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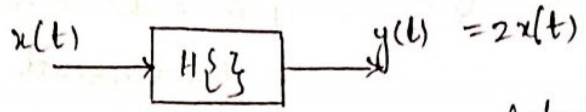
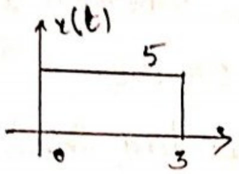


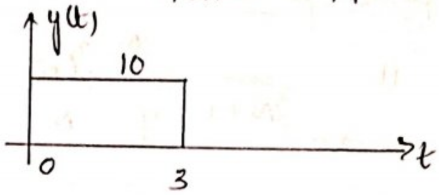
Actual i/p

fig ①



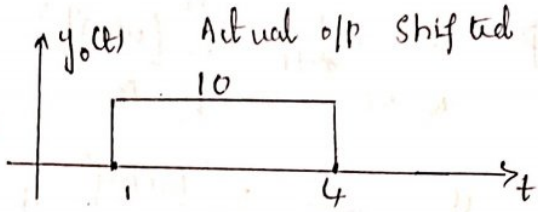
Actual o/p

fig ②



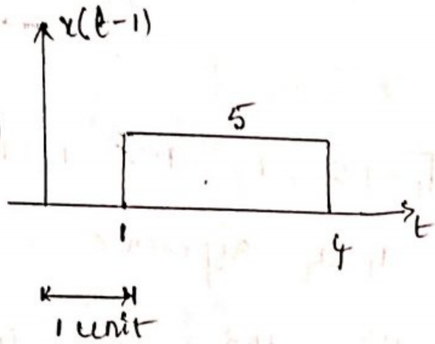
Actual o/p shifted

fig ③



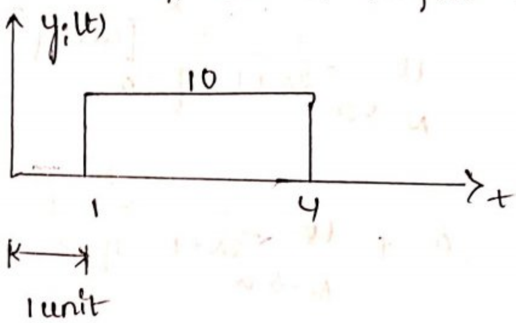
shifted i/p

fig ④



o/p due to shifted i/p

fig ⑤



The system is time invariant because:

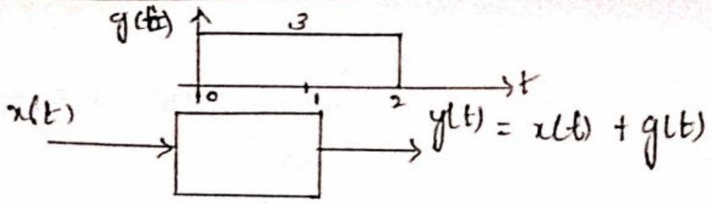
shifted o/p = o/p due to shifted i/p

i.e., $y_o(t) = y_i(t)$; compare fig ③ & ⑤

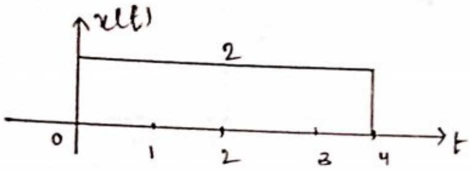
OR

Shift w.r.t resulted in identical shift in the o/p.

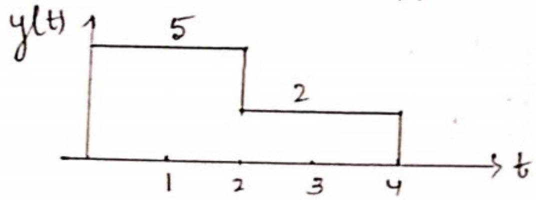
compare fig ④ and 5 ; 1unit shift in both.



Actual i/p

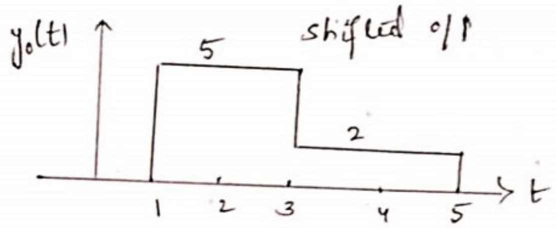


Actual o/p

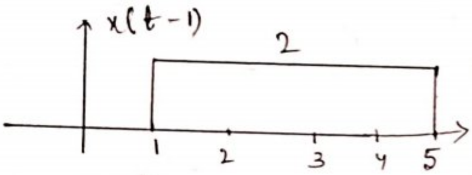


shif

shifted o/p

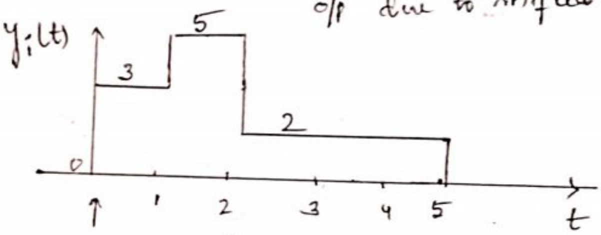


shifted i/p



1 unit shift

o/p due to shifted i/p



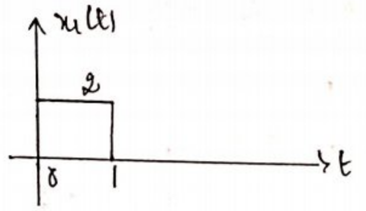
0 unit shift

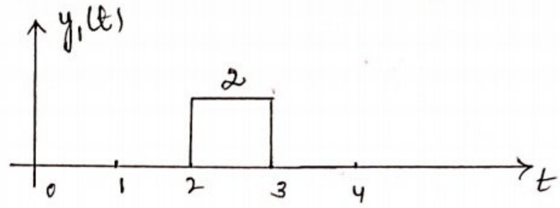
$$y_0(t) \neq y_i(t)$$

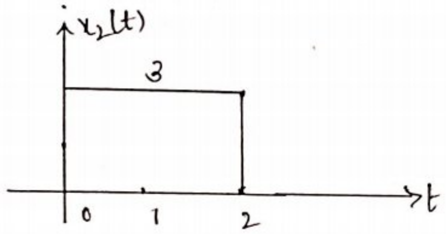
OR

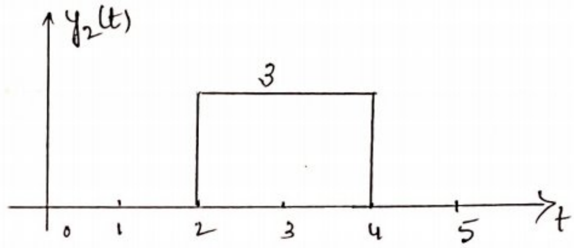
Shift w.r.t i/p has not resulted identical shift in o/p

