $x(t) = u(t+2) - u(t-1)$
$h(t) = u(-t+2)$.