$x(t) \rightarrow \boxed{H\{\ \}} \rightarrow y(t) = x(t-2)$

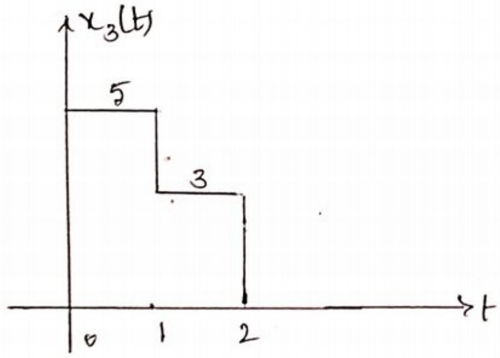


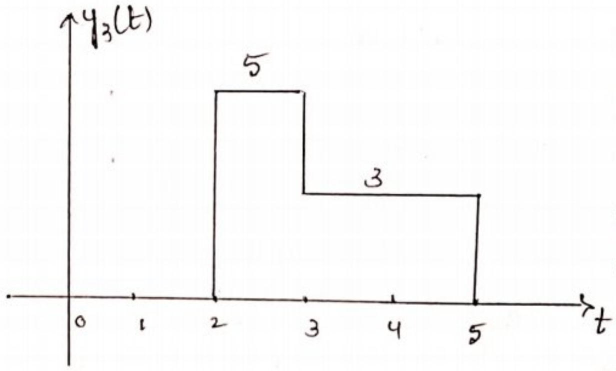






Now let $x_3(t) = x_1(t) + x_2(t)$

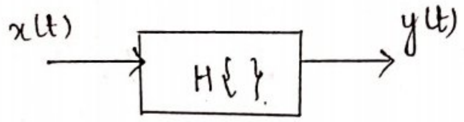




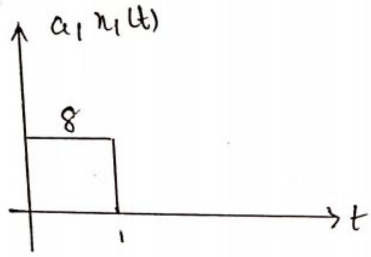
Here it is given that $y_3(t) = y_1(t) + y_2(t)$

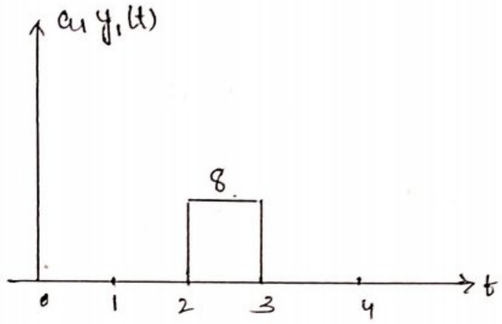
Now scaling the i/p $x_1(t)$ by $a_1 = 4$

$x_2(t)$ by $a_2 = 7$.

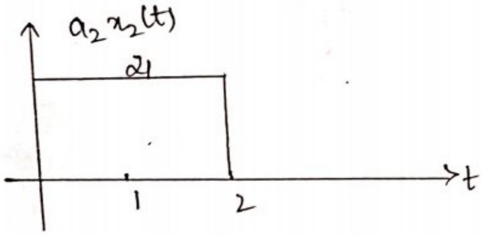


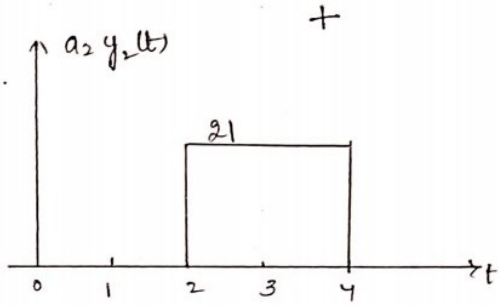
Here $a_1 = 4$ $\quad a_2 = 7$





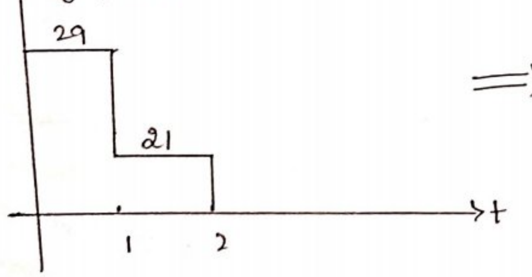
+



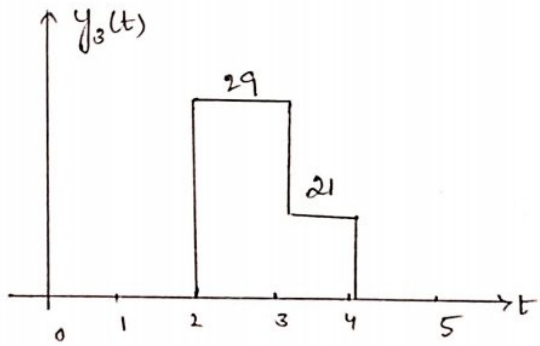


Now let

$x_3(t) = a_1 x_1(t) + a_2 x_2(t)$



$\Longrightarrow$



Here it is seen that

$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$

$$x(t) \longrightarrow \boxed{H\{\}} \longrightarrow y(t) = \log_{10} x(t)$$

i/p $x_1(t)$ o/p $y_1(t) = \log_{10} x_1(t)$

i/p $x_2(t)$ o/p $y_2(t) = \log_{10} x_2(t)$

let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

Now

i/p $x_3(t)$ o/p $y_3(t) = \log_{10} x_3(t)$

$$= \log [a_1 x_1(t) + a_2 x_2(t)]$$

$$y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$$

$$y_3(t) \neq a_1 \log_{10} x_1(t) + a_2 \log_{10} x_2(t)$$

Not a linear Sym.

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SYSTEM

A System consists of operators related to dependent & independent variables.

It manipulates one or more signals & produces an output signal. The overall system operator is denoted by symbol H or T

$$\text{input signal} \quad \xrightarrow[x(n)]{x(t)} \boxed{\text{H or T}}^{\text{System}} \xrightarrow[y(n)]{y(t)} \quad \text{output signal or Response}$$

The o/p is related to i/p by means of system operator as

$$y(t) = H\{x(t)\} \quad \text{or} \quad y(t) = T\{x(t)\}$$

$$y(n) = H\{x(n)\} \quad \text{or} \quad y(n) = T\{x(n)\}$$

The time shift operator of a continuous time system and discrete time system are denoted as

S^{t_0} & S^{k} respectively.

where t_0 is a real constant & k is an integer.

$$x(t) \longrightarrow \boxed{S^{t_0}} \longrightarrow y(t) = x(t-t_0)$$

$$x(n) \longrightarrow \boxed{S^{k}} \longrightarrow y(n) = x(n-k)$$

$$x(t) \longrightarrow \boxed{S^{1/2}} \longrightarrow y(t) = x(t-1/2)$$

$$x(n) \longrightarrow \boxed{S^{2}} \longrightarrow y(n) = x(n-2)$$

1) Find the system operator if the o/p of the system is given by $y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$

Soln

$$y(n) = \frac{1}{3}\left[S^{-1}x(n) + x(n) + S^{1}x(n)\right]$$

$$= \frac{1}{3}\left[S^{-1} + 1 + S\right]x(n)$$

$$y(n) = H\{x(n)\}$$

where the system operator $H = \frac{1}{3}\left[S^{-1} + 1 + S\right]$

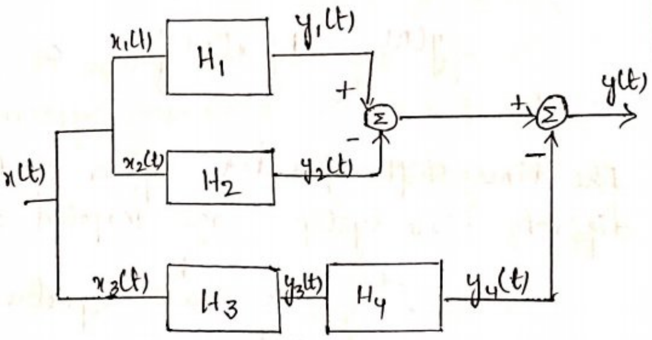
2) A system consists of several subsystems, constructed as shown in figure. Find the operator H relating $x(t)$ to $y(t)$ for the subsystem operator given by

H_1 : $y_1(t) = x_1(t)\, x(t-1)$

H_2 : $y_2(t) = |x_2(t)|$

H_3 : $y_3(t) = 1 + 2x_3(t)$

H_4 : $y_4(t) = \cos[x_4(t)]$



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PROPERTIES OF SYSTEM

The different properties of a system are

1) Memory
2) Causal
3) Stability
4) Linearity
5) Time invariance
6) Invertiblity

1) **MEMORY** : A system is said to be memory LESS if the o/p at any time instant depends upon the i/p at that instant of time. only.
If the o/p depends on past or future i/p value or both, the system is said to be memory system.

Ex: 1) $y(t) = 3x(t)$

At $t=-1$ $\quad y(-1) = 3x(-1)$

$t=0$ $\quad y(0) = 3x(0)$ $\quad$ Etc.

In this case o/p at any instant of time depends upon the i/p at that instant of time.

Ex 2: $y(t) = x(t-1)$

$t=-1$ $\quad y(-1) = x(-2)$

$t=0$ $\quad y(0) = x(-1)$ Since the present O/p depends on past i/p, it is a memory system.

2) **CAUSAL**

A system is said to be causal, if the o/p at any instant of time depends upon present or PAST or present past inputs. [OR system is causal if o/p does not depend on future i/ps]

A system is said to be non causal, if the o/p at any instant of time depends upon the future i/p.

Ex i) $y(t) = 3x(t)$; o/p depends on present i/p only not on future i/p values ∴ it is causal system.

ii) $y(t) = x(t-2)$; o/p depends on past i/p; it is causal.

iii) $y(t) = x(t)\,x(t-1)$; o/p depends on present & past i/p. so it is causal system.

iv) $$y(n) = \frac{1}{3}\left[x(n+1) + x(n) + x(n-1)\right]$$

for $n=0$ $y(0) = \frac{1}{3}\left[x(1) + x(0) + x(-1)\right]$

Since present o/p $y(0)$ depends on future i/p $x(1)$, it is a non causal system.

⇒ $y(t) = \log[x(t)]$; it is a memoryless & causal system.

$y(t) = e^{x(t)}$; —— " —— " ——

$y(t) = \underbrace{\cos}_{\text{sym operator}}(t+1)\; x(t)$

for $t=0$ $y(0) = \cos(1)\; x(0)$

present o/p $y(0)$ depends on present i/p $x(0)$ only ∴ it is memoryless & causal. (∵ we should consider only timing of i/p, not the system operator)

③ STABILITY

A system is said to be stable if the o/p of the system is FINITE for FINITE input.

If $|x(t)| \le M_x < \infty$ results in $|y(t)| \le M_y < \infty$; the system is said to be BOUNDED i/p BOUNDED o/p stable or BIBO stable. Here M_x & M_y are some finite consts.

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Ex: $y(t) = 3x(t)$

Let $|x(t)| = M_x = 5$, then o/p

$|y(t)| = |3x(t)|$

$= 3|x(t)|$

$= 3M_x$

$= 3 \times 5$

$|y(t)| = 15 = M_y < \infty$

$\therefore$ the system is stable.

$y(t) = \frac{5}{x(t)}$

if $|x(t)| = 0$

$y(t) = \frac{6}{0}$

$= \infty$

non stable sys.

Ex: $y(t) = e^t x(t)$

Let $|x(t)| = M_x = 5$

the o/p

$|y(t)| = |e^t x(t)|$

$= |e^t| \, |x(t)|$

$= |e^t| \, 5$

$|y(t)| = 5 e^t = M_y$

As $t \to \infty$, $M_y \to \infty$

$\therefore$ the system is unstable

4) LINEARITY

A system is said to be linear if it satisfies the principle of superposition.

Statement of superposition: weighted sum of several i/p giving rise to weighted sum of corresponding o/p.

$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = H\{x(t)\}$

Explanation:

For the i/p $x(t) = x_1(t)$, let the o/p be $y(t) = y_1(t)$
$= H\{x_1(t)\}$

Similarly for

$x(t) = x_2(t)$ $\quad y(t) = y_2(t) = H\{x_2(t)\}$

Now let the i/p $x(t) = x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

let the o/p be $y(t) = y_3(t)$

i.e.,

$y_3(t) = H\{x_3(t)\}$

$y_3(t) = H\{a_1 x_1(t) + a_2 x_2(t)\}$

If $y_3(t) = a_1 y_1(t) + a_2 y_2(t)$ then the system is said to be stable.

Ex: $y(t) = 2x(t)$

$y_1(t) = 2x_1(t)$

$y_2(t) = 2x_2(t)$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$ then

$y_3(t) = 2x_3(t)$

$y_3(t) = 2[a_1 x_1(t) + a_2 x_2(t)]$

$y_3(t) = a_1 2x_1(t) + a_2 2x_2(t)$

It is of the form

$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$

$\therefore$ the system is linear.

Ex $y(t) = 2x(t) + 3$

$y_1(t) = 2x_1(t) + 3$

$y_2(t) = 2x_2(t) + 3$

Now let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

then o/p

$y_3(t) = 2x_3(t) + 3$

$y_3(t) = 2\left[a_1 x_1(t) + a_2 x_2(t)\right] + 3$

$y_3(t) = a_1 2x_1(t) + a_2 2x_2(t) + 3$

$\cancel{y_3(t) = a_1 y_1(t) + a_2 y_2(t) + 3}$

Since $y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$

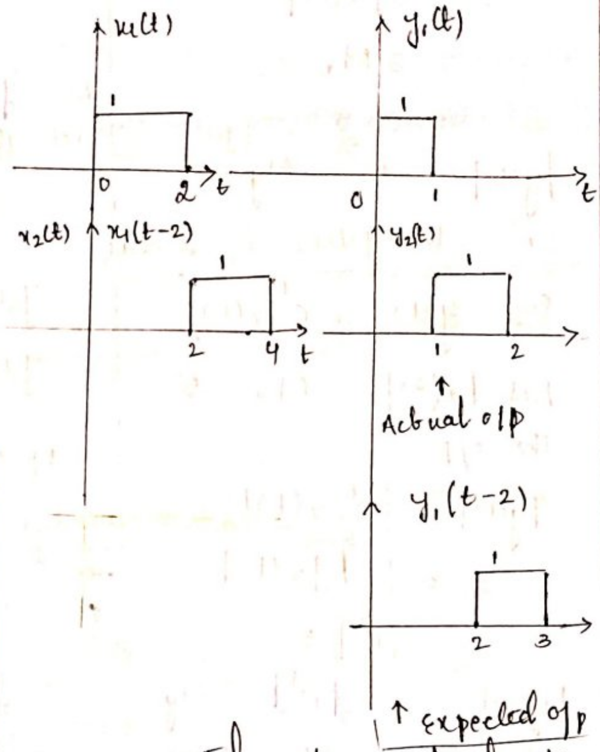
$\therefore$ It is a non linear sym.

Alternate way to interprete TIME INVARIANCE

$x(t) \rightarrow \boxed{H} \rightarrow y(t) = x(2t)$

$x_1(t)$ $y_1(t)$

$x_2(t)$ $y_2(t)$



Since expected o/p & actual o/p are not same; it is not a time variant sym.

5 TIME INVARIANCE

A system is said to be time invariant, if a time delay or time advance of the i/p results in a identical time shift in the o/p signal.

consider a system as shown below.

$x(t) \rightarrow \boxed{H} \rightarrow y(t)$

i) Shift the Signal before system operation (sbso)

$x(t) \rightarrow \boxed{S^{t_0}} \xrightarrow{x(t-t_0)} \boxed{H} \rightarrow y_{sbso}(t) = H\{x(t-t_0)\}$

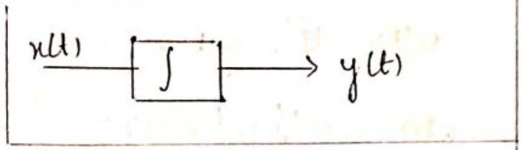
ii) Shift the signal after system operation (saso)

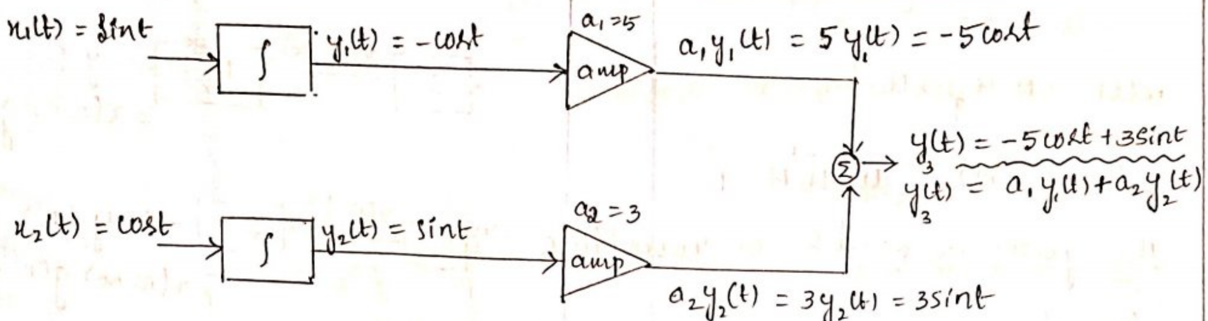
$x(t) \rightarrow \boxed{H} \xrightarrow{y(t)} \boxed{S^{t_0}} \rightarrow y_{saso}(t) = y(t-t_0)$

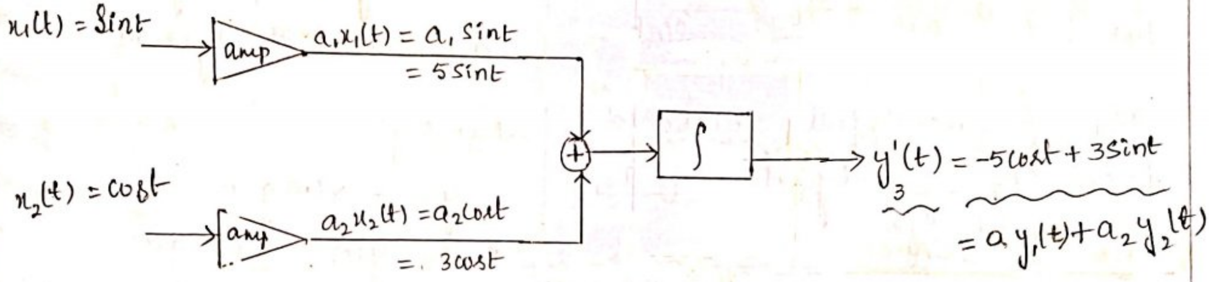
If $y_{saso}(t) = y_{sbso}(t)$, the system is time invariant.

Example for Linearity

Consider a system, which is an integrator







Since $y_3(t) = y'_3(t)$; the system is Linear.

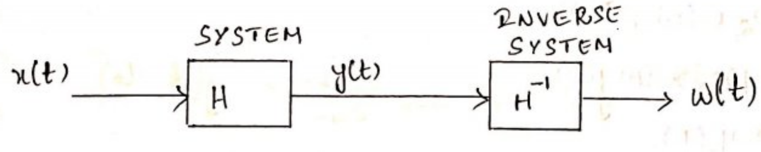
Note: In figure 1 amplification is done after sum operation
In figure 2 amplification is done before sum operation

6) INVERTIBLTY:

A system is said to be invertible, if the i/p of the system can be recovered from the systems o/p.

OR

A system is said to be invertible if it produces unique o/p for every input.



$y(t) = H\{x(t)\}$.

$$w(t) = H^{-1}\{y(t)\}$$

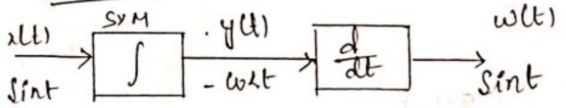
$$w(t) = H^{-1}\{H\{x(t)\}\}$$

$$w(t) = H^{-1}H\{x(t)\}$$

$w(t) = x(t)$ If $H^{-1}H = 1$,

the system is said to be Invertible

Example:



I/p is successfully recovered from o/p ∴ Integrator is a invertible sym.

Problems:

Verify whether the following systems are linear, memoryless, stable, causal & time invariant.

1) $y(n) = x(n)\, g(n)$

a) Linearity:

$$y_1(n) = x_1(n)\, g(n)$$

$$y_2(n) = x_2(n)\, g(n)$$

$$y_3(n) = x_3(n)\, g(n) \quad \text{where}$$

$$x_3(n) = a_1 x_1(n) + a_2 x_2(n)$$

$$y_3(n) = [a_1 x_1(n) + a_2 x_2(n)]\, g(n)$$

$$= a_1 x_1(n) g(n) + a_2 x_2(n) g(n)$$

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n).$$

∴ it is linear.

b) Time invariance

Sym Eqn: $y(n) = x(n)\, g(n) \rightarrow (1)$

I. o/p due to shifted i/p: $y_i(n)$

Place $-n_0$ inside $x(\,)$ in (1)

$$y_i(n) = x(n-n_0)\, g(n) \rightarrow (2)$$

II shifted o/p $y_0(n)$:

Replace $n \rightarrow n-n_0$ in (1)

$$y_0(n) = y(n-n_0) = x(n-n_0)\, g(n-n_0) \rightarrow (3)$$

From (2) & (3)

$$y_i(n) \neq y_0(n)$$

Sym not time invariant.

c) Memory:

$$y(n) = x(n)\, g(n)$$

$n=0 \quad y(0) = x(0)\, g(0)$

present o/p does not depend on past or future i/p value. ∴ it is memoryless sym.

d) Causality:

$$y(n) = g(n)\, x(n)$$

$n=0 \quad y(0) = g(0)\, x(0)$

present o/p does not depend on future i/p ∴ it is Causal System.

e) Stability

Let $|x(n)| \leq M_x < \infty$

$y(n) = |x(n)\, g(n)|$

$= |x(n)|\,|g(n)|$

$y(n) = M_x\, |g(n)|$

As $n \to \infty$ if $g(n)$ converges (attains a finite value), $y(n)$ is bounded or finite.

$\therefore$ the system is BIBO stable.

② $y(n) = x(n)\, u(n)$

a) Linearity:

$y_1(n) = x_1(n)\, u(n)$

$y_2(n) = x_2(n)\, u(n)$

$y_3(n) = x_3(n)\, u(n)$

where $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$\Rightarrow$

$y_3(n) = [a_1 x_1(n) + a_2 x_2(n)]\, u(n)$

$= a_1 x_1(n)\, u(n) + a_2 x_2(n) u(n)$

$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

$\therefore$ it is linear system.

b) time invariance:

sym Eqn: $y(n) = x(n) u(n) \longrightarrow ①$

o/p due to shifted i/p is $y_i(n)$

~~x(n)~~ — * replace $-n_0$ inside $x(\ )$ in ①

$y_i(n) = x(n-n_0)\, u(n) \longrightarrow ②$

Shifted o/p is $y_0(n)$

* replace $n \longrightarrow n-n_0$ in ①

$y_0(n) = y(n-n_0) = x(n-n_0)\, u(n-n_0) \longrightarrow ②$

$y_i(n) \neq y_0(n)$

$\therefore$ sym is not time invariant.

c) Memory:

$y(n) = x(n)\, u(n)$

Since present o/p depends on present i/p only, it is memoryless sym.

d) Causality

It is a causal sym.

e) Stability

Let $|x(t)| = M_x < \infty$

$y(t) = |x(t)\, u(n)|$

$y(t) = |x(t)|\, |u(n)|$

Here $|x(t)|$ is finite. Also $|u(n)| = 1$ (finite) as $n \to \infty$. $\therefore$ it is stable sym

③

$\Rightarrow y(n) = x(n) + u(n-2)$

a) Linearity:

$y_1(n) = x_1(n) + u(n-2)$

$y_2(n) = x_2(n) + u(n-2)$

$y_3(n) = x_3(n) + u(n-2)$

Where $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$y_3(n) = a_1 x_1(n) + a_2 x_2(n) + u(n-2)$

$y_3(n) = a_1 y_1(n) + a_2 y_2(n) + u(n-2)$

Since $y_3(n) \neq a_1 y_1(n) + a_2 y_2(n)$

it is a non linear system.

b) time invariance

sym Eqn: $y(n) = x(n) + u(n-2) \rightarrow ①$

o/p due to shifted i/p is $y_i(n)$

* place $-n_0$ inside $x(\ )$ in ①

$y_i(n) = x(n-n_0) + u(n-2) \longrightarrow ②$

shifted o/p: $y_0(n)$

$n \longrightarrow n-n_0$ in ①

$y_0(n) = y(n-n_0) = x(n-n_0) + u(n-n_0-2) \longrightarrow ③$

$\therefore y_i(n) \neq y_0(n)$

Sym is not time invariant.

c) memory

$y(n) = x(n) + u(n-2)$

$n=0 \quad y(0) = x(0) + u(-2)$

Since the present o/p depends on present i/p only, it is a memory system.

d) Causality:

Since present o/p depends on present i/p only, it is causal sym

Stability:

Let $|x(n)| = M_x < \infty$

$|y(n)| = |x(n) + u(n-2)|$

$= |x(n)| + |u(n-2)|$

$= M_x + |u(n-2)|$

As $n \rightarrow \infty \quad |u(n-2)| = 1$

$\therefore |y(n)| \leq M_y < \infty$

It is BIBO stable.

$x(n<2)$

4) $y(t) = \cos[x(t)]$

a) linearity:

$y_1(t) = \cos[x_1(t)]$

$y_2(t) = \cos[x_2(t)]$

$y_3(t) = \cos[x_3(t)]$

let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$y_3(t) = \cos[a_1 x_1(t) + a_2 x_2(t)]$

$y_3(t) \neq a_1 y_1(t) + a_2 y_2(t)$

$\therefore$ it is not a linear system.

b) <u>time</u> <u>invariance</u>

Sym Eqn : $y(t) = \cos[x(t)] \rightarrow (1)$

To find $y_i(n)$, place $-t_0$ inside $x(\,)$

$y_i(t) = \cos[x(t-t_0)] \longrightarrow (2)$

To find $y_0(t)$, $t \longrightarrow t-t_0$ in (1)

$y_0(t) = y(t-t_0) = \cos[x(t-t_0)] \longrightarrow (3)$

Since $y_i(t) = y_0(t)$, it is a time invariant sym.

c) It is memoryless sym $\because$ present o/p depends on present i/p only.

d) It is a causal system as it does not depend on future i/p value.

e) <u>Stability</u>

$|x(t)| = M_x < \infty$

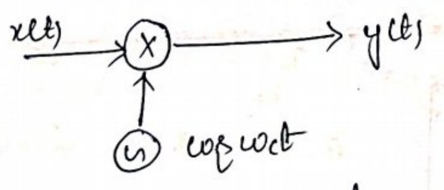
$|y(t)| = |\cos[x(t)]|$

$|y(t)| = \cos M_x$

$|y(t)| = M_y < \infty$

$\therefore$ it is BIBO stable sym.

$\Rightarrow$ (5)



Consider the system shown in fig determine whether the o/p of the system is stable, linear causal.

* $y(t) = x(t)\cos\omega_c t$

$y_1(t) = x_1(t)\cos\omega_c t$

$y_2(t) = x_2(t)\cos\omega_c t$

$y_3(t) = x_3(t)\cos\omega_c t$

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$y_3(t) = [a_1x_1(t) + a_2x_2(t)]\cos\omega_c t$

$= a_1x_1(t)\cos\omega_c t + a_2x_2(t)\cos\omega_c t$

$y_3(t) = a_1y_1(t) + a_2y_2(t)$

$\therefore$ it is linear sym

* Let $|x(t)| = M_x < \infty$

$|y(t)| = |x(t)\cos\omega_c t|$

$= |x(t)|\,|\cos\omega_c t|$

$|y(t)| = M_x\,|\cos\omega_c t|$

$= M_y < \infty$

Bounded i/p $x(t)$ multiplied with another finite value gives a bounded value $\therefore$ it is stable sym.

* <u>causality</u>:

It is a causal sym.

* memoryless sym.

$y(t) \overset{\text{sym Eqn}}{=} x(t)\cos\omega_c t \longrightarrow (1)$

To find o/p due to shifted i/p $x(t-t_0)$ place $-t_0$ inside of $x(\ )$ in (1)

$y_i(t) = x(t-t_0)\cos\omega_c t \longrightarrow (2)$

To find shifted o/p, $t \to t-t_0$ in (1)

$y_0(t) = y(t-t_0) = x(t-t_0)\cos\omega_c(t-t_0) \longrightarrow (3)$

$y_i(t) \neq y_0(t)$

$\therefore$ it is time variant sym.

Since $y_i(n) = y_0(n)$

it is a time invariant sym

c) memoryless sym

d) causal sym.

e) Stability:

Let $|x(n)| = M_x < \infty$

$|y(n)| = 10|\log|x(n)||$

$= 10\log M_x$

$|y(n)| = M_y < \infty$

$\therefore$ it is a stable sym.

(6) $\Rightarrow y(n) = 10\log_{10}|x(n)|$

a) Linearity:

$y_1(n) = 10\log_{10}|x_1(n)|$

$y_2(n) = 10\log_{10}|x_2(n)|$

$y_3(n) = 10\log_{10}|x_3(n)|$

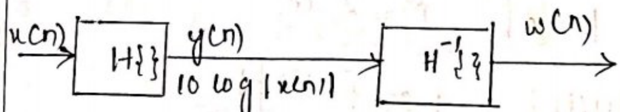
$x_3(n) = a_1x_1(n) + a_2x_2(n)$

$y_3(n) = 10\log_{10}|a_1x_1(n) + a_2x_2(n)|$

$y_3(n) \neq a_1y_1(n) + a_2y_2(n)$

$\therefore$ it is not a linear sym.

f) Invertibility



$w(n) = \text{antilog}[10\,y(n)]$

$= \text{antilog}[10\frac{1}{10}\log|x(n)|]$

$w(n) = |x(n)|$

$\therefore$ it is a invertible.

b) Time invariance

sym Eqn: $y(n) = 10\log|x(n)| \longrightarrow (1)$

o/p due to shifted i/p is $y_i(n)$

* place $-n_0$ inside $x(\ )$ in (1)

$y_i(n) = 10\log|x(n-n_0)| \longrightarrow (2)$

shifted o/p, $y_0(n)$

$n \to n-n_0$ in (1)

$y_0(n) = y(n-n_0) = 10\log|x(n-n_0)|$

⇒

$$y(n) = x[n^2]$$

$$x(n) \longrightarrow \boxed{H} \longrightarrow y(n) = x[n^2]$$

a) **Linearity**

for i/p $x_1(n)$, the o/p is

$y_1(n) = x_1[n^2]$

Ill.ly for the i/p $x_2(n)$ the o/p is

$y_2(n) = x_2[n^2]$

for i/p $x_3(n)$, the o/p is

$y_3(n) = x_3[n^2]$

Let $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$$x_3(n) \longrightarrow \boxed{H} \longrightarrow y_3(n)$$

the sym replaces n by n^2

$\therefore$ o/p is

$y_3(n) = a_1 x_1(n^2) + a_2 x_2(n^2)$

$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

$\therefore$ it is linear.

b) Time invariance

c) **Memory**

$y(n) = x(n^2)$

$n=0 \quad y(0) = x(0)$

$n=1 \quad y(1) = x(1)$

$n=-1 \quad y(-1) = x(1)$

$n=2 \quad y(2) = x(4)$

since present o/p depends on future i/p $\therefore$ it is memory sym.

d) **Causality:**

$n = 2 \quad y(2) = x(2^2) = x(4)$

It is Non causal sym.

e) **Stability:**

$|x(n)| = M_x < \infty$

$|y(n)| = |x(n^2)|$

$= M_y < \infty$

since $x(n)$ is bounded, the o/p $x(n^2) = y(n)$ is also bounded.

$\therefore$ the o/p of sym does not alter the amplitude of signal. $\therefore$ sym is BIBO stable.

b) Time invariance

sym Eqn: $y(n) = x(n^2) \longrightarrow (1)$

To find o/p due to shifted i/p, place $-n_0$ inside $x(\ )$ in (1)

$y_i(n) = x(n^2 - n_0) \longrightarrow (2)$

To find shifted o/p, $n \longrightarrow n - n_0$ in (1)

$y_0(n) = y(n-n_0) = x((n-n_0)^2)$

Since $y_i(n) \neq y_0(n)$; it is a time variant / not a time invariant sym.

(7)

$\Rightarrow y(t) = \frac{d\,x(t)}{dt}$

a) **Linearity**

$$y_1(t) = \frac{dx_1(t)}{dt}$$

$$y_2(t) = \frac{d\,x_2(t)}{dt}$$

$$y_3(t) = \frac{dx_3(t)}{dt}$$

Let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$$\therefore y_3(t) = \frac{d}{dt}\left[a_1x_1(t) + a_2x_2(t)\right]$$

$$y_3(t) = a_1\frac{d\,x_1(t)}{dt} + a_2\frac{dx_2(t)}{dt}$$

$$y_3(t) = a_1y_1(t) + a_2y_2(t)$$

$\therefore$ it is linear sym

b) **Time invariance:**

Sym eqn: $y(t) = \frac{d\,x(t)}{dt} \longrightarrow (1)$

I. o/p due to shifted i/p is $y_i(t)$; place $-t_0$ inside $x(\ )$ in (1)

$$y_i(t) = \frac{d\,x(t-t_0)}{dt} \longrightarrow (2)$$

II. Shifted o/p $y_0(t)$; $t \longrightarrow t-t_0$ in (1)

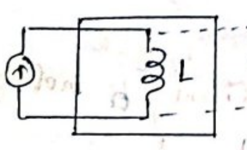
$$y_0(t) = y(t-t_0) = \frac{d\,x(t-t_0)}{dt} \longrightarrow (3)$$

$$y_i(t) = y_0(t)$$

Sym is time invariant.

c) **Memory**

i/p



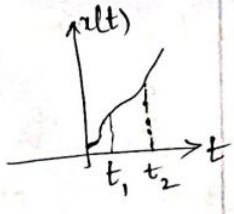
The definition of differentiation by first principle is given as

$$y(t) = \frac{d\,x(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t) - x(t-\Delta t)}{\Delta t}$$

(OR)

$\frac{dx(t)}{dt}$: rate of change of $x(t)$

$$\frac{dx(t)}{dt} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$



t_2 : present time

t_1 : past time

Since present o/p depends on past i/p Sym is memory sym

$$y(t) = \frac{d}{dt}\left[e^{-t}\,x(t)\right]$$

Linearity:

$$y_1(t) = \frac{d}{dt}\left[e^{-t}\,x_1(t)\right]$$

$$y_2(t) = \frac{d}{dt}\left[e^{-t}\,x_2(t)\right]$$

$$y_3(t) = \frac{d}{dt}\left[e^{-t}\,x_3(t)\right]$$

let $x_3(t) = a_1x_1(t) + a_2x_2(t)$

$$\Rightarrow y_3(t) = \frac{d}{dt}\left[e^{-t}(a_1x_1(t) + a_2x_2(t))\right]$$

$$y_3(t) = a_1\frac{d}{dt}\left[e^{-t}x_1(t)\right] + a_2\frac{d}{dt}\left[e^{-t}x_2(t)\right]$$

$$y_3(t) = a_1y_1(t) + a_2y_2(t)$$

∴ the sym is linear.

Sym Eqn: $y(t) = \frac{d}{dt}[e^{-t} x(t)] \longrightarrow (1)$

I. o/p due to shifted i/p is $y_i(n)$; place $-t_0$ inside $x(\,)$ in (1)

$$y_i(t) = \frac{d}{dt}[e^{-t} x(t-t_0)]$$

II Shifted o/p; $y_o(n)$. $t \longrightarrow t - t_0$ in (1)

$$y_o(t) = \frac{d}{dt}\left[e^{-(t-t_0)} x(t-t_0)\right]$$

$y_i(t) \neq y_o(t)$

Not a time invariant.

the sym is time variant.

Qn. (9)

$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

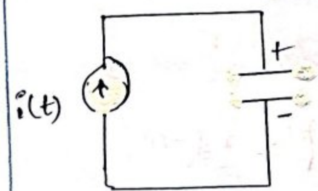
(OR)

Consider capacitor as shown in figure. Let $x(t) = i(t)$ & $y(t) = V_c(t)$.

i) Find the i/p o/p relation

ii) Determine whether the sym is

a) Linear b) time invariant c) memory d) causal e) stable.



soln

a) Linearity:

$$y_1(t) = \int_{-\infty}^{t} x_1(\tau)\, d\tau$$

$$y_2(t) = \int_{-\infty}^{t} x_2(\tau)\, d\tau$$

$$y_3(t) = \int_{-\infty}^{t} x_3(\tau)\, d\tau$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$\Rightarrow$

$$y_3(t) = \int_{-\infty}^{t} [a_1 x_1(\tau) + a_2 x_2(\tau)]\, d\tau$$

$$y_3(t) = a_1 \int_{-\infty}^{t} x_1(\tau)\, d\tau + a_2 \int_{-\infty}^{t} x_2(\tau)\, d\tau$$

$$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$$

∴ it is linear.

b) Time invariance: given $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \rightarrow (1)$

I. o/p due to shifted i/p; place $-t_0$ inside $x(\,)$

$$y_i(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau \longrightarrow (2)$$

II. shifted o/p; $t \rightarrow t - t_0$ in (1)

$$y_o(t) = y(t-t_0) = \int_{-\infty}^{t-t_0} x(\tau)\, d\tau \rightarrow (2')$$

Consider $y_i(t) = \int_{-\infty}^{t} x(\tau - t_0)\, d\tau$

Put $\tau - t_0 = m$

$d\tau = dm$

when $\tau = -\infty$ $\quad m = -\infty$

when $\tau = t$ $\quad m = t - t_0$

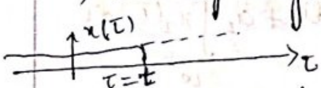
$\therefore y_i(t) = \int_{-\infty}^{t-t_0} x(m)\,dm$

$$y_i(t) = \int_{-\infty}^{t-t_0} x(\tau)\,d\tau \longrightarrow ②$$

from ① & ②

$y_i(t) = y_0(t)$ ∴ the system is time invariant.

c) memory: $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$



* Since the o/p depends on the past value of the i/p. The sym is memory sym.
* It is causal sym.

~~Capacitor performing integration, which is example for memory~~ device.

d) Stability:

Let $|x(t)| \leq M_x < \infty$

$$|y(t)| = \left| \int_{-\infty}^{t} x(\tau)\,d\tau \right|$$

$$= \int_{-\infty}^{t} M_x\,d\tau$$

$$= M_x \int_{-\infty}^{t} 1\,d\tau$$

$$= M_x \left[\tau\right]_{-\infty}^{t}$$

$$= M_x\left[t - (-\infty)\right]$$

$$= \infty$$

Since o/p is not bounded ∴ the sym is not stable.

Qn 10

$y(n) = x(-n)$

a) Linearity:

$y_1(n) = x_1(-n)$

$y_2(n) = x_2(-n)$

$y_3(n) = x_3(-n)$

Let $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$$x(n) \longrightarrow \boxed{H\{\ \}} \longrightarrow y(n) = x(-n)$$

$y(n) = H\{x(n)\}$

$y_3(n) = H\{x_3(n)\} = x_3(-n)$

$y_3(n) = a_1 x_1(-n) + a_2 x_2(-n)$

$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

∴ It is linear sym.

Sym eqn $y(n) = x(-n) \longrightarrow ①$

I o/p due to shifted i/p is denoted as $y_i(n)$; to find it place $-n_0$ inside $x(\ )$ in ①

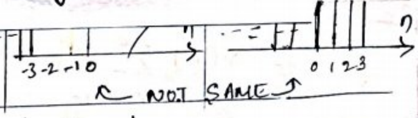
$y_i(n) = x(-n-n_0) \longrightarrow ②$

II o/p shifted is $y_o(n)$, $n \longrightarrow n-n_0$ in ①

$y_o(n) = y(n-n_0) = x(-(n-n_0)) \longrightarrow ③$
$= x(-n+n_0)$

from ② & ③

$y_i(n) \neq y_o(n)$ ∴ the sym is time VARIANT (Not a time invariant)



Memory:

$y(n) = x(-n)$

$n=0 \quad y(0) = x(0)$
$1 \quad y(1) = x(-1)$
$-1 \quad y(-1) = x(1)$

Present o/p depends on previous i/p value. ∴ it is memory sym.

Causality:

$y(-1) = x(1)$
$y(-2) = x(2)$
$y(-3) = x(3)$

Since present o/p depends on future i/p value; it is a causal sym.

Stability:

$y(n) = x(-n)$

Let $|x(n)| = M_x < \infty$

$|y(n)| = |x(-n)|$

As the reflection operation doesnot affect the amplitude of the o/p of the system, it is BIBO stable sym

Qn ⑪

$\Rightarrow y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

a) **Linearity**:

$y_1(n) = x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$y_2(n) = x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

$y_3(n) = x_3(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$

Let $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$y_3(n) \Longrightarrow$

$$y_3(n) = [a_1 x_1(n) + a_2 x_2(n)] \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$y_3(n) = a_1 x_1(n) \sum_{k=-\infty}^{\infty} \delta(n-2k) + a_2 x_2(n) \sum_{k=-\infty}^{\infty} \delta(n-2k)$$

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

$\therefore$ it is linear sym.

Time invariance

$$y(n) = x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k) \longrightarrow (1)$$

o/p due to shifted i/p is $y_i(n)$; place $-n_0$ inside $x(\,)$ in (1)

$$y_i(n) = x(n-n_0) \sum_{k=-\infty}^{\infty} \delta(n-2k) \longrightarrow (2)$$

shifted o/p $y_o(n)$; $n \longrightarrow n-n_0$ in (1)

$$y_o(n) = y(n-n_0) = x(n-n_0) \sum_{k=-\infty}^{\infty} \delta(n-n_0-2k) \longrightarrow (3)$$

Since $y_i(n) \neq y_o(n)$; It is a time Variant sym.

*** Memory:**

$$y(1) = x(1) \sum_{k=-\infty}^{\infty} \delta(1-2k)$$

It is memoryless sym.

* It is causal sym.

*** Stability:**

$$|x(n)| = M_x < \infty$$

$$|y(n)| = \left| x(n) \sum_{k=-\infty}^{\infty} \delta(n-2k) \right|$$

$$|y(n)| = |x(n)| \left| \sum_{k=-\infty}^{\infty} \delta(n-2k) \right|$$

$$= M_x \left| \sum_{k=-\infty}^{\infty} \delta(n-2k) \right|$$

$\sum_{k=-\infty}^{\infty} \delta(n-2k)$ is a finite value.

$\therefore$ shifted version doesnot affect the amplitude of the signal.

Qn 12

$$y(n) = \sum_{k=n_0}^{n} x(k)$$

* Linearity

$$y_1(n) = \sum_{k=n_0}^{n} x_1(k)$$

$$y_2(n) = \sum_{k=n_0}^{n} x_2(k)$$

$$y_3(n) = \sum_{k=n_0}^{n} x_3(k)$$

Let $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

$n \longrightarrow k$ $\quad x_3(k) = a_1 x_1(k) + a_2 x_2(k)$

$$y_3(n) = \sum_{k=n_0}^{n} [a_1 x_1(k) + a_2 x_2(k)]$$

$$= a_1 \sum_{k=n_0}^{n} x_1(k) + a_2 \sum_{k=n_0}^{n} x_2(k)$$

$$y_3(n) = a_1 y_1(n) + a_2 y_2(n)$$

$\therefore$ it is linear.

* Time Invariance:

Sym Eqn! $y(n) = \sum_{k=n_0}^{n} x(k) \longrightarrow (1)$

I. o/p due to shifted i/p is $y_i(n)$; place $-n_0'$ inside $x(\;)$ in (1)

$$y_i(n) = \sum_{k=n_0}^{n} x(k-n_0') \longrightarrow (?)$$

II o/p shifted is $y_o(n)$; $n \to n-n_0'$ in (1)

$$y_o(n) = y(n-n_0') = \sum_{k=n_0}^{n-n_0'} x(k) \longrightarrow (2)$$

consider $y_i(n) = \sum_{k=n_0}^{n} x(k-n_0')$.

put $k-n_0' = m$

when $k = n_0 \quad = m = n_0 - n_0'$

$k = n \quad m = n - n_0'$

$$\therefore y_i(n) = \sum_{m=n_0-n_0'}^{n-n_0'} x(m)$$

$$y_i(n) = \sum_{k=n_0-n_0'}^{n-n_0'} x(k) \longrightarrow (3)$$

from diagram

$$y_o(n) = \sum_{k=n_0}^{n-n_0} x(k) \longrightarrow (2)$$

from (2) & (3)

$$y_i(n) \neq y_o(n)$$

$\therefore$ it is not time invariant sym.

* Memory

$$y(n) = \sum_{k=n_0}^{n} x(k)$$

for $n > n_0$; Let $n = 3$ $n_0 = 2$

$$y(3) = \sum_{k=2}^{3} x(k)$$

$$y(3) = x(2) + x(3)$$

$\therefore$ it is memory system for $n > n_0$

for $n < n_0$ $n = 4$ $n_0 = 5$

$$y(4) = \sum_{k=5}^{4} x(k) = \sum_{k=4}^{5} x(k)$$

$$y(4) = x(4) + x(5)$$

depends on future value $\therefore$ it is memory sym.

Causality:

$$y(n) = \sum_{k=n_0}^{n} x(k)$$

for $n > n_0$ it depends on past i/p value, for $n < n_0$ it depends on future i/p value $\therefore$ it is non causal sym.

Stability:

$$y(n) = \sum_{k=n_0}^{n} x(k)$$

Let

$$|x(n)| = M_x < \infty$$

$$|y(n)| = \left|\sum_{k=n_0}^{n} M_x\right|$$

$$= M_x \sum_{k=n_0}^{n} 1$$

$$y(n) = M_x(n - n_0 + 1)$$

when n & n_0 are finite, the system is stable, when n & $n_0 \longrightarrow \infty$, the system is unstable.

$\Rightarrow$ ✓ $y(t) = x(2-t)$

a) Linearity:

$$y_1(t) = x_1(2-t)$$

$$y_2(t) = x_2(2-t)$$

$$y_3(t) = x_3(2-t)$$

Let $x_3(t) = a_1 x_1(t) + a_2 x_2(t)$

$$x(t) \longrightarrow \boxed{H} \longrightarrow y(t) = x_2(2-t)$$

$$y_3(t) = a_1 x_1(2-t) + a_2 x_2(2-t)$$

$$y_3(t) = a_1 y_1(t) + a_2 y_2(t)$$

$\therefore$ the sym is linear.

*Time invariance:

Sym eqn $y(t) = x(2-t) \longrightarrow (1)$

o/p due to shifted i/p: place $-t_0$ inside $x(\ )$ in (1)

$$y_i(t) = x(2 - t - t_0) \longrightarrow (2)$$

shifted o/p $y_o(t)$: $t \rightarrow t - t_0$ in (1)

$$y_0(t) = y(t - t_0) = x(2 - (t - t_0))$$

$$y_0(t) = x(2 - t + t_0) \rightarrow (3)$$

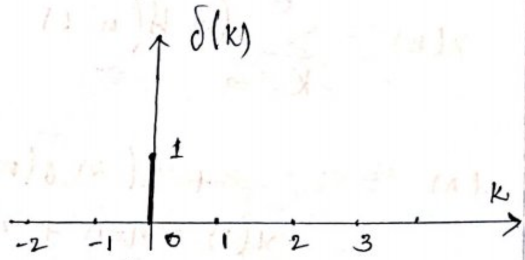
From (2) & (3)

$$y_i(t) \neq y_0(t)$$

Sym is not time invariant.

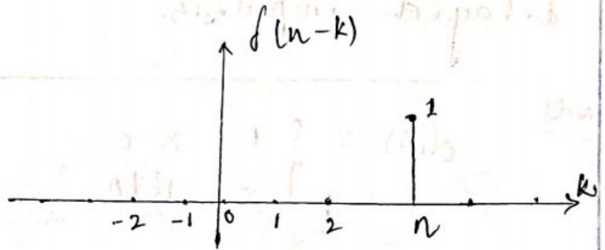
Basics:

$$\delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$



$$\delta(n-k) = \begin{cases} 1 & n-k = 0 \\ 0 & n-k \neq 0 \end{cases}$$

$$\delta(n-k) = \begin{cases} 1 & k = n \\ 0 & k \neq n \end{cases}$$



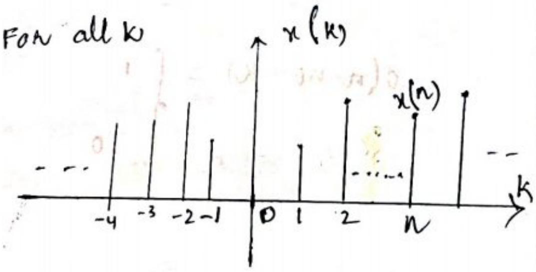
Note: Here n const, k was variable.

Qn

Show that an arbitrary sequence $x(n)$ can be expressed as linear combination of (weighted sum) of delayed (shifted) impulses. i.e,

$$\boxed{x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)}$$

For all k

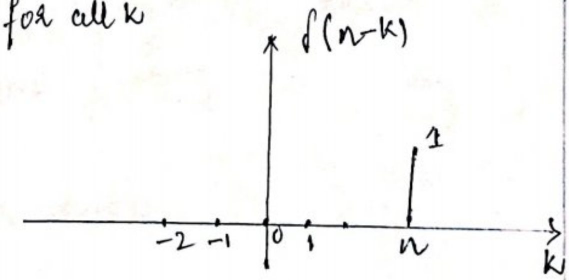


Consider RHS:

$$RHS = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

From figure $x(k)\,\delta(n-k) = x(n)$

for all k



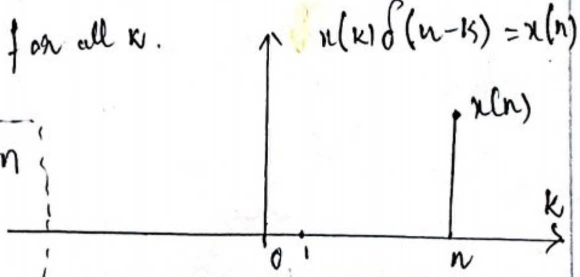
$$RHS = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(n)$$

$= x(n)$ $\because$ no k term in summation; it vanishes

$= LHS.$

for all k.



Thus

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$$

$$x(n) = \cdots + x(-2)\,\delta(n+2) + x(-1)\,\delta(n+1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + \cdots$$

The sequence $x(n)$ is expressed as weighted sum of delayed impulses.

Note:

$$\delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

$$\delta(n-n_0-k) = \begin{cases} 1 & n-n_0-k = 0 \\ 0 & n-n_0-k \neq 0 \end{cases}$$

$$\delta(n-n_0-k) = \begin{cases} 1 & k = n-n_0 \\ 0 & k \neq n-n_0 \end{cases}$$